HYBRID OPTIMIZATION APPROACH FOR MULTI-OBJECTIVE CONSTRAINED MIXED DISCRETE NONLINEAR PROBLEMS

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Introduction and Motivation

• Many engineering design problems require simultaneous optimization of multiple goals or objectives.

• Unlike single objectives, a multi-objective has no single solution. It returns a number of trade-off optimal solutions.

• Due to competing objectives, an optimum solution with respect to one objective may not produce acceptable result with respect to the other objectives.
Introduction and Motivation

• Most optimization algorithms are adept at handling continuous variables.
• Fewer can handle discrete variables like technology selection, material choice, configuration selection etc.
• Genetic Algorithm (GA) can handle both types of variables and explores the entire design space. But performs poorly at locating the exact minimum and is computationally expensive.
Introduction and Motivation

- Gradient based Sequential Quadratic Programming (SQP) is computationally efficient but stalls at local minima and cannot deal directly with discrete variables.
- Combining these two algorithms can lead to a hybrid approach that can improve the overall optimization process.
Introduction and Motivation

• Handling constraints in GA is also difficult.
• Constraints in GA are generally handled by adding them to the objectives using a penalty multiplier which is difficult to predict in advance.
• In my approach GA is hybridized with SQP in a way that makes the constrained multi-objective problem appear unconstrained to GA.
Technical Approach

• Hybridization is carried out using GA (both binary and real coded) and the gradient based goal attainment method which uses the Sequential Quadratic Programming (SQP) available in ‘fgoalattain’ of MATLAB.

• The goal attainment method strives for achieving a set of user defined goal values by minimizing a slack variable $\gamma$. 
The algorithm tries to minimize $\gamma$, satisfying the constraints $f(x) - a_i \gamma \leq f_i^G$. 

Technical Approach
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- This method pre-defines a set of goal points that would lead to points on the Pareto front.
Technical Approach

- The overall problem statement can be posed in two phases:
  - Phase I: As seen by Genetic Algorithm
    - Minimize $\gamma$ (slack variable)
    - Subject to:
      - $(x_c)_i^L \leq (x_c)_i \leq (x_c)_i^U$ (Continuous Variable)
      - $(x_d)_i \in \{1, 2, 3, 4\}$ (Discrete choice)
Technical Approach

– Phase II: As seen by ‘fgoalattain’ of MATLAB
  
  • Minimize $\gamma$ (Slack Variable)
  
  • Subject to:

$$f_i(x) - a_i \gamma \leq f_i^G$$

$$g_i(x) \leq 0$$

$$h_i(x) = 0$$

$$(x_c)_i^L \leq (x_c)_i \leq (x_c)_i^U \text{, given } (x_d)_i$$

$f_i(x)$ – objective value, $a_i$ – weight vector

$f_i^G$ – objective goal, $g_i(x)$ – inequality constraints

$h_i(x)$ – equality constraints
Objectives are to minimize the weight of the truss system and to minimize the maximum deflection of the free node.
Problem Description: Design Variables & Constraints (Mixed-Discrete Non-linear)

- Continuous variables are the radii for the links.
- Discrete variables are the material choices. (Aluminum, Titanium, Steel & Nickel).
- A simple FEM code has been used to compute the stresses on each link.

- Maximum allowable stress that a given material can withstand defines the constraints for this problem.
Summary of results: 3-bar Truss Problem

- The Pareto frontier seems to be aligned with the coordinate axes.

- This is due to the large change of the objective values at the extreme ends of the Pareto front. Ends are (2.87kg, 14mm), (85290kg, 1.55e-04mm)
Summary of results: 3-bar Truss Problem

Pareto Frontier between the mass range 0-5 Kg

Pareto Frontier between the mass range 20,000-100,000 Kg
Summary of results: 3-bar Truss Problem: Customized Pareto Front

Pareto Frontier between the mass range 150 - 15000 Kg
10-bar Truss Problem

- A similar version of the previous 3-bar truss problem, but puts GA on a more rigorous test (It has $4^{10} = 1048576$ possible material choices, compared to only 64 in 3-bar truss problem).
10-bar Truss Result

Complete Pareto front representation of the 10-bar Truss problem.
[ Extreme points: (1063.5 lbs, 10.9 in), (46438.8 lbs, 0.329 in) ]
10-bar Truss Result

- Each point on the Pareto front has unique design variable values.
- This enables the analysts to decide which design configuration to choose, based on their requirements.
Goal To Accomplish

• The method has not being implemented beyond two objectives.

• Further visualizing and addressing a problem having an objective space with number of objectives beyond three makes it very difficult and challenging.
Possible Future applications

• **Aeroelastic Wing**: Designing a simple wing structure of an aircraft for maximum flutter speed and minimum wing weight. Discrete material choice would drive the location of C.G which in turn would influence both weight and flutter speed.

• **Domestic Daily Aircraft Routing**: Assigning a group of airplanes into various cities to come up with a daily flight schedule that would maximize the revenue for the airline.