ITAP Toolbox

Integrated Test Analysis Processor Toolbox
User's Guide

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Foreword

The overall concept of the Integrated Test Analysis Processor (ITAP) evolved over a period of 23 years. From 1975 to 1983, while employed at The Aerospace Corporation, I was involved (with colleagues at that organization) in the planning, witnessing and evaluation of launch vehicle and spacecraft modal testing and in flight data analysis. In addition, from 1979 to 1983, Sheldon Rubin and I conducted technology studies aimed at in-situ structural health monitoring of fixed offshore oil platforms. Technical demands associated with these activities pointed to the need for a rigorous, organized and disciplined test-analysis philosophy, which is the foundation of ITAP. One key feature of ITAP, namely the Simultaneous Frequency Domain (SFD) technique for identification of vibration mode parameters from measured data was introduced in 1980. SFD has been enhanced and refined over the years as a result of its continued application in laboratory and field test activities.

From 1983 to 1987, ITAP philosophy was formally introduced by a consortium led by three individuals, namely Julius Bendat, Richard Stroud and the writer. During that period, the talents and experience of these principals in Random Data Analysis (Bendat), Dynamic Test Philosophy (Stroud) and Integration of (Finite Element) Analysis and Test Disciplines (Coppolino) were combined to establish an early “incarnation” of ITAP. Our technical approach and software was successfully applied (and refined) in a variety of combined experimental and analytical projects sponsored by Ford Motor Company and the National Aeronautics and Space Administration.

From 1987 to 1998, I was associated with Measurement Analysis Corporation, which provides system engineering and technical assistance to the U.S. Government and private industry. During this time frame, ITAP philosophy was applied to a variety of aerospace, marine, automotive and advanced machinery systems. Naturally, technical innovations and systematic procedures were upgraded to meet requirements presented by these projects. In addition to the above noted engineering project commitments, I collaborated with Julius Bendat and Paul Palo (of the U.S. Navy, NFESC) on U.S. Navy sponsored research devoted to identification of nonlinear systems from random data. Successful developments in this area have been incorporated in ITAP.

In June 1998, I joined Universal Analytics Incorporated (UAI), a leading supplier of technical software and engineering services. ITAP now joins UAI’s family of products and services.

Robert N. Coppolino
January 1999

Preface

During the past ten years, I have worked extensively with Dr. Robert Coppolino, the developer of the Integrated Test Analysis Processor (ITAP). This was done under research contracts received by the J. S. Bendat Company from the Naval Facilities Engineering Service Command (NFESC) in Port Hueneme, California to develop new practical techniques to analyze and identify nonlinear system properties for oceanographic applications. Dr. Paul Palo of NFESC provided guidance and assistance for this work. Under subcontract with my company, Dr. Coppolino participated in these research activities which included formulation of nonlinear models, analysis of simulated nonlinear system data, and analysis of measured data from two experimental test programs conducted at the Hydrodynamics Laboratory at the US Naval Academy.

All of the data from both the simulation studies and the two test programs were analyzed with ITAP. Preliminary analysis techniques were used to detect nonlinear behavior. This was followed by detailed data analyses using new direct and reverse multiple input-single output (MI/SO) techniques to estimate and identify nonlinear system properties. During these studies special features were added to ITAP to achieve research goals. New significant frequency domain results were obtained to identify nonlinear system response properties of scale models of a frigate and a barge from measured tow basin data. Results of the NFESC sponsored research and other studies are discussed in my latest book, *Nonlinear System Techniques and Applications*, Wiley, New York 1998. Earlier work for NFESC where ITAP was used appears in my book with Allan Piersol, *Engineering Applications of Correlation and Spectral Analysis*, 2nd Edition, Wiley, New York, 1993.

I am now involved in other work related to biomedical and automotive applications, where ITAP is helping to define and validate different proposed linear and nonlinear models through analysis of measured data. Thus speaking as a user who appreciates good software that properly applies random data analysis procedures in my books, it is a pleasure for me to recommend ITAP to people who want to analyze and identify dynamic properties of linear and nonlinear physical systems.

Julius S. Bendat
January 1999

What is ITAP?

The Integrated Test Analysis Processor (ITAP) is a software suite, which supports and coordinates dynamic test planning, measured data analysis and test-analysis correlation activities. ITAP consists of two toolboxes, namely ITAP-A and ITAP-T, in the MATLAB computation, visualization and programming environment (**MATLAB 5.2 or higher and the MATLAB Signal Processing Toolbox 4.0 or higher are required for ITAP operations**).

The ITAP-A toolbox addresses dynamic test planning and test/analysis correlation tasks. Interfaces with UAI/NASTRAN finite element output files and processed test data files provide the engineer efficient and consistent means for coordination of predictive analysis and laboratory or field test activities.

The ITAP-T toolbox provides the engineer with practical, state-of-the-art means for preliminary test data evaluation, detailed signal analysis and modal parameter estimation. ITAP-T interfaces with time history test data files from popular data acquisition systems and produces test results files used by ITAP-A for test/analysis correlation.

Test-analysis model (TAM) definition and test/analysis reconciliation methodology in the UAI/NASTRAN software environment complement the ITAP software suite. As a complete package, ITAP and UAI/NASTRAN provide a unified capability for effective and efficient treatment of dynamic testing and analysis.

While ITAP provides comprehensive tools for support of vibration mode laboratory testing, it includes features supporting ambient field measurement and analysis and nonlinear dynamic system estimation.

The ITAP-A Toolbox

ITAP-A processes UAI/NASTRAN (finite element model) output data to define a Test-Analysis Model (TAM) which is ultimately employed in test/analysis correlation. The TAM is a condensed finite element model described in terms of selected response measurement degrees of freedom (dof).

A response measurement dof array is developed and validated with a unique test planning algorithm. Adequacy of a single or multiple exciter array intended to stimulate dynamic mode responses is evaluated by review of TAM frequency responses.

TAM geometry and local coordinate systems for the test instrumentation array are automatically defined and stored for use in test-analysis computation and display functions.

Modal test data, which is developed by the ITAP-T toolbox (described in the next section), is validated and compared with TAM predictions (test-analysis correlation) utilizing orthogonality, cross-orthogonality and modal coherence criteria. Moreover rigorous visual and tabular displays describe predicted and measured modal data in terms of frozen and animated mode shapes, motion and kinetic energy distribution bar charts and optional numerical output data.

ITAP-A capabilities are arranged in standardized procedures for TAM development, test planning and test-analysis correlation. For the more advanced user, a library of function modules is included to facilitate definition of customized dynamic model analysis and test results evaluation procedures. The function library includes real and complex modal analysis, steady-state sinusoidal, transient, shock and random response analysis of linear systems, test data adjustment procedures and a variety of graphical display routines.

The ITAP-T Toolbox

The ITAP-T toolbox processes test data recorded by a selected data acquisition system. The test data set consists of a matrix array of excitation and response channels, descriptive channel names (optionally defined automatically by the ITAP-A TAM) and a sampling rate (Δt). Three vital steps for successful measured data processing are provided in standardized procedures, namely, (1) preliminary data review, (2) single and multiple input/output spectral analysis and (3) modal parameter estimation.

Preliminary data review provides channel-by-channel capability for data quality and content evaluation, including time-frequency spectrograms, power spectra, probability distributions, shock spectra and random decrement signatures. For overview of the complete data ensemble, spectral density based envelopes and overall channel response intensities are employed.

Input/output spectral analysis, automated for an arbitrary number of input and output channels, produces estimated frequency response functions and associated cumulative coherence functions. Highly descriptive graphical displays are included for detailed review of frequency response data.

Frequency response data associated with single or multiple excitations, which may be applied forces or imposed foundation motions, are processed by the latest version of the Simultaneous Frequency Domain (SFD) algorithm to estimate system natural frequencies, damping parameters and mode shapes. The estimated mode shapes may be real or complex. SFD, an efficient and highly automated procedure includes extensive graphical displays which assist in data quality and content evaluations during the estimation process.

For the more advanced user, a library of function modules is included to facilitate definition of customized measured data analysis and evaluation procedures. The function library includes single- and multiple-channel data analysis and graphical display routines, powerful multiple input/multiple output (MI/MO) functions which may be employed in nonlinear system identification, and individual SFD function routines (which when combined with random decrement functions may be exploited for ambient data based modal estimation).

UAI/NASTRAN in the Laboratory

A specialized DMAP-Alter was developed to facilitate TAM definition and test planning in conjunction with the ITAP-A toolbox. Frequency response functions and modal parameters may be passed to UAI/NASTRAN for automated Analysis Test Reconciliation (ATR). Reconciliation of predicted finite-element model and test data is accomplished with state-of-the-art multi-disciplinary-optimization (MDO) methodology in UAI/NASTRAN.

ITAP-A, ITAP-T, and UAI/NASTRAN software, when combined with user selected data acquisition hardware, provides end-to-end integration of dynamic testing and analysis. With these resources, the integrated dynamic test and analysis process is completed in a cost-effective and timely manner during a laboratory or field test.

Installation

The ITAP toolboxes are available for Windows, NT and UNIX operating environments. To install the ITAP-A and/or ITAP-T toolbox, see the *ITAP Installation Guide* section for your particular system.

To determine if ITAP-A and/or ITAP-T are already installed on your system, check for the subdirectory named ITAP within the main MATLAB toolbox directory or folder.

1.0 ITAP-A Theory

1.1 Introductory Remarks

This chapter presents the theoretical basis for the ITAP-A Toolkit. Fundamentals of Single Degree-of-Freedom (SDOF) system dynamics are first addressed, in Section 1.2, with emphasis on linear system response to dynamic excitation. Characteristics of linear system response to harmonic, impulsive, transient and random dynamic environments are discussed. A brief note on treatment of SDOF nonlinear systems is provided.

Section 1.3 addresses foundations of Multiple Degree-of-Freedom (MDOF) system dynamics with an emphasis on (1) the basis for development of relevant mathematical models of structures and mechanical systems and (2) concepts in theoretical and practical modal analysis. Appropriateness and limitations of finite element models are discussed. Fundamentals of finite element based formulations, particularly in the NASTRAN software environment, are reviewed. Imposition of constraints and model DOF condensation on finite element models is summarized. Theory and mathematics of real (normal) and complex modes is outlined. Finally, issues of modal truncation and preparation of truncated models for dynamic response analysis are addressed.

Section 1.4 focuses on analysis of MDOF system response to harmonic, impulsive, transient and random dynamic environments. Methodologies employing MDOF system descriptions in terms of real and complex modes are discussed along with direct analysis strategies. A note on treatment of MDOF nonlinear systems employing direct and modal descriptions is provided.

Section 1.5 addresses concepts employed in evaluation of theoretical and experimental modal data. Emphasis is placed on means for evaluation of modal data quality, content and correlation analysis. Use of geometric rigid body kinematics for mathematical model checkout and review is described. Means for assessment of measured mode set quality, completeness and correlation with analytical predictions are developed on the basis of modal orthogonality, cross-orthogonality and coherence relationships. Finally, a variety of tools for adjusting test modal data (and/or theoretical mass and stiffness matrices) to improve quality are discussed.

Section 1.6 presents a theoretical basis for modal test planning. A strategy for unambiguous determination of an accelerometer array for mapping of experimental modes is described and demonstrated. This approach also provides a rigorous basis for definition of a relevant test-analysis model (TAM), which is employed in modal test quality, content and correlation evaluations.

1.2 Dynamics of Single Degree-of-Freedom Systems

1.2.1 Fundamental Single-Degree-of-Freedom Equations

Many structural dynamic and mechanical dynamic systems may be described in terms of a single degree of freedom (SDOF) linear mechanical system or a multi degree of freedom (MDOF) assembly of such systems (Reference 1). The dynamic response, u(t), of a SDOF system which is excited by an applied force, F(t) and base (foundation) motion input, $u_0(t)$, is governed by the ordinary differential equation (see figure below)

$$m\ddot{u}(t) + b\dot{u}(t) + ku(t) = F(t) + b\dot{u}_0(t) + ku_0(t)$$

where m, b, and k are the constant mass, viscous damping and elastic stiffness coefficients, respectively. By defining the relative displacement variable, $u_R(t) = u(t) - u_0(t)$, the dynamic equation simplifies to

$$m\ddot{u}_R(t) + b\dot{u}_R(t) + ku_R(t) = F(t) - m\ddot{u}_0(t)$$

or

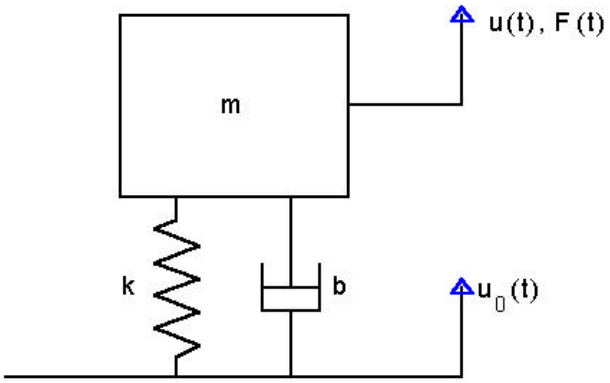
$$\ddot{u}_R(t) + 2\zeta_n\omega_n\dot{u}_R(t) + \omega_n^2 u_R(t) = F(t)/m - \ddot{u}_0(t)$$

where the undamped natural frequency (rad/sec) and critical damping ratio, respectively, are

$$\omega_n = \sqrt{k/m}\ , \quad \zeta_n = b/(2m\omega_n)$$

In addition, the damped natural frequency (rad/sec) is defined as

$$\omega_d = \omega_n\sqrt{1-\zeta_n^2}$$



1.2.2 Response to Simple Harmonic Excitation

Steady-state response of a SDOF system subjected to harmonic force excitation, $F(t) = F(\omega)e^{i\omega t}$, is of the form $u(t) = u(\omega)e^{i\omega t}$ where the following frequency response function (FRF) quantities are derived (Reference 2), after substitutions into the system differential equation:

$$H(\omega) = \frac{ku(\omega)}{F(\omega)} = \frac{1}{(1-(\omega/\omega_n)^2 + 2i\zeta_n(\omega/\omega_n))} \qquad \text{(displacement FRF)}$$

$$H_A(\omega) = \frac{m\ddot{u}(\omega)}{F(\omega)} = \frac{-(\omega/\omega_n)^2}{(1-(\omega/\omega_n)^2 + 2i\zeta_n(\omega/\omega_n))} \qquad \text{(acceleration FRF)}$$

$$TR(\omega) = \frac{b\dot{u}(\omega) + ku(\omega)}{F(\omega)} = \frac{(1+2i\zeta_n(\omega/\omega_n))}{(1-(\omega/\omega_n)^2 + 2i\zeta_n(\omega/\omega_n))} \qquad \text{(transmissibility)}$$

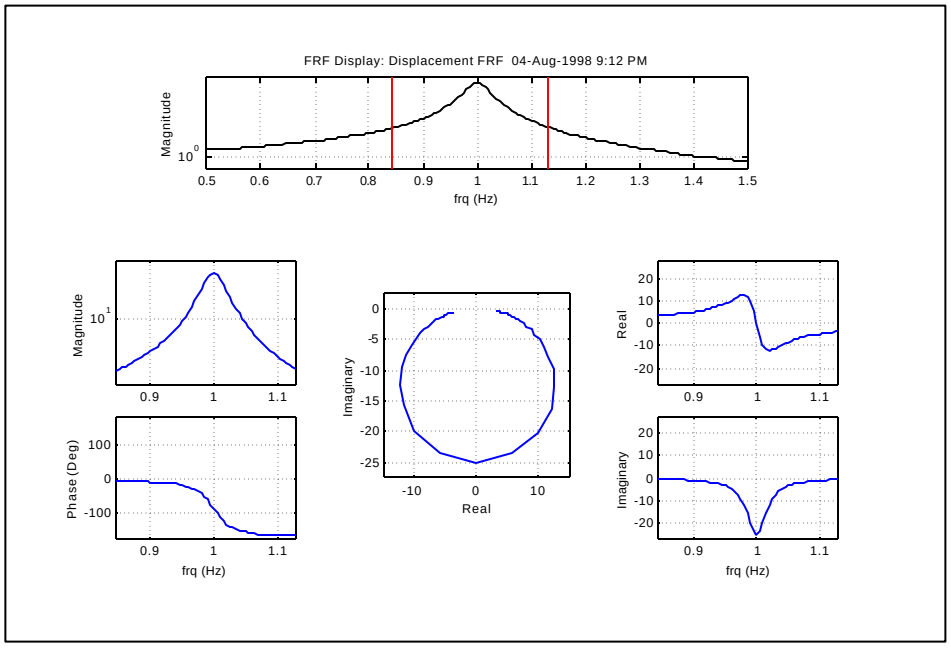
Steady-state response of a SDOF system subjected to harmonic base motion excitation, $\ddot{u}_0(t) = \ddot{u}_0(\omega)e^{i\omega t}$, is of the form $u_R(t) = u_R(\omega)e^{i\omega t}$ where the following frequency response function (FRF) quantities are derived after substitutions into the system differential equation:

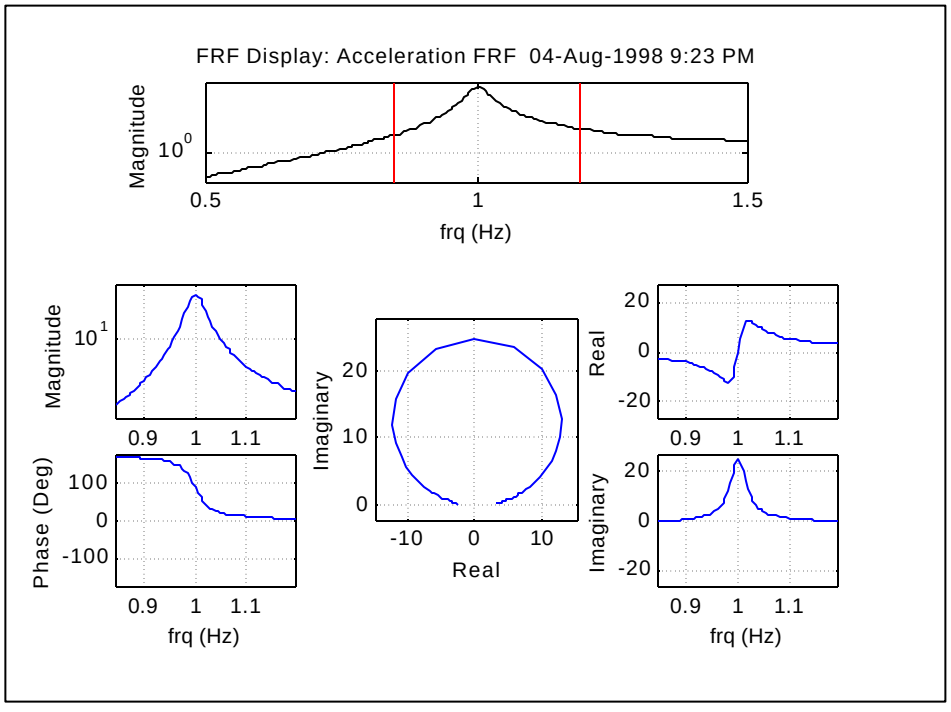
$$H_A(\omega) = \frac{-\ddot{u}_R(\omega)}{\ddot{u}_0(\omega)} = \frac{-(\omega/\omega_n)^2}{(1-(\omega/\omega_n)^2 + 2i\zeta_n(\omega/\omega_n))} \qquad \text{(relative accel. FRF)}$$

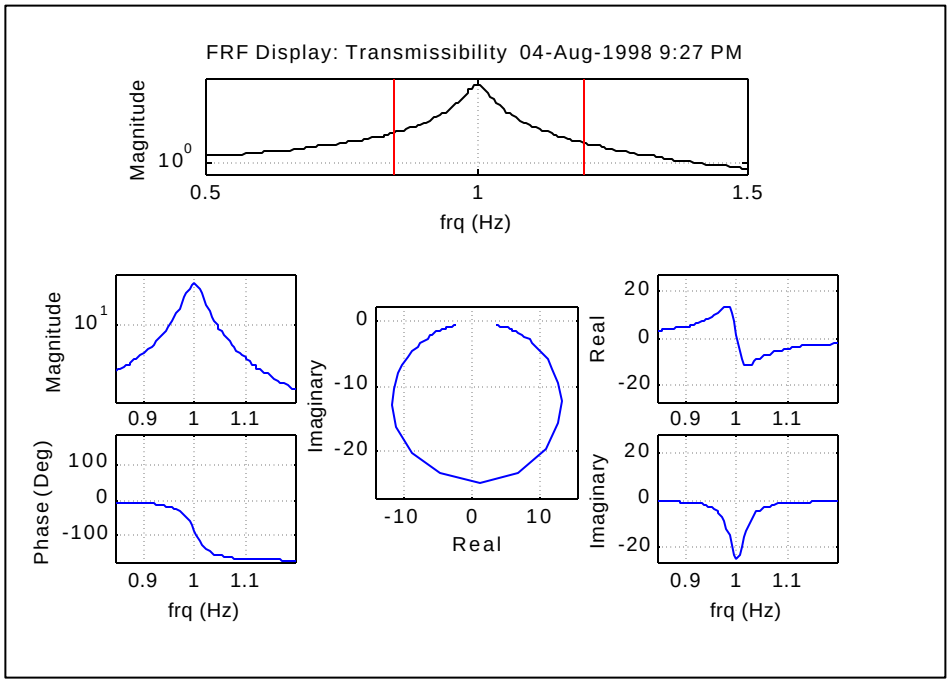
$$TR(\omega) = \frac{\ddot{u}(\omega)}{\ddot{u}_0(\omega)} = \frac{(1+2i\zeta_n(\omega/\omega_n))}{(1-(\omega/\omega_n)^2 + 2i\zeta_n(\omega/\omega_n))} \qquad \text{(transmissibility)}$$

Since all of the above frequency response functions are dependent on the frequency ratio, ω/ω_n, they may be expressed in terms of the ratio, f/f_n (where $\omega=2\pi f$).

Each of the above frequency response functions exhibits peak response amplification when the excitation occurs near the natural frequency. Moreover, the phase of each frequency response function shifts 180 degrees as the excitation frequency increases from below to above the natural frequency. Shown below are plots of the three types of frequency response functions in terms of magnitude and phase, real and imaginary parts versus frequency (f/f_n) and real versus imaginary parts (where f_n=1, ζ_n=.02).







1.2.3 Response to Impulsive and Transient Excitations

A pure impulsive force, $\hat{F}$, is defined as the finite integral

$$\hat{F} = \int_{0-}^{0+} F(t)dt$$

over which the force approaches infinity as the time interval approaches zero duration. When such a loading is applied to a system, which is initially at rest, the velocity response immediately after the impulse (t=0+) is

$$\dot{u}(0+) = \hat{F}/m$$

and the ensuing free decay response is

$$m\omega_n u(t)/\hat{F} = h(t) = e^{-\zeta_n \omega_n t}\sin(\omega_d t).$$

An interesting property of the normalized free decay function, h(t), is that it is the Fourier transform of the displacement FRF (H(ω)).

Response of the SDOF system to an impulsive force, $\hat{F}(\tau) = F(\tau)d\tau$, is

$$u(t) = 0 \qquad \text{for } t<\tau$$

$$u(t) = (1/m\omega_n)h(t-\tau)F(\tau)d\tau \quad \text{for } t\geq\tau$$

This function forms the basis of the Duhamel integral formula for SDOF system response to a general transient force excitation (Reference 1), which is the summation or integral of responses to a continuous train of pulses,

$$u(t) = \int_0^t (1/m\omega_n)h(t-\tau)F(\tau)d\tau$$

The ITAP-A function **sDOF** (See Chapter 5) calculates response of a second order SDOF system (with unit mass) to a general transient force excitation. Using the nomenclature adopted in this document, the call statement for **sDOF** is

$$[u(t), \dot{u}(t), \ddot{u}(t)] = \text{sdof}(F(t), \omega_n, \zeta_n, dt, n1)$$

where the parameter n1 indicates a user selected initial condition, which is either

$$u(0) = \dot{u}(0) = 0 \qquad \text{for n1=0}$$

or

$$u(0) = F(0)/k \ , \ \dot{u}(0) = 0 \qquad \text{for n1=1}$$

[Note: Additional SDOF transient response routines **sDOF0** and **sDOF1** are described in Chapter 5.]

1.2.4 Response Spectrum and Shock Spectrum

Two closely related "signature" functions for SDOF system response to transient excitations are the response spectrum and shock spectrum (Reference 2).

The response spectrum is associated with response of a unit mass SDOF system excited by an applied force, F(t), i.e.,

$$\ddot{u}(t) + 2\zeta_n\omega_n\dot{u}(t) + \omega_n^2 u(t) = F(t)$$

The maximum and minimum response map SDOF systems to a specific force history, F(t), with selected value of ζ_n for the range of natural frequency, $0<\omega_n<\omega_{max}$ is defined as the **response spectrum**. The response spectrum is defined for extrema in displacement, velocity and acceleration response. Normalized response spectra are defined (using $F_0=\max(|F(t)|)$) as follows:

$$RD(\omega_n, \zeta_n, F(t)) = [\max(\omega_n^2 u(t)/F_0), \min(\omega_n^2 u(t)/F_0)]$$
$$RV(\omega_n, \zeta_n, F(t)) = [\max(\omega_n \dot{u}(t)/F_0), \min(\omega_n \dot{u}(t)/F_0)]$$
$$RA(\omega_n, \zeta_n, F(t)) = [\max(\ddot{u}(t)/F_0), \min(\ddot{u}(t)/F_0)]$$

The shock spectrum is associated with response of a unit mass SDOF system excited by an applied base acceleration, i.e.,

$$\ddot{u}_R(t) + 2\zeta_n\omega_n\dot{u}_R(t) + \omega_n^2 u_R(t) = -\ddot{u}_0(t)$$

where the relationship between relative and absolute response is

$$u_R(t) = u(t) - u_0(t)$$

The maximum and minimum response map SDOF systems to a specific base acceleration history, $\ddot{u}_0(t)$, with selected value of ζ_n for the range of natural frequency, $0<\omega_n<\omega_{max}$ is defined as the **shock spectrum**. The shock spectrum is defined for extrema in relative displacement, relative velocity and absolute acceleration response. Normalized response spectra are defined (using $\ddot{U}_0 = \max(|\ddot{u}_0(t)|)$) as follows:

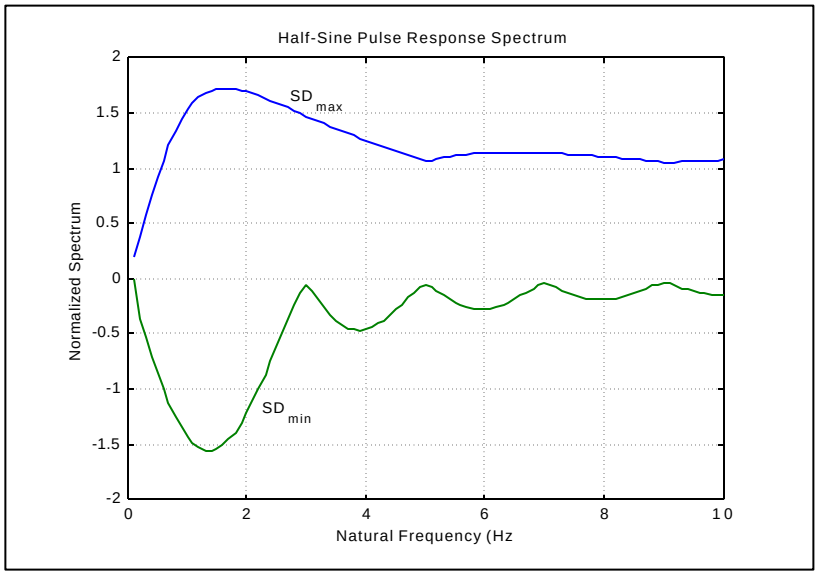
$$SD(\omega_n, \zeta_n, \ddot{u}(t)) = [\max(\omega_n^2 u_R(t)/\ddot{U}_0), \min(\omega_n^2 u_R(t)/\ddot{U}_0)]$$
$$SV(\omega_n, \zeta_n, \ddot{u}(t)) = [\max(\omega_n \dot{u}_R(t)/\ddot{U}_0), \min(\omega_n \dot{u}_R(t)/\ddot{U}_0)]$$
$$SA(\omega_n, \zeta_n, \ddot{u}(t)) = [\max(\ddot{u}(t)/\ddot{U}_0), \min(\ddot{u}(t)/\ddot{U}_0)]$$

Illustrative Example:

Consider the half-sine wave force excitation,

$F(t) = \sin(2\pi t)$, $0 \le t \le 0.5$ seconds
$F(t) = 0$, $t > 0.5$ seconds.

The normalized displacement response spectrum (assuming ζ_n=0.02) for this excitation is shown below:



A useful property of the displacement response spectrum is that its asymptotic value, as natural frequency (f_n) increases to a value (f*) above the "dynamic bandwidth" of the excitation force, approaches unity. Systems with $f_n > f^*$ respond "quasi-statically to the excitation environment, F(t); i.e., the excitation is "slowly varying" for a system with relatively high natural frequency. This characteristic also applies to relative displacement and absolute acceleration shock spectra.

The ITAP-A function **shock** (see Chapter 5) calculates response and shock spectra.

1.2.5 Response to Random Excitation

A random excitation environment may be thought of as a transient environment, which has specific statistical properties. For a thorough exposition on the fundamentals of random processes, Reference 3 is highly recommended. If one considers a random process, which is measured repeatedly under similar conditions, an ensemble of separate test records is collected. If statistical properties (e.g., mean and standard

deviation, autocorrelation function) are roughly the same across separate test records as well as for sub-records within each test record, the statistical process is called **ergodic** [For a more thorough discussion on this matter, see Section 6.2]. The present discussion is limited to, ergodic excitation environments.

Some fundamental properties of a random time history, F(t), are:

mean value: $$\mu_F = \int_{-\infty}^{\infty} F(t)dt$$

variance: $$\sigma_F^2 = \int_{-\infty}^{\infty} (F(t) - \mu_F)^2 dt$$

Standard deviation, σ_F, is defined as the square root of the variance.

The power spectral density function or autospectrum, $G_{FF}(f)$, describes frequency distribution of variance (or mean square value of F(t) with mean value biasing removed) for a time history function, F(t). In particular,

$$\sigma_F^2 = \int_0^{\infty} G_{FF}(f)df$$

Integration of the power spectral density over a finite frequency band, $f_1 \le f \le f_2$, yields the contribution of activity in that frequency band to the total variance.

The response power spectral density, $G_{UU}(f)$, for a single degree of freedom system excited by a random force, F(t), is

$$G_{UU}(f) = |H(f)|^2 G_{FF}(f)$$

In addition, the power spectral density of any particular response function for a SDOF linear system follows the same type of relationship.

The variance of SDOF system response, σ_U^2, is calculated as

$$\sigma_U^2 = \int_0^{\infty} G_{UU}(f)df = \int_0^{\infty} |H(f)|^2 G_{FF}(f)df$$

Since H(f) is often a narrow band function (the greatest magnitude is near the natural frequency), nearly all of the response variance is associated with the frequency band, $(1-\zeta_n)f_n \le f \le (1+\zeta_n)f_n$. This characteristic of

random response leads to simplified formulae for response variance. However, in Section 1.4.4, which describes MDOF system response to random excitation, a general strategy for numerical calculation of random response is described.

1.2.6 Nonlinear Single-Degree-of-Freedom Systems

The above discussion has described relationships and procedures for computation and analysis of linear system dynamic response. If a SDOF system has nonlinear restoring forces, its dynamic response is governed by the general, second order differential equation

$$m\ddot{u}(t) = R(F(t), u(t), \dot{u}(t), t)$$

where the function, $R(F(t), u(t), \dot{u}(t), t)$, can be any nonlinear function of the enclosed variables. The only effective, general way to evaluate response of nonlinear systems is through numerical integration. MATLAB has very effective tools for numerical analysis of nonlinear ordinary differential equations (Reference 4).

1.3 Structural and Mechanical System Dynamic Models

1.3.1 Guidelines for Definition of Relevant Finite Element Models

In order to develop a relevant dynamic model, general requirements should be addressed based on

(1) Frequency band, $0<f<f^*$, and intensity (F^*) of anticipated dynamic environments.

(2) General characteristics of structural or mechanical components.

Typical dynamic environments include seismic excitations applied to building and bridge foundations, road roughness and ambient noise imposed on automobiles, gusts and turbulent flow fields experienced by aircraft, impacts experienced by cargo during handling, and liftoff, engine shutdown and stage separation events for launch vehicles. Dynamic environments are generally (a) harmonic, (b) transient, (c) impulsive or (d) random. For all categories, the cut-off frequency (f^*) is reliably determined by shock or response spectrum analysis (see section 1.2.4 for details). The overall intensity level of a dynamic environment is described by a peak amplitude for harmonic, transient and impulsive events or by a statistical amplitude (e.g., mean plus a multiple of standard deviation) for a long duration random environment.

With the cut-off frequency (f^*) established, the shortest relevant wavelength of forced vibration for components in a structural assembly may be calculated. For finite element modeling, the quarter wavelength (L/4) is of particular interest, since it defines the grid spacing requirement needed to accurately model dynamics (Reference 5).

The table provided below summarizes guidelines for typical structural components:

Component	Mode Type	L/4	Additional Data
String	Lateral	$(\sqrt{T/\rho A})/(4f^*)$	T=tension, A=area ρ = mass density
Rod	Axial	$(\sqrt{E/\rho})/(4f^*)$	E = elastic modulus
Rod	Torsion	$(\sqrt{G/\rho})/(4f^*)$	G = shear modulus
Beam	Bending	$(\pi/2)(EI/\rho A)^{1/4}/\sqrt{2\pi f^*}$	EI = flexural stiffness
Membrane	Lateral	$(\sqrt{N/\rho h})/(4f^*)$	N = stress resultant
Plate	Bending	$(\pi/2)(D/\rho h)^{1/4}/\sqrt{2\pi f^*}$	D=plate flexural stiffness h=plate thickness
3-D Elastic	Dilational	$(\sqrt{E/\rho})/(4f^*)$	
3-D Elastic	Shear	$(\sqrt{G/\rho})/(4f^*)$	
Acoustic	Dilational	$(\sqrt{B/\rho})/(4f^*)$	B=Bulk Modulus

In addition to the above grid spacing guidelines, the engineer must also consider limitations associated with beam and plate technical theories. In particular, if the wavelength to thickness ratio (L/h) is less than about 10, a higher order theory (than pure flexure) or 3-D elasticity modeling should be considered (Reference 6). Moreover, modeling requirements for capture of stress concentration details may call for finer grid meshing than suggested by the cut-off frequency. Finally, if the dynamic environment is of sufficiently high amplitude (F^*), nonlinear modeling may be required, e.g., if plate deflections are greater than thickness, h (Reference 7).

1.3.2 Modal Density and the Effectiveness of Finite Element Models

Finite element modeling is an effective approach for study of structural and mechanical system dynamics as long as individual vibration modes have “sufficient frequency spacing” or “low modal density”. Modal density is typically described as the number of modes within a 1/3 octave frequency band ($f_0<f<1.26f_0$). When modal density of a structural component or structural assembly is greater that 10 modes per 1/3 octave band, details of individual vibration modes are not of significance and statistical vibration response characteristics are of primary importance. In such a situation, the Statistical Energy Analysis (SEA) method applies (Reference 8).

The table provided below (See Reference 8) gives formulae for modal density (as a mathematical derivative, $dn/d\omega$ (n=number of modes, ω=frequency in radians/sec), for typical structural components:

Component	Motion	Modal Density, $dn/d\omega$	Additional Data
String	Lateral	$L/(\pi\sqrt{T/\rho A})$	T=tension, A=area ρ = mass density L=length
Rod	Axial	$L/(\pi\sqrt{E/\rho})$	E = elastic modulus
Rod	Torsion	$L/(\pi\sqrt{G/\rho})$	G = shear modulus
Beam	Bending	$L/(2\pi)(\omega\sqrt{EI/\rho A})^{-1/2}$	EI = flexural stiffness
Membrane	Lateral	$A_S\omega/(2\pi)(N/\rho h)$	N = stress resultant A_S=surface area
Plate	Bending	$A_S/(4\pi)\sqrt{D/\rho h}$	D=plate flexural stiffness h=plate thickness
Acoustic	Dilational	$V_O\omega^2/(2\pi^2)(\sqrt{B/\rho})^3$	B=Bulk Modulus V_O=enclosed volume

1.3.3 Fundamental Dynamic Formulations

In the previous subsection, guidelines for definition of relevant structural and mechanical system dynamic models were described. Finite element models of linear dynamic systems with constant coefficients take the **conventional linear structural dynamic formulation** (Reference 1)

$$[M]\{\ddot{U}(t)\}+[B]\{\dot{U}(t)\}+[K]\{U(t)\}=[\Gamma]\{F(t)\}.$$

Systems which include complicating effects due to elastomeric materials, hydraulic subsystems (e.g., shock absorbers), and interaction with surrounding fluid media (e.g., surface ships), are more appropriately described in terms of state variables or integro-differential equations. The **general linear system state variable equations** are of the form (Reference 9)

$$\{\dot{q}(t)\}-[A]\{q(t)\}=[B]\{F(t)\}$$

$$\begin{Bmatrix}\dot{U}(t)\\ U(t)\end{Bmatrix}=[C]\{q(t)\}+[D]\{F(t)\}$$

where [A], [B], [C], and [D] are constant coefficient matrices and {q(t)} are generalized coordinates.

The most general type of dynamic system includes nonlinear forces, which are dependent upon state variables, time and applied excitation loading. The **general nonlinear system state variable equations** are of the form (Reference 9)

$$\{\dot{q}(t)\} = \{Q(\{q(t)\}, t, \{F(t)\})\}$$

$$\begin{Bmatrix} \dot{U}(t) \\ U(t) \end{Bmatrix} = [C]\{q(t)\} + [D]\{F(t)\}$$

While the general linear and nonlinear system state variable equations describe a very broad spectrum of dynamic systems, primary attention in ITAP-A focuses on the **conventional linear structural dynamic formulation.**

1.3.4 Constraints and Model Condensation

In UAI/NASTRAN (and other NASTRAN versions) finite element models are assembled in terms of an unconstrained "g" (displacement) set, (Reference 10) denoted here as

$$[M_{gg}]\{\ddot{U}_g(t)\} + [B_{gg}]\{\dot{U}_g(t)\} + [K_{gg}]\{U_g(t)\} = [\Gamma_g]\{F(t)\}.$$

A series of standardized constraints (e.g., multi-point, single point, static condensation) are imposed on the finite element model by the user to produce dynamic equations in terms of the analysis set or "a" set. Explicitly, the constraint transformation from "g" set to "a" set displacements is

$$\{U_g\} = [G_{ga}]\{U_a\}$$

and the constrained "a" set dynamic equations are

$$[M_{aa}]\{\ddot{U}_a(t)\} + [B_{aa}]\{\dot{U}_a(t)\} + [K_{aa}]\{U_a(t)\} = [\Gamma_a]\{F(t)\}$$

where

$$[M_{aa}] = [G_{ga}]^T [M_{gg}][G_{ga}], \; [B_{aa}] = [G_{ga}]^T [B_{gg}][G_{ga}]$$

$$[K_{aa}] = [G_{ga}]^T [K_{gg}][G_{ga}], \; [\Gamma_a] = [G_{ga}]^T [\Gamma_g]$$

In ITAP-A it is preferred that static condensation constraints are used to eliminate massless degrees of freedom from the dynamic equation set (not for a Guyan reduction approximation). An additional user defined “y” set is defined to describe instrumented degrees of freedom for an approximate Test-Analysis Model (TAM). The relationships describing this further reduction are of the same form as shown above for the “g” to “a” set reduction (with “y” substituted for the “a” subscript).

ITAP-A does not include capabilities, which address general finite element model assembly. Matrix data from UAI/NASTRAN (as well as other NASTRAN versions), written to an OUTPUT4 file, may be accessed and converted to a MATLAB .mat file using the ITAP-A module, **out4read**.

1.3.5 Description of Normal Modes

The homogeneous form for the **conventional linear structural dynamic formulation** equations, with damping ignored, defines the real eigenvalue problem (with “a” set subscripts not shown for simplicity)

$$[K]\{\Phi_n\} - [M]\{\Phi_n\}\omega_n^2 = \{0\}$$

where

$$\{U(t)\} = \{\Phi_n\}\sin(\omega_n t)$$

There are as many distinct eigenvectors or modes, $\{\Phi_n\}$, as “a” set degrees of freedom for a well-defined undamped dynamic system. The eignevalues, ω_n^2 (ω_n = natural frequency of mode “n”), however, are not necessarily all distinct. Individual modes or mode shapes represent displacement patterns of arbitrary amplitude. It is convenient to normalize the mode shapes (to unit modal mass) as follows:

$$\{\Phi_n\}^T [M]\{\Phi_n\} = 1$$

Assembly of all or a truncated set of normalized modes into a modal matrix, $[\Phi]$ defines the (orthonormal) modal transformation

$$\{U\} = [\Phi]\{q\}$$

where

$$[\Phi]^T [M][\Phi] = [OR] = [I] \qquad \text{(identity matrix),}$$

$$[\Phi]^T [K][\Phi] = [\Lambda] \qquad \text{(diagonal eigenvalue matrix)}$$

The modal transformation produces a diagonal matrix for $[\Phi]^T[B][\Phi]$ only for special forms of the damping matrix; it is assumed in many situations that this matrix triple product is diagonal.

Application of the modal transformation to the dynamic equations results in the uncoupled SDOF dynamic equations

$$\ddot{q}_n + 2\varsigma_n\omega_n\dot{q}_n + \omega_n^2 q_n = [\Phi_n^T\Gamma]\{F(t)\} = [\Gamma_{qn}]\{F(t)\} = Q_n(t),$$

where ς_n is the critical damping ratio and $[\Gamma_{qn}] = [\Phi_n^T\Gamma]$ is the modal excitation gain array.

1.3.6 Useful Normal Mode Relationships

The character and content of an individual normal mode, $[\Phi_n]$, is described fundamentally by the geometric distribution of the displacement degrees of freedom. Utilizing the mass matrix, [M], the modal momentum distribution is

$$\{P_n\} = [M]\{\Phi_n\}$$

and the modal kinetic energy distribution is

$$\{E_n\} = \{P_n\} \otimes \{\Phi_n\} = ([M]\{\Phi_n\}) \otimes \{\Phi_n\}$$

where $\otimes$ denotes term-by-term multiplication. The sum of terms in the modal kinetic energy vector, $\{E_n\}$, is 1.0 when the mode is normalized to unit modal mass.

Internal structural loads and stresses, relative displacements and strains and other user defined terms are calculated as recovery variables. In many cases the recovery variables, {S} are related to physical displacements, {U}, through a load transformation matrix, $[K_S]$,

$$\{S\} = [K_S]\{U\}$$

A modal (displacement based) load transformation matrix, defined by substitution of the modal transformation is

$$\{S\} = [\Phi_{KS}]\{q\} \text{ where } [\Phi_{KS}] = [K_S][\Phi]$$

1.3.7 ITAP-A Normal Mode Calculations

The ITAP-A routine **modalr** calculates a normal mode solution following the relationships described in the above discussion. Using the nomenclature adopted in this document, the function call statement for **modalr** is

$$[\Phi, f, OR, E, \Gamma_{qn}, \Phi_{KS}] = \textbf{modalr}(K, M, \Gamma, K_S)$$

The output column array, f, is the system natural frequencies in "Hz" type units ($f_n = \omega_n/(2\pi)$) sequenced in ascending order.

This routine calculates the complete set of system modes. It is the user's responsibility to select a truncated modal set, if desired.

1.3.8 Description of Complex Modes for a Structural Dynamic System

As noted previously, the normal mode transformation produces a diagonal modal damping matrix, $[\Phi]^T[B][\Phi]$, only for special forms of [B]. When an explicit damping matrix, [B], is formed, the off-diagonal diagonal terms are ignored (if use of real, normal modes is preferred) and uncoupled modal critical damping ratios are assumed to be

$$\zeta_n \approx \left(\Phi_n^T B \Phi_n\right)/(2\omega_n)$$

It should be noted, however, that for most theoretical structural dynamic models, damping is assumed to be a small value (e.g., $\zeta_n \leq 0.05$ for all modes based on engineering judgement and experience.

In situations for which damping is modeled and the normal mode transformation does not produce a diagonal modal damping matrix, the system modes are mathematically complex (Reference 10). Calculation of a modal solution for this situation is performed on a first order (state variable) form of the structural dynamic equations which is

$$\begin{Bmatrix} \ddot{U} \\ \dot{U} \end{Bmatrix} + \begin{bmatrix} M^{-1}B & M^{-1}K \\ -I & 0 \end{bmatrix} \begin{Bmatrix} \dot{U} \\ U \end{Bmatrix} = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} \{F(t)\}$$

The algebraic eigenvalue problem for this situation is of the form

$$[A]\{\Phi_n\} = \{\Phi_n\}\lambda_n$$

where

$$\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix} = \{\Phi_n\}e^{\lambda_n t}, \quad [A] = \begin{bmatrix}-M^{-1}B & -M^{-1}K\\ I & 0\end{bmatrix}.$$

For the present case of a real, unsymmetric dynamic matrix, [A], the eigenvalues, λ_n, and eigenvectors (modes), $\{\Phi_n\}$, are either real or occur in complex conjugate pairs.

Assembly of all modes into a matrix, $[\Phi]$ defines the modal transformation

$$\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix} = [\Phi]\{q\}$$

where

$[\Phi^{-1}][A][\Phi] = [\Lambda]$ (diagonal matrix of eigenvalues)

The inverse of the complete modal matrix, $[\Phi^{-1}]$, is commonly referred to as the left-hand modal matrix, $[\Phi_L]$.

Application of the complex modal transformations to the first order (state variable) form of the structural dynamic equations results in the uncoupled set of modal equations

$$\dot{q}_n - \lambda_n q_n = \left[\Phi_n^{-1}\begin{bmatrix}\Gamma\\ 0\end{bmatrix}\right]\{F(t)\} = [\Gamma_n]\{F(t)\}$$

1.3.9 ITAP-A Complex Structural Dynamic Mode Calculations

The ITAP-A routine **modalc** calculates a complex mode solution following the relationships described in the above discussion. Using the nomenclature adopted in this document, the function call statement for **modalc** is

$$[\Phi, \Phi_L, \Lambda, \Gamma_n] = \textbf{modalc}(K, M, B, \Gamma)$$

The output column array, Λ, is the array of system eigenvalues sequenced in ascending order and $[\Phi_L]$ is the left hand modal matrix.

This routine calculates the complete set of system modes. It is the user's responsibility to select a truncated modal set, if desired.

It should be noted that for treatment of a system formulated in terms of more **general linear system state variable equations,** the MATLAB standard routine, **eig,** may be employed (see Reference 4).

1.3.10 Truncated Mode Set Descriptions

As described in previous sections, the **conventional linear structural dynamic formulation** for MDOF systems with constant coefficients is

$$[M]\{\ddot{U}(t)\} + [B]\{\dot{U}(t)\} + [K]\{U(t)\} = [\Gamma]\{F(t)\}.$$

Application of the (orthonormal) modal transformation,

$$\{U\} = [\Phi]\{q\},$$

results in the uncoupled SDOF modal dynamic equations

$$\ddot{q}_n + 2\varsigma_n\omega_n\dot{q}_n + \omega_n^2 q_n = [\Phi_n^T\Gamma]\{F(t)\} = [\Gamma_{qn}]\{F(t)\} = Q_n(t).$$

Techniques discussed in subsections 1.2.1-1.2.5 are used to calculate each modal response, $q_n(t)$, and the physical responses, {U(t)}, are "recovered" using the modal transformation.

A common practice in structural dynamics analysis is to describe system response, {U(t)}, in terms of a truncated set of (lowest frequency) modes. Selection of an appropriate truncated mode set is accomplished by response spectrum analysis of each force component in the excitation environment, {F(t)}, and establishment of the cut-off frequency, f*. All modal responses for systems with natural frequency, $f_n=\omega_n/(2\pi)>f^*$, will respond "quasi-statically". Therefore, dynamic response will be governed

by the truncated set of modes, $[\Phi_L]$, with natural frequency below f*. The remaining set of "high frequency" modes is denoted as $[\Phi_H]$. Therefore the partitioned modal relationships are

$$\{U\} = [\Phi_L]\{q_L\} + [\Phi_H]\{q_H\},$$

$$\{\ddot{q}_L\} + [2\varsigma_L \omega_L]\{\dot{q}_L\} + [\omega_L^2]\{q_L\} = [\Phi_L^T \Gamma]\{F(t)\}, \quad [\omega_H^2]\{q_H\} \approx [\Phi_H^T \Gamma]\{F(t)\}.$$

Since the high frequency modal equations are algebraic, the modal transformation becomes

$$\{U\} = [\Phi_L]\{q_L\} + [\Psi_\rho]\{F(t)\},$$

where $[\Psi_\rho]$ is the residual flexibility matrix defined as

$$[\Psi_\rho] = [\Phi_H][\omega_H^2]^{-1}[\Phi_H]^T[\Gamma].$$

1.3.11 Residual Mode Vectors

The significance of residual flexibility (quasi-static response of high frequency modes) is well established in the literature (Reference 11), as are methods for efficient definition of residual vectors. The basic definition for residual flexibility, using all of the high frequency modal vectors, is computationally inefficient for large order models. Therefore procedures, which do not explicitly require knowledge of the high frequency modes, have been developed.

The most fundamental procedure for deriving residual vectors forms residual shape vectors as the difference between a complete static solution and a static solution based on the low frequency mode subset (Reference 12,13). The complete static solution, for unit applied loads, using a shifted stiffness (allowing treatment of an unconstrained structure) is

$$[\Psi_S] = [K + \lambda_S M]^{-1}[\Gamma]$$

The corresponding truncated, low frequency mode static solution is

$$[\Psi_L] = [\Phi_L][\omega_L^2 + \lambda_S]^{-1}[\Phi_L]^T[\Gamma].$$

Therefore, the residual vectors are

$$[\Psi_\rho] = [\Psi_S] - [\Psi_L] = [K + \lambda_S M]^{-1}[\Gamma] - [\Phi_L][\omega_L^2 + \lambda_S]^{-1}[\Phi_L]^T[\Gamma] \quad .$$

While the residual vectors are orthogonal to the low frequency modes, they are not necessarily orthogonal to one another. Orthogonalization of the residual vectors is best achieved by solution of the residual eigenvalue problem

$$[K_\rho]\{\varphi_\rho\} = [M_\rho]\{\varphi_\rho\}\omega_\rho^2$$

where

$$[K_\rho] = [\Psi_\rho]^T [K][\Psi_\rho], \quad [M_\rho] = [\Psi_\rho]^T [M][\Psi_\rho].$$

The orthogonalized residual modes are therefore

$$[\Phi_\rho] = [\Psi_\rho][\varphi_\rho].$$

Augmentation of the truncated low frequency modal transformation with the residual modes results in a truncated dynamic formulation which has the standard form used in modal response analysis as well as the benefit associated with residual flexibility. The augmented modal dynamic equations are

$$\{U\} = \begin{bmatrix}\Phi_L & \Phi_\rho\end{bmatrix}\begin{Bmatrix}q_L \\ q_\rho\end{Bmatrix},$$

$$\{\ddot{q}_L\} + [2\zeta_L\omega_L]\{\dot{q}_L\} + [\omega_L^2]\{q_L\} = [\Phi_L^T\Gamma]\{F(t)\}$$

$$\{\ddot{q}_\rho\} + [2\zeta_\rho\omega_\rho]\{\dot{q}_\rho\} + [\omega_\rho^2]\{q_\upsilon\} = [\Phi_\rho^T\Gamma]\{F(t)\}$$

where ζ_ρ are the assumed critical damping ratios for the residual modes.

The ITAP-A function **modres** calculates residual mode matrices and augments a selected set of low frequency modal data. Using the nomenclature adopted in this document, the call statement for **modres** is

[[Φ],{f_n},[ORR]]=**modres**([K],[M],[Φ_L],{f_L},[Γ],λ_S)

where [Φ] is the augmented modal matrix, {f_n} is the augmented set of natural frequencies, and [ORR] is the orthogonality check matrix for the augmented mode set.

1.3.12 The Mode Acceleration Method

An alternative strategy which automatically compensates for modal truncation is the mode acceleration method (Reference 14). The basis for this strategy is substitution of truncated expressions for acceleration and velocity in the system dynamic equations which results in

$$[K]\{U(t)\} = [\Gamma]\{F(t)\} - [M][\Phi_L]\{\ddot{q}_L\} - [B][\Phi_L]\{\dot{q}_L\}$$

In most applications, the term with modal velocity is ignored. The “static” solution of the above equation, at each time point, produces physical displacements which include the quasi-static effects of all high frequency modes.

1.4 Response of Multi-Degree of Freedom Systems

1.4.1 Normal Modes and Complex Modes

The conventional linear structural dynamic formulation is expressed by the following equation set:

$$[M]\{\ddot{U}(t)\}+[B]\{\dot{U}(t)\}+[K]\{U(t)\}=[\Gamma]\{F(t)\}.$$

Utilizing the orthonormal real mode transformation, uncoupled second order dynamic equations are obtained:

$$\ddot{q}_n + 2\varsigma_n\omega_n\dot{q}_n + \omega_n^2 q_n = [\Phi_n^T\Gamma]\{F(t)\}$$

Physical displacements, {U}, and other physical quantities, {S}, are recovered using

$$\{U\}=[\Phi]\{q\}$$

$$\{S\}=[K_S]\{U\} \text{ or } \{S\}=[\Phi_{KS}]\{q\} \text{ where } [\Phi_{KS}]=[K_S][\Phi]$$

When a normal mode approximation is not accurate, complex modes resulting from solution of the state variable matrix equation set

$$\begin{Bmatrix}\ddot{U}\\ \dot{U}\end{Bmatrix}+\begin{bmatrix}M^{-1}B & M^{-1}K\\ -I & 0\end{bmatrix}\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix}=\begin{bmatrix}\Gamma\\ 0\end{bmatrix}\{F(t)\}$$

The complex modal transformation

$$\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix}=[\Phi]\{q\}$$

produces the uncoupled first order differential equations

$$\dot{q}_n - \lambda_n q_n = \left[\Phi_n^{-1}\begin{bmatrix}\Gamma\\ 0\end{bmatrix}\right]\{F(t)\}$$

Physical responses and other physical quantities, {S}, are then recovered using

$$\begin{Bmatrix} \dot{U} \\ U \end{Bmatrix} = [\Phi]\{q\}$$

and

$$\{S\} = [K_S]\{U\}$$

1.4.2 Response to Base Motion Excitation

Dynamic excitation environments sometimes are described in terms of specified foundation or “interface” motions. In such situations, the physical displacement array is partitioned into two subsets as follows:

$$\{U(t)\} = \begin{Bmatrix} U_1(t) \\ U_2(t) \end{Bmatrix} = \begin{Bmatrix} \text{Re sponses} \\ \text{Excitations} \end{Bmatrix}$$

The conventional linear structural dynamic formulation is expressed in partitioned form as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}\begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} 0_1 \\ F_2(t) \end{Bmatrix}$$

Using the partitioned stiffness matrix, the transformation from absolute to relative response displacements is

$$\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{bmatrix} I_{11} & -K_{11}^{-1}K_{12} \\ 0_{21} & I_{22} \end{bmatrix}\begin{Bmatrix} U_{1R} \\ U_2 \end{Bmatrix} = \begin{bmatrix} I_{11} & \Psi_{12} \\ 0_{21} & I_{22} \end{bmatrix}\begin{Bmatrix} U_{1R} \\ U_2 \end{Bmatrix}$$

Substitution of this transformation into the partitioned dynamic equation set results in

$$\begin{bmatrix} M_{11} & M_{12R} \\ M_{21R} & M_{22R} \end{bmatrix}\begin{Bmatrix} \ddot{U}_{1R} \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12R} \\ B_{21R} & B_{22R} \end{bmatrix}\begin{Bmatrix} \dot{U}_{1R} \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & 0_{12} \\ 0_{21} & K_{22R} \end{bmatrix}\begin{Bmatrix} U_{1R} \\ U_2 \end{Bmatrix} = \begin{Bmatrix} 0_1 \\ F_2(t) \end{Bmatrix}$$

In many situations, the excitation motions correspond to a statically determinate (or rigid foundation) subset. Under these conditions, the partitions, $[B_{12R}]$, $[B_{21R}]$, $[B_{22R}]$, and $[K_{22R}]$ are null, and the following dynamic equations are obtained:

$$[M_{11}]\{\ddot{U}_{1R}\}+[B_{11}]\{\dot{U}_{1R}\}+[K_{11}]\{U_{1R}\}=[-M_{12R}]\{\ddot{U}_2\}$$ (relative response)

$$\{\ddot{U}_1\}=\begin{bmatrix} I_{11} & -K_{11}^{-1}K_{12}\end{bmatrix}\begin{Bmatrix}\ddot{U}_{1R}\\ \ddot{U}_2\end{Bmatrix}$$ (absolute accelerations)

$$\{F_2\}=[M_{21R} \quad M_{22R}]\begin{Bmatrix}\ddot{U}_{1R}\\ \ddot{U}_2\end{Bmatrix}$$ (foundation load recovery)

The relative response equation is of the same form as the conventional linear structural dynamic formulation with $[-M_{12R}]$ serving as the load allocation matrix, $[\Gamma]$, and $\{\ddot{U}_2\}$ serving as the applied loading, {F(t)}. Therefore, all of the forced response relationships described in the sections which follow apply to the situation of base motion excitation.

1.4.3 Response to Simple Harmonic Excitation

Steady-state response of a MDOF system subjected to a unit amplitude harmonic force excitation, $F(t)=e^{i\omega t}$, is of the form $\{U(t)\}=\{U(\omega)\}e^{i\omega t}$. Direct substitution of these expressions into the conventional linear structural dynamic formulation results in:

$$[K+i\omega B-\omega^2 M]\{U(\omega)\}=[\Gamma]$$

or

$$\{U(\omega)\}=[K+i\omega B-\omega^2 M]^{-1}[\Gamma]$$

The complex velocity and acceleration amplitude solutions are

$$\{\dot{U}(\omega)\}=i\omega\{U(\omega)\}$$

$$\{\ddot{U}(\omega)\}=-\omega^2\{U(\omega)\}$$

The ITAP-A routine **fresp** directly calculates the complex frequency response following the relationships described above. Using the nomenclature adopted here, the function call statement for **fresp** is

$$[[\ddot{U}(\omega)],[\dot{U}(\omega)],[U(\omega)]\,] = \textbf{fresp}([K],[M],[B],\Gamma,\{f\});$$

where {f} is the array of frequencies for which the frequency response solution is to be calculated and $[\ddot{U}(\omega)],[\dot{U}(\omega)],[U(\omega)]$ are solution matrices with each column corresponding to the solution evaluated at a specific frequency, $\omega=2\pi f$.

Additional physical quantities may be recovered using the matrix product

$$[S(\omega)] = [K_S][U(\omega)]\,.$$

Direct solution for MDOF frequency response using fresp is appropriate for a system which has either real (normal) or complex modes.

If the system can be expressed in terms of orthonormal modes,

$$\{U(\omega)\} = [\Phi]\{q(\omega)\}$$

a modal frequency response solution may be computationally efficient if a truncated set of modes (augmented by an appropriate residual mode vector) is used. Employing the modal substitution, uncoupled modal equations of the form

$$(\omega_n^2 + 2i\zeta_n\omega_n\omega - \omega^2)q_n(\omega) = \Phi_n^T\Gamma$$

are obtained. Recalling results from SDOF frequency response analysis, each modal displacement solution is

$$q(\omega) = \omega_n^2(\Phi_n^T\Gamma)H(\omega)$$

where

$$H(\omega) = \frac{1}{(1-(\omega/\omega_n)^2 + 2i\zeta_n(\omega/\omega_n))}$$

Physical solutions are then recovered employing the orthonormal modal transformation.

The ITAP-A routine **mfresp** calculates the modal complex frequency response following the relationships described above. Using the nomenclature adopted here, the function call statement for **mfresp** is

$$[[\ddot{U}(\omega)],[\dot{U}(\omega)],[U(\omega)]] = \textbf{mfresp}([\Phi],\{f_n\},\{\varsigma_n\},[\Gamma],\{f\});$$

where $\{f_n\}$ is the array of natural frequencies, $\{\varsigma_n\}$ is the array of modal critical damping ratios, frequencies $\{f\}$ is the array of frequencies for which the frequency response solution is to be calculated and $[\ddot{U}(\omega)],[\dot{U}(\omega)],[U(\omega)]$ are solution matrices with each column corresponding to the solution evaluated at a specific frequency, $\omega=2\pi f$.

Additional physical quantities may be recovered using the matrix product

$$[S(\omega)] = [K_S][U(\omega)] .$$

All of the above defined frequency response functions have narrow band peaks concentrated around each modal frequency. Therefore, numerical evaluation of all frequency responses should be performed with incremental $df \le \zeta_n f_n/4$ in order to accurately trace significant peaks.

1.4.4 Response to Random Excitation

As in the discussion of response of SDOF system to random excitation, it is assumed that the excitation environment is ergodic. Moreover, the particular situation of a single applied loading is assumed. Therefore, relationships for response to random excitation similar to that developed for SDOF systems apply, namely,

$$G_{UU}(f) = \left|\{U(f)\}\right|^2 G_{FF}(f)$$
$$G_{\ddot{U}\ddot{U}}(f) = \left|\{\ddot{U}(f)\}\right|^2 G_{FF}(f)$$

where $G_{FF}(f)$ is the excitation loading autospectrum.

In addition, the autospectrum of any particular response function for a MDOF linear system follows the same type of relationship when the frequency response function associated with the output quantity of interest is employed.

The variance of MDOF system response, σ_U^2, is calculated as

$$\sigma_U^2 = \int_0^\infty G_{UU}(f)df$$

The frequency distribution of the output autospectrum, $G_{UU}(f)$, has narrow band peaks concentrated around each modal frequency. Therefore, numerical evaluation of the response variance must be performed with incremental $df \leq \zeta_n f_n/4$.

The ITAP-A routine **rresp** directly calculates the response and excitation function variances. Using the nomenclature adopted here, the function call statement for **rresp** is

$$[\{\sigma_U^2\}\sigma_F^2,[G_{UU}(f)],[G_{FF}(f)]] = \textbf{rresp}(f,[U(f)],ff,G_{FF}(ff));$$

where {f} is the array of frequencies at which the frequency response solution, [U(f)], is calculated (with **fresp** or **mfresp**) and {ff} and $\{G_{FF}(ff)\}$ are the frequency "break" points and associated values of the input autospectrum. Within the **rresp** routine, interpolation of the specified input autospectrum to conform with {f} is performed. The interpolation is carried out on the logarithms of frequency and autospectrum yielding interpolated autospectrum values, which vary linearly on a log-log plot.

1.4.5 Response to Transient Excitation (Orthonormal Real Mode Case)

The conventional linear structural dynamic formulation,

$$[M]\{\ddot{U}(t)\}+[B]\{\dot{U}(t)\}+[K]\{U(t)\}=[\Gamma]\{F(t)\},$$

is transformed using, the orthonormal real modes into the series of uncoupled second order dynamic equations,

$$\ddot{q}_n + 2\zeta_n\omega_n\dot{q}_n + \omega_n^2 q_n = [\Phi_n^T\Gamma]\{F(t)\}.$$

Physical displacements, {U}, and other physical quantities, {S}, are recovered using

$$\{U\}=[\Phi]\{q\}$$

$$\{S\}=[K_S]\{U\} \text{ or } \{S\}=[\Phi_{KS}]\{q\} \text{ where } [\Phi_{KS}]=[K_S][\Phi].$$

The majority of computational steps required to perform MDOF transient response analysis, using orthonormal real modes, have been established in previous sections. The ITAP-A routine **mtresp** calculates transient response (using normal mode data calculated with **modalr** and **modres,** and SDOF transient responses calculated with **sDOF**). Using the nomenclature adopted in this document, the function call statement for **mtresp** is

$$\left[\left[\ddot{U}(t)\right],\left[\dot{U}(t)\right],[U(t)]\right] = \textbf{mtresp}([\Phi],\{f_n\},\{\zeta_n\},[\Gamma],[F(t)],dt);$$

where the rows of $\left[\ddot{U}(t)\right]$, $\left[\dot{U}(t)\right]$ and $[U(t)]$ represent individual response variable time histories, [F(t)] are applied force time histories and dt is the applied force record sampling time.

1.4.6 Response to Transient Excitation (Complex Mode Case)

The conventional linear structural dynamic formulation,

$$[M]\{\ddot{U}(t)\}+[B]\{\dot{U}(t)\}+[K]\{U(t)\}=[\Gamma]\{F(t)\},$$

is expressed in state variable form as

$$\begin{Bmatrix}\ddot{U}\\ \dot{U}\end{Bmatrix}+\begin{bmatrix}M^{-1}B & M^{-1}K\\ -I & 0\end{bmatrix}\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix}=\begin{bmatrix}\Gamma\\ 0\end{bmatrix}\{F(t)\}.$$

Using the complex mode transformation,

$$\{X\}=\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix}=[\Phi]\{q\}$$

produces the series of uncoupled first order dynamic equations,

$$\dot{q}_n-\lambda_n q_n=\left[\Phi_n^{-1}\begin{bmatrix}\Gamma\\ 0\end{bmatrix}\right]\{F(t)\}$$

Physical displacements and velocities are recovered using the complex modal transformation, and other physical quantities, {S}, are recovered using

$$\{S\}=[K_S]\{U\} \text{ or } \{S\}=[\Phi_{KS}]\{q\} \text{ where } [\Phi_{KS}]=[K_S][\Phi].$$

The majority of computational steps required to perform MDOF transient response analysis, using orthonormal real modes, have been established

in previous sections. The ITAP-A routine **mtrespc** calculates transient response (using complex mode data calculated with **modalc** and first order SDOF system transient responses calculated with **sDOF1**). Using the nomenclature adopted in this document, the function call statement for **mtrespc** is

$$[[\dot{X}],[X]] = \text{mtrespc}([\Phi],[\Phi_L],\{\lambda_n\},[\Phi_L\Gamma],[F(t)],dt);$$

where the time history response time history rows represent individual response variable time histories and dt is the applied force record sampling time.

1.4.7 Summary of Key Linear Dynamic System Analysis Modules

The key ITAP-A modules employed in analysis of linear structural dynamic and mechanical systems are summarized in the following table:

Task	Sections	ITAP-A Routine
Establish Cut-off Frequency, f*	1.2.4	shock
Model Definition	1.3.1-1.3.4	out4read
Modal Analysis (real)	1.3.5-1.3.7	modalr
Modal Analysis (complex)	1.3.8-1.3.9	modalc
Modal Truncation/Residuals	1.3.10-1.3.11	modres
Frequency Response (direct)	1.4.3	fresp
Frequency Response (modal)	1.4.3	mfresp
Random Response	1.4.4	rresp
Transient Response (real modes)	1.4.5	mtresp
Transient Response (complex modes)	1.4.6	mtrespc

1.4.8 Response of Nonlinear Dynamic Systems

"The universe is composed of bananas and things that are not bananas. Linear systems are the bananas". This metaphor, shared by Harry Himelblau at a Los Angeles ASME meeting in 1986, describes the central difficulty associated with categorizing, modeling, understanding and analyzing nonlinear systems.

If a MDOF structural or mechanical dynamic system has nonlinear restoring forces, its dynamic response is governed by the general, second order differential equation set

$$[M]\{\ddot{U}(t)\} = \{R(\{F(t)\},\{U(t)\},\{\dot{U}(t)\},t)\},$$

where the function, $\{R(\{F(t)\},\{U(t)\},\{\dot{U}(t)\},t)\}$, can be any nonlinear function of the enclosed variables. The only effective, general way to evaluate response of nonlinear systems is through numerical integration. MATLAB has very effective tools for numerical analysis of nonlinear ordinary differential equations (see Using MATLAB, Chapter 8).

Dynamic systems, which have localized nonlinear forces, may be described by the following equation set:

$$[M]\{\ddot{U}(t)\} + [B]\{\dot{U}(t)\} + [K]\{U(t)\} = [\Gamma_N]\{R_N(\{U_N(t)\},\{\dot{U}_N(t)\},t)\} + [\Gamma]\{F(t)\},$$

$$\{U_N(t)\} = [\Gamma_N^T]\{U(t)\},$$

where $[\Gamma_N]$ is the allocation matrix for the localized degrees of freedom associated with nonlinear restoring forces, $\{R_N\}$. This type of system (assuming that [B] permits a real mode approximation) may be expressed in terms of a truncated set of orthonormal modes for the linear portion of the system (augmented by residual modes defined on the basis of the allocation matrix $[\Gamma_N\,\Gamma]$) as

$$\ddot{q}_n(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = [\Phi_n^T\Gamma_N]\{R_N\} + [\Phi_n^T\Gamma]\{F(t)\},$$

$$\{U_N(t)\} = [\Gamma_N^T\Phi]\{q(t)\}, \quad \{U(t)\} = [\Phi]\{q(t)\}.$$

1.5 Model Quality, Content and Correlation Analysis

1.5.1 Rigid Body Relationships

When a finite element model of a structural or mechanical dynamic system is defined, it is recommended that several fundamental calculations be performed on the assembled mass and stiffness matrices. Within UAI/NASTRAN (and other NASTRAN versions), rigid body mass checks are optionally performed (Reference 10).

The geometric rigid body vectors, $[\Psi_{RB}]_g$, represent an assembly of six column vectors associated with x, y, z, θ_x, θ_y, and θ_z displacement translations of the unconstrained ("g" set) finite element model. Rigid body mass of the complete model is calculated as

$$[M_{RB}]_g = [\Psi_{RB}]_g^T [M_{gg}][\Psi_{RB}]_g$$

This calculation may be repeated for the various steps in the model constraint and reduction process. Errors and inconsistencies in constraints (especially multi-point constraints) will often yield anomalies in reduced set calculations of rigid body mass. Consider, for example, the following calculations performed on analysis or "a" set matrices:

$$[M_{RB}]_a = [\Psi_{RB}]_a^T [M_{aa}][\Psi_{RB}]_a \quad \text{('a" set rigid body mass)}$$

$$[F_{RB}]_a = [K_{aa}][\Psi_{RB}]_a \quad \text{('a" set rigid body support reactions)}$$

The following assessments may be made from $[M_{RB}]_a$ and $[F_{RB}]_a$:

(a) If the "a" set is defined for an unconstrained model, the components of "g" set and "a" set rigid body mass matrices should be the same. If the "a" set is defined for a "supported" system, "a" set rigid body mass terms will have decreased magnitudes if the constrained degrees of freedom have non-zero mass contributions.

(b) In the case of an unconstrained model, all rigid body support reactions should be zero. If the model is supported and if all support attachment degrees of freedom are included in the "a" set, $[F_{RB}]_a$ should have non-zero terms only at the support attachment degrees of freedom. Finite reactions at any other degrees of freedom will indicate the presence of modeling inconsistencies or errors.

1.5.2 Modal Participation Factors and Modal Effective Mass

The theoretical basis for modal participation factors and modal effective mass is rooted in the formulation for a linear structural dynamic system partitioned into two displacement subsets as follows:

$$\{U(t)\} = \begin{Bmatrix} U_1(t) \\ U_2(t) \end{Bmatrix} = \begin{Bmatrix} \text{Responses} \\ \text{Excitations} \end{Bmatrix}$$

The structural dynamic equation set is expressed in partitioned form as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} 0_1 \\ F_2(t) \end{Bmatrix}$$

Using the partitioned stiffness matrix, the transformation from absolute to relative response displacements is

$$\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{bmatrix} I_{11} & -K_{11}^{-1}K_{12} \\ 0_{21} & I_{22} \end{bmatrix} \begin{Bmatrix} U_{1R} \\ U_2 \end{Bmatrix} = \begin{bmatrix} I_{11} & \Psi_{12} \\ 0_{21} & I_{22} \end{bmatrix} \begin{Bmatrix} U_{1R} \\ U_2 \end{Bmatrix}$$

Substitution of this transformation into the partitioned dynamic equation set results in

$$\begin{bmatrix} M_{11} & M_{12R} \\ M_{21R} & M_{22R} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{1R} \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12R} \\ B_{21R} & B_{22R} \end{bmatrix} \begin{Bmatrix} \dot{U}_{1R} \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & 0_{12} \\ 0_{21} & K_{22R} \end{bmatrix} \begin{Bmatrix} U_{1R} \\ U_2 \end{Bmatrix} = \begin{Bmatrix} 0_1 \\ F_2(t) \end{Bmatrix}$$

When the excitation degrees of freedom are associated with six degrees of freedom at a reference point, $[M_{22R}]$, is the system rigid body mass and $[B_{22R}]= [K_{22R}]=[0]$.

The orthonormal modes associated with the response partition,

$$\{U_{1R}\} = [\Phi_1]\{q_1\},$$

when applied to the above dynamic equation set results in

$$\begin{bmatrix} I_{11} & P_{12} \\ P_{21} & M_{22R} \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} 2\zeta_1\omega_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} \omega_1^2 & 0_{12} \\ 0_{21} & K_{22R} \end{bmatrix} \begin{Bmatrix} U_{1R} \\ U_2 \end{Bmatrix} = \begin{Bmatrix} 0_1 \\ F_2(t) \end{Bmatrix}$$

(assuming $[B_{22R}]= [K_{22R}]=[0]$, $[B_{12R}]=[0]$).

The modal participation factor matrix, $[P_{12}]$, is defined as

$$[P_{12}] = [\Phi_1]^T [M_{12}],$$

and the modal effective mass matrix, $[M_{eff}]$, is defined as

$$[M_{eff}] = [P_{12}] \otimes [P_{12}].$$

Each row of the modal effective mass matrix yields qualitative and quantitative information regarding the overall activity in a mode. Across a row, the terms indicate the relative level of excitation and delivered by base acceleration in each successive direction (as well as base reaction loading). Column summation of modal effective mass terms indicates the level of completeness in truncated mode representations where

$$\sum_n (M_{eff})_{n,j} \le (M_{RB})_{j,j}$$

1.5.3 Modal Orthogonality Evaluation

Normal vibration modes associated with the algebraic eigenvalue problem,

$$[K]\{\Phi_n\} - [M]\{\Phi_n\}\omega_n^2 = \{0\}$$

satisfy the orthonormal relationship (as calculated with **modalr**),

$$[\Phi]^T [M][\Phi] = [OR] = [I].$$

When a modal matrix, $[\Phi_T]$, comes from either another mathematical model (typically a variation of the reference [M], [K] based model) or from test data, each mode should be unit mass normalized as follows:

$$\{\Phi_n\}_{normalized} = \{\Phi_n\} / \sqrt{\{\Phi_n\}^T [M]\{\Phi_n\}}$$

The normalized “test” mode orthogonality check yields,

$$[OR_T] = \begin{bmatrix} 1 & or_{12} & .. & or_{1n} \\ or_{21} & 1 & .. & : \\ : & : & 1 & : \\ or_{n1} & .. & .. & 1 \end{bmatrix},$$

which is a measure of test mode “purity”. A common criterion used in the evaluation of test based modal data is that the coupling terms should be

less than 0.10 (or 10 percent) in order for modes to have been adequately estimated/identified.

The ITAP-A routine **modnorm** (see Chapter 5) performs normalization and orthogonality calculations on a modal matrix. Using the nomenclature adopted in this document, the function call statement for **modnorm** is

[[Φ],[OR]] = **modnorm**([M],[Φ]);

where the input mode matrix is not necessarily normalized and the output mode matrix is normalized.

1.5.4 Modal Cross-Orthogonality and Coherence Evaluation

The relationship between a reference mode set, [Φ], and a "test" mode set, $[\Phi_T]$, is described as a coordinate transformation,

$$[\Phi_T] = [\Phi][COR] + [R],$$

or

$$[R] = [\Phi_T] - [\Phi][COR],$$

[COR] is the cross-orthogonality matrix and [R] is the residual error, defined as the least squares solution for the mass weighted quadratic residual function,

$$\delta([R^T M R]) = 0,$$

which yields,

$$[COR] = [OR]^{-1}[\Phi^T M \Phi_T],$$

where

$$[OR] = [\Phi]^T [M][\Phi].$$

The orthogonality matrix associated with the test mode set is

$$[OR_T] = [\Phi_T]^T [M][\Phi_T].$$

Substitution of the above results into the quadratic residual function yields

$$[R^T M R] = [OR_T] - [COR]^T [OR][COR].$$

Further manipulation of this result leads to definition of the modal coherence matrix,

$$[COH] = [I] - [OR_T]^{-1/2}[R^T MR][OR_T]^{-1/2} = [OR_T]^{-1/2}[COR]^T[OR][COR][OR_T]^{-1/2}$$

Since the "test" modes, $[\Phi_T]$, represent a projection onto the "modal space" spanned by, $[\Phi]$, the modal coherence matrix provides a measure of completeness associated with that projection. Note that if the residual error matrix, [R], is null, modal coherence is an identity matrix.

The ITAP-A routine **modcoh** (see Chapter 5) performs normalization, orthogonality, and coherence calculations on "test" and reference modal data. Using the nomenclature adopted in this document, the function call statement for **modcoh** is

[[COH],[OR],[OR_T],[COR]] = **modcoh**($[\Phi_T]$,$[\Phi]$,[M]).

1.5.5 Mode Set Orthogonalization

When dealing with test based modal data, a common practice is to orthogonalize the modal matrix, $[\Phi_T]$. Motivations for orthogonalization include (a) reduction of measurement/analysis inconsistencies associated with the entire test process, and (b) purification of the modal matrix for its subsequent use as part of an empirical dynamic model. There are several strategies for mode orthogonalization, which may be selected based on user assumptions regarding the test data. In all cases, orthogonalization should only be applied to modal data which is nearly orthogonal (i.e., small off-diagonal terms in $[OR_T]$).

If "errors" in the modal matrix are viewed as "evenly distributed", it has been proven (Reference 15, 16) that the optimally corrected modal matrix is

$$[\Phi_T]_{Corrected} = [\Phi_T][OR_T]^{-1/2} \quad \text{(spectral method).}$$

If "errors" in the modal matrix are assumed to grow steadily for successive modes (Reference 17), the optimally corrected modal matrix is

$$[\Phi_T]_{Corrected} = [\Phi_T][L]^{-T} \quad \text{(Cholesky method),}$$

where [L] is the upper triangular matrix calculated by Cholesky decomposition of $[OR_T]$.

The ITAP-A routine **modorth** (see Chapter 5) performs orthogonalization calculations on a modal matrix with respect to a mass matrix, [M]. Using the nomenclature adopted in this document, the function call statement for **modorth** is

$$[\,[\Phi_T]_{\text{corrected}}, [OR_T]\,] = \textbf{modorth}([M],\,[\Phi_T], \text{opt}).$$

If opt=1, the Cholesky method is employed

If opt=2, the spectral method is employed.

$[OR_T]$ is the orthogonality check matrix after modal adjustments.

In some situations, greater confidence is assigned to a subset of modal vectors. For example, consider a modal matrix composed of two modal partitions,

$$[\Phi_T] = [\Phi_1 \quad \Phi_2]$$

where $[\Phi_1]$ is assumed of superior quality to $[\Phi_2]$. The modal subset, $[\Phi_2]$, may be block-orthogonalized relative to $[\Phi_1]$ by a block Gram-Schmidt calculation (Reference 18),

$$[\Phi_2]_{\text{corrected}} = [\Phi_1] - [\Phi_1][Q_{12}] \qquad [\Phi_2]_{\text{modified}} = [\Phi_2] - [\Phi_1][Q_{12}]$$

with

$$[Q_{12}] = \left[\Phi_1^T M \Phi_1\right]^{-1}\left[\Phi_1^T M \Phi_2\right].$$

The ITAP-A routine **modsweep** (see Chapter 5) performs block orthogonalization calculations on a modal matrix partitions with respect to a mass matrix, [M]. Using the nomenclature adopted in this document, the function call statement for **modsweep** is

$$[\,[\Phi_2]_{\text{corrected}}\,] = \textbf{modsweep}([M],\,[\Phi_1],\,[\Phi_2]).$$

1.5.6 Apparent Mass and Stiffness Matrices

An interesting alternative to “test” mode matrix adjustment was introduced by Berman (Reference 19). In this approach, the theoretical TAM mass matrix, [M], is optimally modified in order to obtain a perfect orthogonality check matrix. Inherent in this strategy is the assumption that the “test” modes are error free. Further work by Berman and Wei (Reference 20) defined an optimally adjusted TAM stiffness matrix. The relationships defining the optimally adjusted (apparent) mass and stiffness matrices are,

$$[M_T] = [M] + [M\Phi_T][OR_T]^{-1}[I - OR_T][OR_T]^{-1}[\Phi_T^T M]$$

$$[K_T] = [K] + [M\Phi_T][\Phi_T^T K\Phi_T + \Lambda_T][\Phi_T^T M] - [K\Phi_T\Phi_T^T M + M\Phi_T\Phi_T^T K]$$

where Λ_T is the diagonal matrix composed of test eigenvalues.

The apparent mass and stiffness matrices yield numerically perfect orthogonality of test modes and eigenvalues in agreement with test natural frequencies. When the test modes are a truncated subset of TAM modes, those modal frequencies, which are not represented in the test mode set may be interspersed with the test modal frequencies. The ITAP-A user is cautioned that application of this procedure may result in misleading apparent mass and stiffness matrices.

The ITAP-A routine **berman** (see Chapter 5) calculates apparent mass and stiffness matrices following the relationships defined above. Using the nomenclature adopted in this document, the function call statement for **berman** is

[$[M_T]$,$[K_T]$]=**berman**([M],[K],$[\Phi_T]$,$\{f_T\}$);

where $\{f_T\}$ is the array of test modal frequencies.

1.6 DOF Selection for Mapping of Experimental Normal Modes

1.6.1 Overview

A direct method for response "measurement" degree-of-freedom (DOF) selection to map experimental normal modes is described and demonstrated (Reference 21). The method utilizes a detailed pre-test finite element model of the subject structure. An initial response "measurement" DOF set is first selected for mapping modes in the frequency band of interest. The remaining DOFs of the structural model are assumed to respond in a quasi-static manner following a Guyan reduction transformation (Reference 22). Modes formed from the "measurement" partition of the FEM modal vectors and expanded using the Guyan reduction transformation comprise the "approximate" normal mode set. Residual error vectors are formed based on differences between complete FEM normal modes in the frequency band of interest and the set of corresponding "approximate" modes. A residual kinetic energy matrix directly identifies additional DOFs (or DOF groupings) which must be instrumented in order to experimentally map the normal modes of interest.

1.6.2 Selection of DOFs for Modal Mapping

The most commonly used approach for development of a test-analysis model (TAM) is Guyan reduction in which the "measurement" DOFs are retained in the "analysis" set. Using commonly accepted NASTRAN notation, the matrix dynamic equations for free vibration (with all physical constraints applied) are

$$\begin{bmatrix} K_{AA_1} & K_{AO} \\ K_{OA} & K_{OO} \end{bmatrix} \begin{Bmatrix} \Phi_A \\ \Phi_O \end{Bmatrix} - \begin{bmatrix} M_{AA_1} & M_{AO} \\ M_{OA} & M_{OO} \end{bmatrix} \begin{Bmatrix} \Phi_A \\ \Phi_O \end{Bmatrix} \lambda = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix},$$

where the FEM modal matrix is expressed in terms of "analysis" and "omitted" partitions. The Guyan reduction transformation, which is used to form the approximate TAM is

$$[T_{RED}] = \begin{bmatrix} I \\ -K_{OO}^{-1} K_{OA} \end{bmatrix}$$

If the "analysis" set partition of the modal matrix is assumed to represent "measured" modal DOFs, then the approximate TAM modal matrix is

$$\begin{bmatrix} \Phi_A \\ \Psi_O \end{bmatrix} = \begin{bmatrix} I \\ -K_{OO}^{-1} K_{OA} \end{bmatrix} [\Phi_A]$$

where the partition, Ψ_O, represents the approximate "omitted" DOF partition.

The residual displacement error matrix is therefore defined as

$$[R_\Phi] = \begin{bmatrix} 0 \\ \Phi_O - \Psi_O \end{bmatrix}$$

Note that the residual error associated with the "measured" or "analysis" DOF partition is null. The modal kinetic energy distribution for the complete system is

$$[E_\Phi] = [M\Phi] \otimes [\Phi]$$

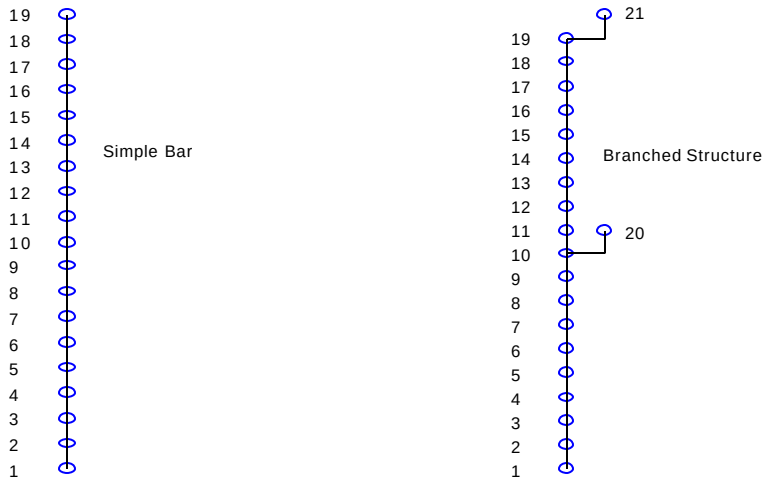
where the column sum for each individual mode is unity (if the modes are normalized to unit modal mass). The residual kinetic energy matrix is now defined, in a similar manner, as

$$[E_R] = [MR] \otimes [R]$$

Like the residual displacement error, [R], the residual kinetic energy matrix is exactly null at the rows corresponding to measurement DOFs. The expected characteristic that residual energy is pronounced at "omitted" yet dynamically significant DOFs in any particular mode will be demonstrated in the illustrative examples which follow.

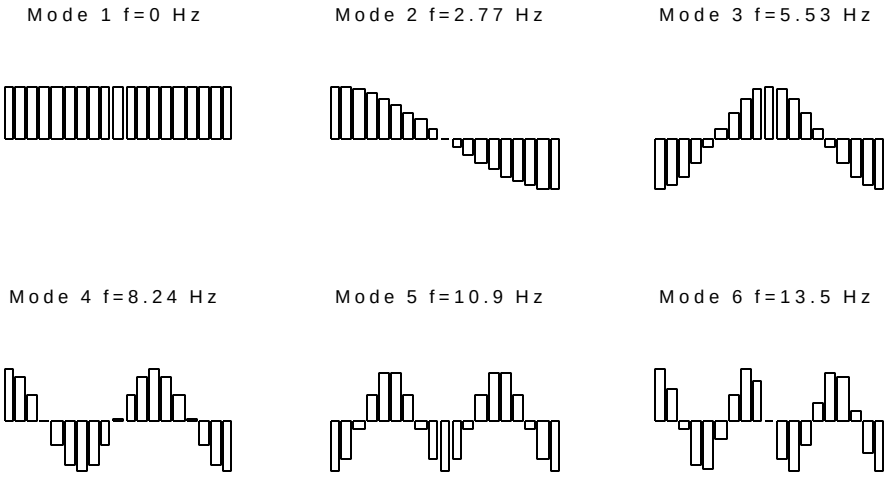
1.6.3 Illustrative Examples

Two simple structures, used to illustrate the measurement DOF selection procedure are shown in the figure below:

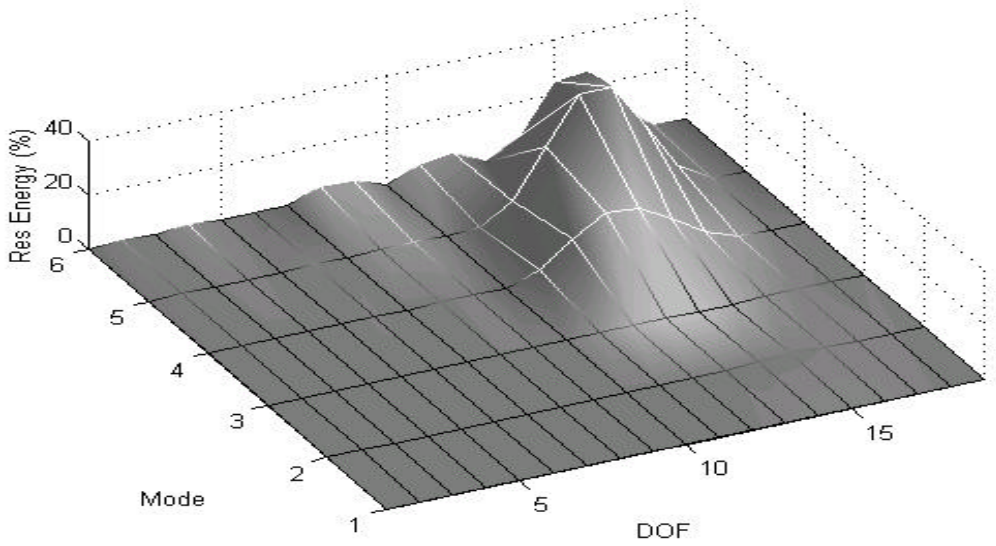


The first illustrative example (shown on the left) consists of a uniform, free-free bar modeled as a 19 node dynamic model with 19 axial DOFs. The second illustrative example (shown on the right) is the free-free bar augmented with two attached substructures (a branched structure).

The first six (6) axial vibration modes of the uniform bar are shown below:



Six DOFs (1,3,5,7,10,19), which are insufficient for mapping modal behavior on the "upper portion" of the bar, are first selected as response "measurement" DOFs. The residual kinetic energy associated with this selected response DOF set is illustrated below as a surface with the horizontal axes corresponding to model DOF number and mode number, respectively. The surface clearly indicates that "measurement" DOFs are adequate for the first three modes. Pronounced residual kinetic energy terms in modes 4-6 suggest that at least two to three more DOFs on the "upper portion" of the bar structure must be instrumented to adequately map modes 4-6.

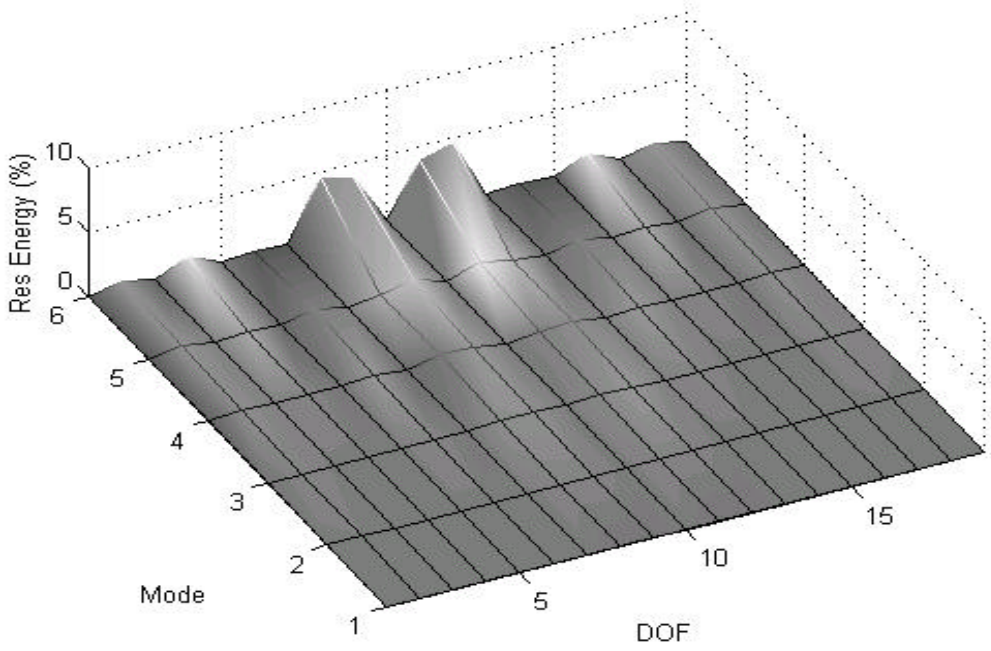


It is of interest to note the orthogonality (normalized such that on-diagonal terms are 100) of the approximate "measured" modes, shown below, for the present DOF selection (1,3,5,7,10,19)

$$[OR_{\Psi}]=\begin{bmatrix} 100 & 10 & -1 & -55 & -58 & -46 \\ 10 & 100 & -5 & 44 & 51 & 45 \\ -1 & -5 & 100 & -18 & 3 & 29 \\ -55 & 44 & -18 & 100 & 23 & 60 \\ -58 & 51 & 3 & 23 & 100 & 33 \\ -46 & 45 & 29 & 60 & 33 & 100 \end{bmatrix}$$

The good orthogonality among modes 1-3 confirms adequacy of the selected DOF set for these modes, while the extremely poor orthogonality of the remaining modes indicates a need for additional measurement DOFs.

Following the recommendation for additional “measured” DOFs implied by the residual kinetic energy, illustrated in the above figure, DOFs 13,15 and 17 are added to the measurement set. The residual kinetic energy reduces substantially as illustrated below:



The improvement in orthogonality of “approximate” measured modes, provided below, indicates adequacy of the refined response measurement DOF set.

$$[OR_{\Psi}]=\begin{bmatrix} 100 & 0 & -2 & 0 & 5 & 0 \\ 0 & 100 & 0 & 2 & 0 & -6 \\ -2 & 0 & 100 & 0 & 7 & 0 \\ 0 & 2 & 0 & 100 & 0 & 20 \\ 5 & 0 & 7 & 0 & 100 & 0 \\ 0 & -6 & 0 & 20 & 0 & 100 \end{bmatrix}$$

In order to illustrate application of the response “measurement” DOF selection method to more complicated structures, the branched structure is investigated. Four (4) substructures are defined according to nodal DOFs, namely,

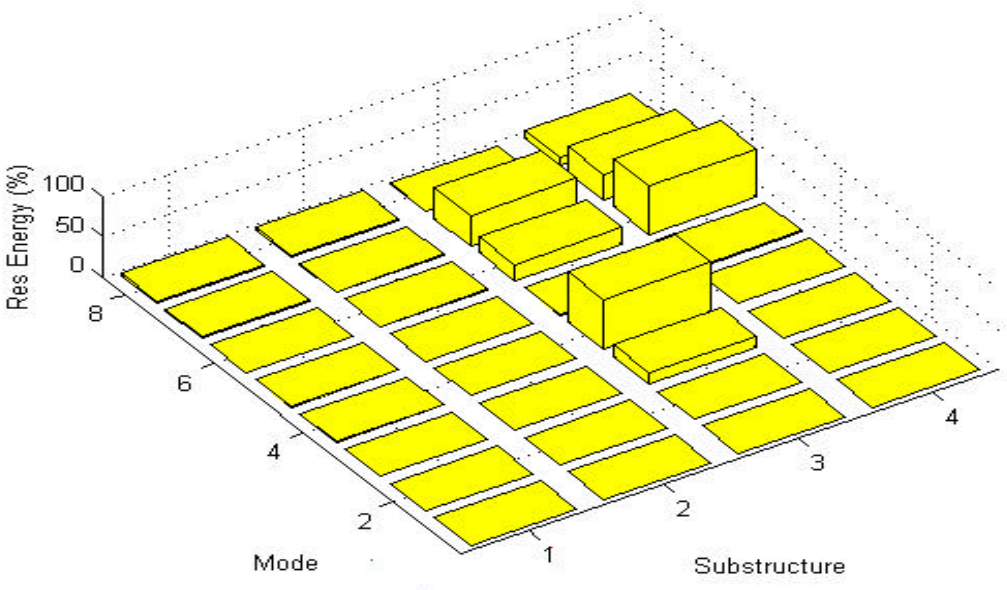
Sub1 (Lower Main Body): DOFs 1-9
Sub2 (Upper Main Body): DOFs 11-18
Sub3 [Middle Branch]: DOFs 10,20
Sub4 [Upper Branch]: DOFs 19,21

The first eight modes of this structure are described in the table provided below according to kinetic energy distribution among substructures. (Additional information is available in details of substructure DOF motions).

Branched Structure Modes

		Percent Kinetic Energy			
Mode	Freq	Sub1	Sub2	Sub3	Sub4
1	0	38	36	23	3
2	2.76	50	44	0	7
3	4.06	22	17	56	4
4	7.16	32	35	26	6
5	8.15	54	37	0	9
6	10.9	11	24	4	61
7	12.0	42	24	8	26
8	13.6	43	47	0	10

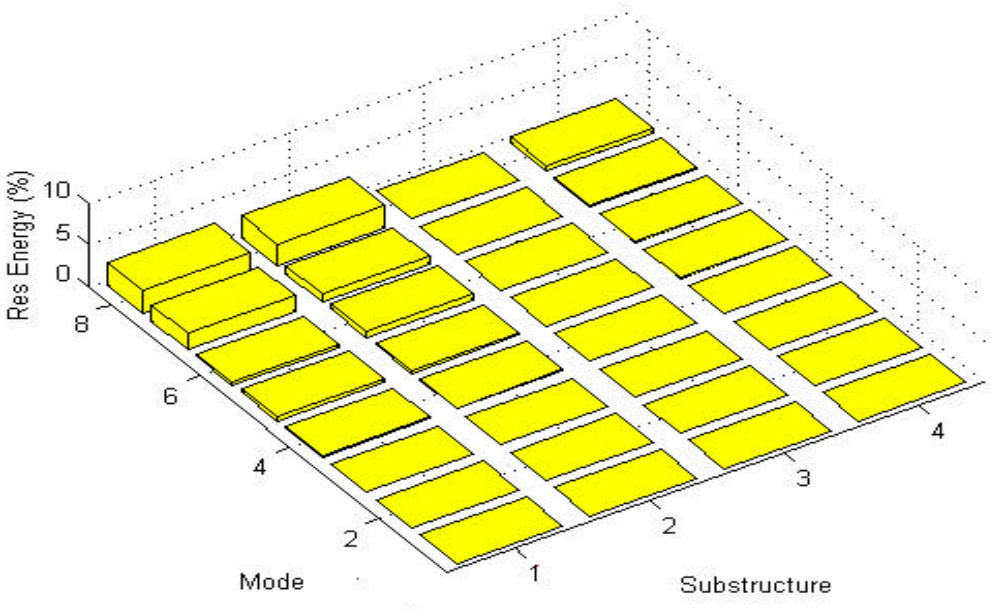
Nine DOFs (2,4,6,8,10,12,14,16,18) are first selected as response "measurement" DOFs. The residual kinetic energy error associated with this selected response DOF set is illustrated below as a 3D bar graph, with the horizontal axes corresponding to substructure number and mode number, respectively. The graph clearly indicates that the two main body substructures (1 and 2) have been adequately instrumented. Pronounced residual kinetic energy terms in modes 4, 6 and 7 indicate the two branch substructures need to be included in the instrumentation array to effectively map these modes.



Orthogonality of the approximate set of “measured” modes for this DOF selection is shown below:

$$[OR_\Psi]=\begin{bmatrix} 100 & 0 & -21 & 38 & -2 & -20 & -33 & -7 \\ 0 & 100 & 1 & 0 & 2 & 16 & -7 & -5 \\ -21 & 1 & 100 & 65 & -7 & -56 & -29 & 0 \\ 38 & 0 & 65 & 100 & -3 & -30 & -1 & 4 \\ -2 & 2 & -7 & -3 & 100 & -24 & 14 & 6 \\ -20 & 16 & -56 & -30 & -24 & 100 & 75 & 25 \\ -33 & -7 & -29 & -1 & 14 & 75 & 100 & -3 \\ -7 & -5 & 0 & 4 & 6 & 25 & -3 & 100 \end{bmatrix}$$

Following the recommendation for additional “measured” DOFs implied by the residual kinetic energy in the above figure, DOFs 20 and 21 are added to the measurement set. The residual kinetic energy reduces substantially as illustrated below.



The improvement in orthogonality of “approximate” measured modes, shown below, indicates adequacy of the refined response measurement DOF set.

$$[\mathrm{OR}_{\Psi}]=\begin{bmatrix} 100 & 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 100 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 100 & -2 & 0 & 2 & -2 & 0 \\ -1 & 0 & -2 & 100 & 0 & 2 & -2 & -1 \\ 0 & 2 & 0 & 0 & 100 & 3 & -2 & -1 \\ 0 & -1 & 2 & 1 & 3 & 100 & -10 & -6 \\ 1 & 1 & 1 & -2 & -2 & -10 & 100 & 4 \\ 0 & 0 & 0 & -1 & -1 & -6 & 4 & 100 \end{bmatrix}$$

The procedure, which has been described and demonstrated with two simple example structures, reliably identifies deficiencies in response “measurement” arrays intended for mapping of experimental normal modes. Pronounced residual kinetic energy provides the test planner with information which effectively assists in the task of completing of the response “measurement” array. For elaborate structural models, residual kinetic energy may be displayed in terms of substructures, providing a more tractable summary than a DOF based display. When instrumentation deficiencies are localized on specific substructures, more in-depth examination may be conducted using a DOF based display for those substructures.

1.7 Closure

Fundamental theoretical considerations employed in development of ITAP-A computational modules and procedures have been presented in this chapter. Chapters 2-4 of the *ITAP User's Guide* describe implementation of ITAP-A theory for test-analysis model (TAM) development, modal test planning and modal test-TAM correlation, respectively. Chapter 5 of the *ITAP User's Guide* provides a comprehensive summary of ITAP-A function modules, which are, for the most part, based on theoretical considerations presented in this chapter.

1.8 References

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19. Berman, “Mass Matrix Correction using an Incomplete Set of Measured Modes”, AIAA Journal, Vol 17, Oct 1979, pp 1147-1148

20. Berman and Wei, “Automated Dynamic Analytical Mode Improvement”, NASA CR3452, July 1981

21. Coppolino, “Automated Response DOF Selection for Mapping of Experimental Normal Modes”, Proceedings of the 16th IMAC, Feb 1998, pp 70-76

22. Guyan, “Reduction of Stiffness and Mass Matrices”, AIAA Journal, Vol 3, No 2, 1965, p 380

2.0 Model Definition and Translation

2.1 Accessing NASTRAN Matrix Data

Within the NASTRAN environment (UAI/NASTRAN and others), static and dynamic linear and nonlinear finite element models are defined and analyzed. A **conventional linear structural dynamic model** is defined in terms of mass, [M], stiffness, [K], and sometimes damping, [B], matrices as well as geometric and response (e.g., internal load and stress) data.

Matrix data from UAI/NASTRAN (as well as other NASTRAN versions), written to an OUTPUT4 file, may be accessed and converted to a MATLAB .mat file using the ITAP-A module, **out4read**. The call statement for out4read is

out4read(namfold,namin,namout);

where

namfold = input file folder (directory/subdirectory)
namin = input file (NASTRAN Output4 file) name
namout = output file (MATLAB .mat file) name.

The OUTPUT4 file must be defined as TYPE=FORMATTED and MSC File Structure. This provides the ITAP-A user with ability to access matrix data from any computational environment and NASTRAN version.

2.2 UAI/NASTRAN Modal Analysis (Rigid Format 3 Alter)

UAI/NASTRAN Rigid Format 3 assembles a finite element dynamic model and performs free vibration analysis on the model. **The conventional linear structural dynamic formulation** equations from which a modal solution is extracted are

$$[K]\{\Phi_n\} - [M]\{\Phi_n\}\omega_n^2 = \{0\}.$$

UAI/NASTRAN Rigid Format 3 analysis imposes user specified constraints and coordinate reductions, which take the finite element model from an unconstrained “g” (grid) set to a constrained “a” (analysis) set description. Static condensation or Guyan reduction is often employed to define an approximate reduced order model.

The Rigid Format 3 Alter described in this section has been developed to define dynamic models, which are convenient for ITAP-A applications. Within the context of this specific application, the “a” set is used primarily for “condensing” massless degrees of freedom from the mass and stiffness matrices (specification of AUTOOMIT=YES in the Case Control Deck is recommended). The user separately defines a “y” set of “instrumented” degrees of freedom (dofs), with DEFUSET and USET1 data card images, which must be comprised of **three translation degrees**

of freedom per instrumented grid point. The instrumented dofs may be defined in a local coordinate system corresponding to the orientation of triaxial accelerometers on a test article.

Matrix data, written to an OUTPUT4 file, always named “ITAPS”, for further analysis in ITAP-A consists of the following variables:

MYY: reduced mass matrix in terms of selected triaxial dofs

KYY: reduced stiffness matrix in terms of selected triaxial dofs

PHIY: partition of the system (“a” set) modal matrix associated with selected triaxial dofs. This is not the solution of an eigenvalue problem based on MYY and KYY.

LAMAT: eigenvalue matrix data from the “a” set eigenvalue problem solution.

PSIRY: geometric rigid body displacement matrix containing information used to extract selected instrumentation grid point x,y,z positions and local triaxial coordinate system transformations.

ERPHG: residual modal kinetic energy distribution matrix in terms of “g” set degrees of freedom. This is used for evaluation of the adequacy of selected triaxial dofs for modal mapping.

EPHIG: modal kinetic energy distribution matrix in terms of “g” set degrees of freedom. This is used for evaluation of the adequacy of selected triaxial dofs for modal mapping.

IGG: matrix used to generate ITAP-A names for instrumented grid points.

The above matrix data are processed by the ITAP-A module, **makmodl**, which translates the NASTRAN OUTPUT4 file and defines an ITAP resident model file. This ITAP model file will include all necessary TAM information for performance of modal test planning, test-analysis correlation analysis and generation of tabular and graphical displays.

2.3 Rigid Format 3 Alter Listing

The following Rigid Format 3 Alter has been developed for UAI/NASTRAN Version 20 to define matrix data for an ITAP-A TAM. The first two lines shown below are case control statements which specify of the OUTPUT4 file named “ITAPS” and turn off the default grid point resequencer.

```
ASSIGN  ITAPS,NEW,USE=OUTPUT4,TYPE=FORMATTED,REALLOC
SEQUENCE=NO

ALTER 160
MATGEN BGPDT,EQEXIN,CSTM/PSIRG/102/0/0 $
UPARTN USET,MAA/MYY1,MZY,MYZ,MZZ/'A'/'Y'/'COMP'/0 $
UPARTN USET,KAA/KYY1,KZY,KYZ,KZZ/'A'/'Y'/'COMP'/0 $
UPARTN USET,PHIA/PHIY,,,/'A'/'Y'/'COMP'/1 $
SOLVE KZZ,KZY/GZY/1/-1 $
MPYAD KYZ,GZY,KYY1/KYY/0 $
MPYAD MYZ,GZY,MYY1/MYY2/0 $
MPYAD GZY,MZY,MYY2/MYY3/1 $
MPY3 GZY,MZZ,MYY3/MYY/0 $
UPARTN USET,PSIRG/PSIRY,,,/'G'/'Y'/'COMP'/1 $
UPARTN USET,PHIG/PHIY,,,/'G'/'Y'/'COMP'/1 $
MPYAD GZY,PHIY,/PHIZ/0 $
UMERGE USET,PHIY,PHIZ/PSIA/'A'/'Y'/'COMP' $
ADD PHIA,PSIA/RPHA/(1.0,0.0)/(-1.0,0.0)/0 $
MPYAD MAA,RPHA,/MRPHA/0 $
ADD MRPHA,RPHA/ERPHA///1 $
UMERGE USET,ERPHA,/ERPHG/'G'/'A'/'COMP' $
SOLVE MYY,MYY/IYY/1 $
UMERGE USET,IYY,/IGG/'G'/'Y'/'COMP' $
MPYAD MGG,PHIG,/MPHIG/0 $
ADD MPHIG,PHIG/EPHIG///1 $
LAMX, ,LAMA/LAMAT//'PREC' $
OUTPUT4 PHIY,MYY,KYY,LAMAT,PSIRY//'ITAPS'//'FULL'////'MSC' $
OUTPUT4 ERPHG,EPHIG,IGG//'ITAPS'//'FULL'////'MSC' $
ENDALTER $
CEND
```

[Note: If using another NASTRAN version, you must develop a version specific DMAP Alter sequence, which generates the appropriate model matrices.]

2.4 ITAP-A Test-Analysis Model (TAM) Definition

The ITAP-A routine **makmodl** performs a variety of calculations aimed at definition of the ITAP based TAM for a structural or mechanical dynamic system.

The major operational steps performed by **makmodl** are as follows:

1. Access the OUTPUT4 file “ITAPS” and translate the matrix files to a MATLAB compatible format. Select a name for the ITAP-A TAM file.

2. Using information contained in the rigid body displacement matrix, **PSIRY**, define (a) the global “x,y,z” position matrix, **XYZ**, for all TAM grid points, (b) the local-to-global triaxial dof displacement transformation matrix, **TMAT**, and (c) the rigid body displacement matrix (in global coordinates), **RBMAT**.

3. Generate a set of descriptive grid point names for the TAM and a dof “roadmap”, **TRP**, for subsequent data processing.

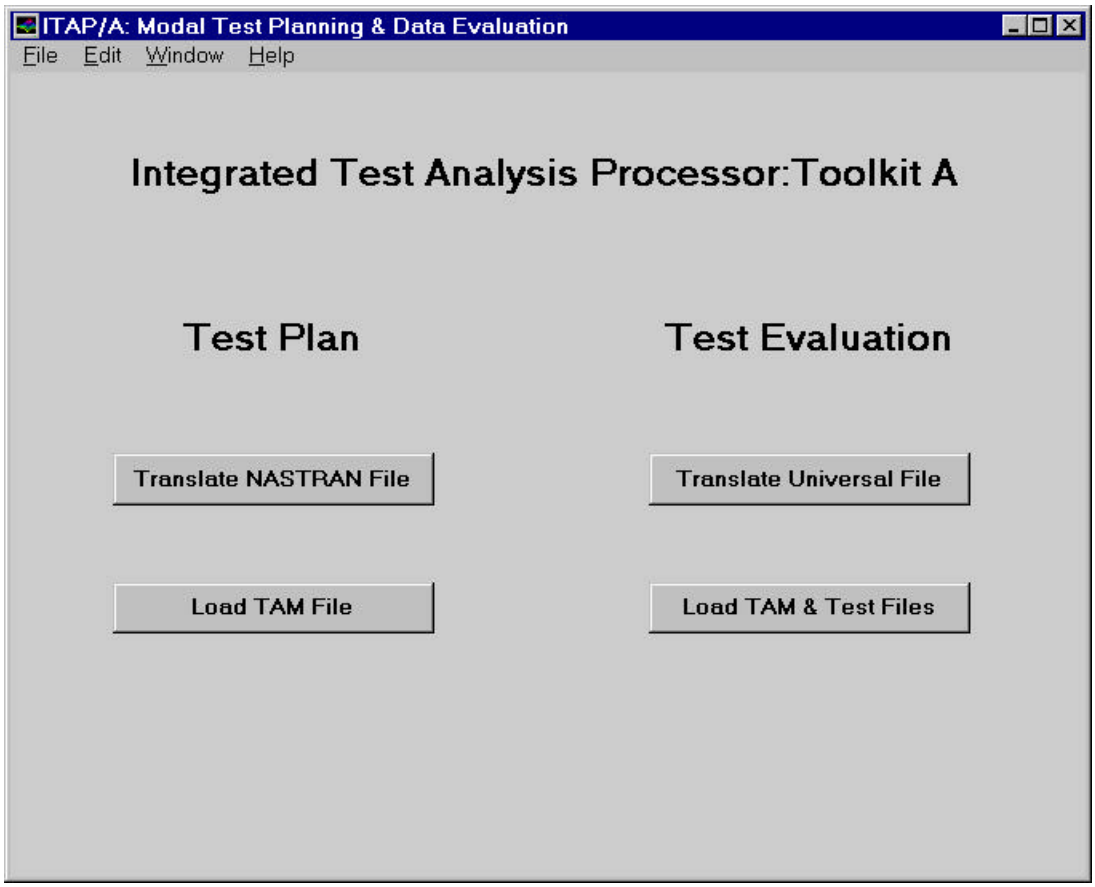
4. Define a TAM model connectivity file, **conn**, containing a grid string to be used in generation of model geometry displays.

5. Optionally define a TAM substructure breakdown in terms of grid point groupings (**sub1**, **sub2**, …), for use in generation of substructure tabular mode shape displays.

6. Save the ITAP-A TAM file.

2.5 Sample TAM Model Definition (a makmodl Session)

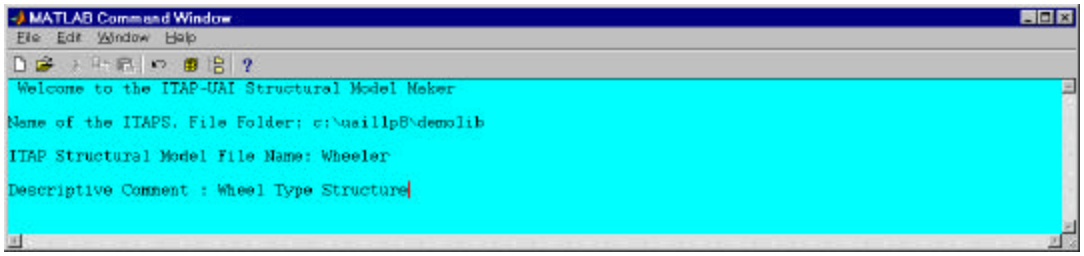
From the MATLAB command window, the user types "**itapa**" to initiate the "**makmodl**" session. A screen with four push-button choices appears as shown below:



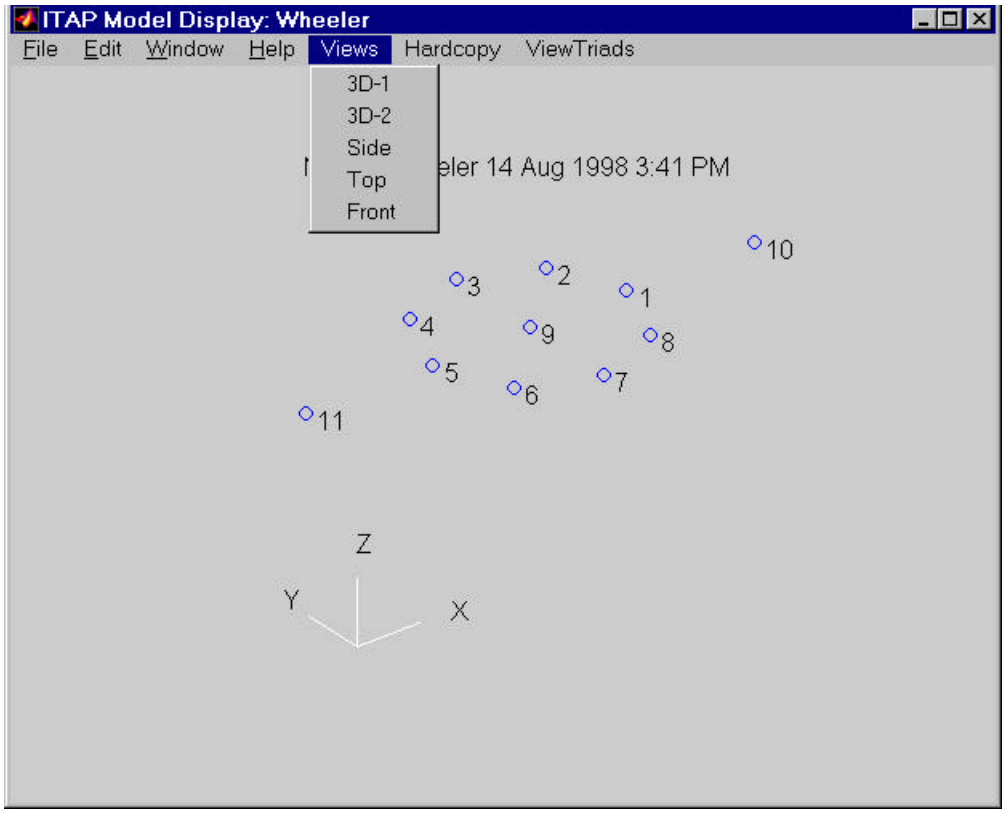
If the upper left button (Translate NASTRAN File) is pressed, the module **makmodl** is invoked, an OUTPUT4 TAM file is translated and remains accessed for further modal test planning operations, which are described in the next chapter.

The following is an illustrative example walk-through of what happens in the **makmodl** process.

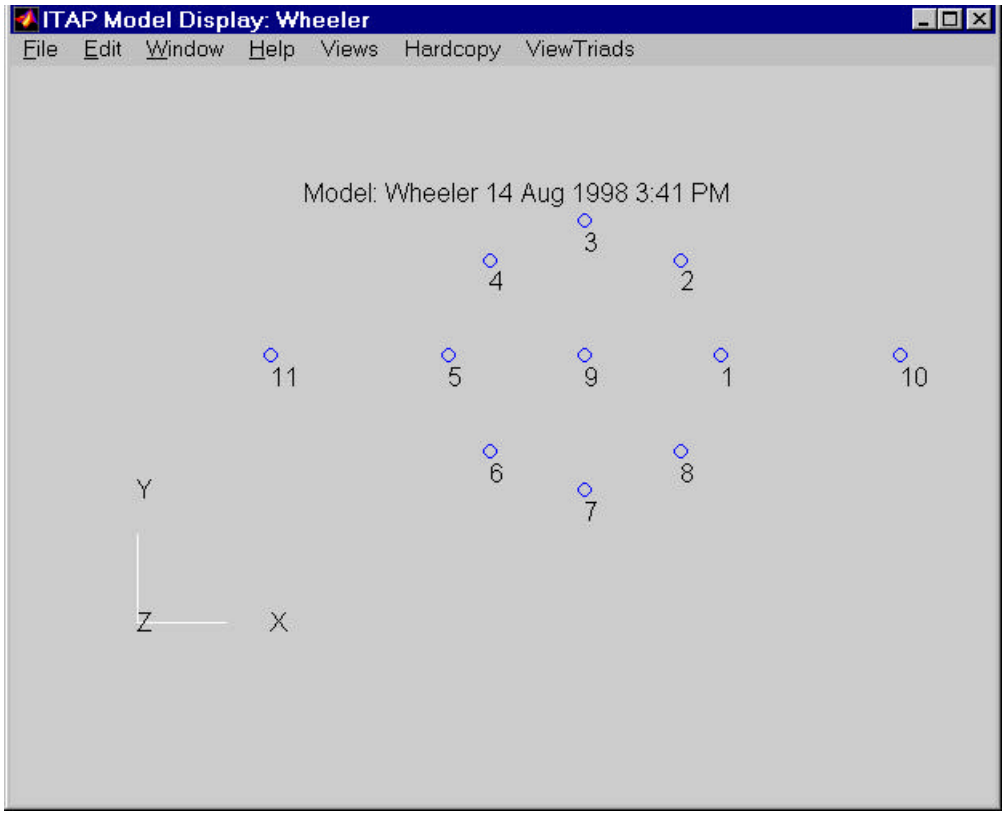
1. Access the OUTPUT4 file “ITAPS” and translate the matrix files to a MATLAB compatible format. Select a name for the ITAP-A TAM file.

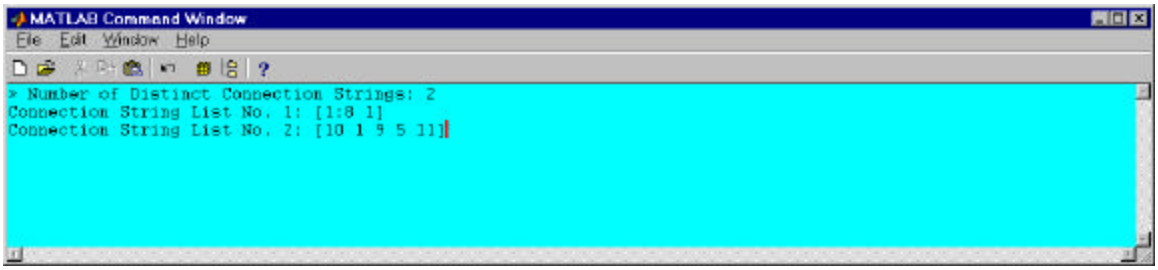


The following display appears (along with the request for connectivity data which is not shown).

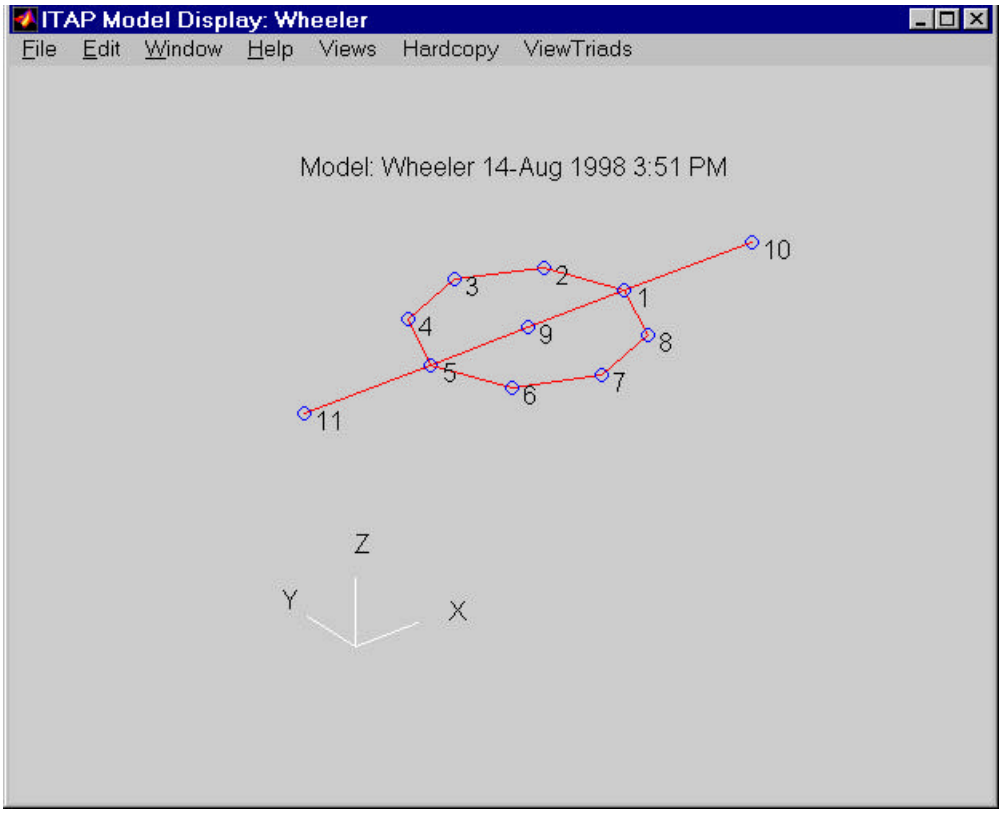


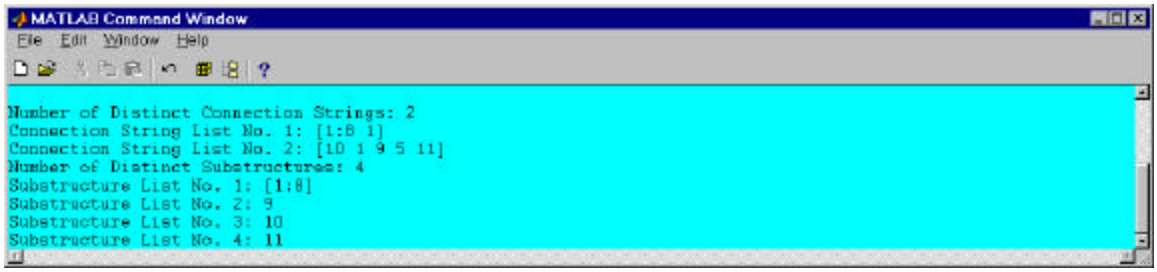
The user may choose a different view to clearer access to grid point geometry. Shown below is a top view and the user specified connectivity data in the MATLAB command window.





After specification of connectivity, the connected display of TAM geometry appears. A request for substructure information appears in the MATLAB command window; the user response is displayed in the MATLAB command window shown below.





If the user specifies “0” or “1” substructures, the TAM file does not save substructuring information.

This completes the “**makmodl**” session. The ITAP-A process automatically continues with a test planning session, which is the subject of the next chapter.

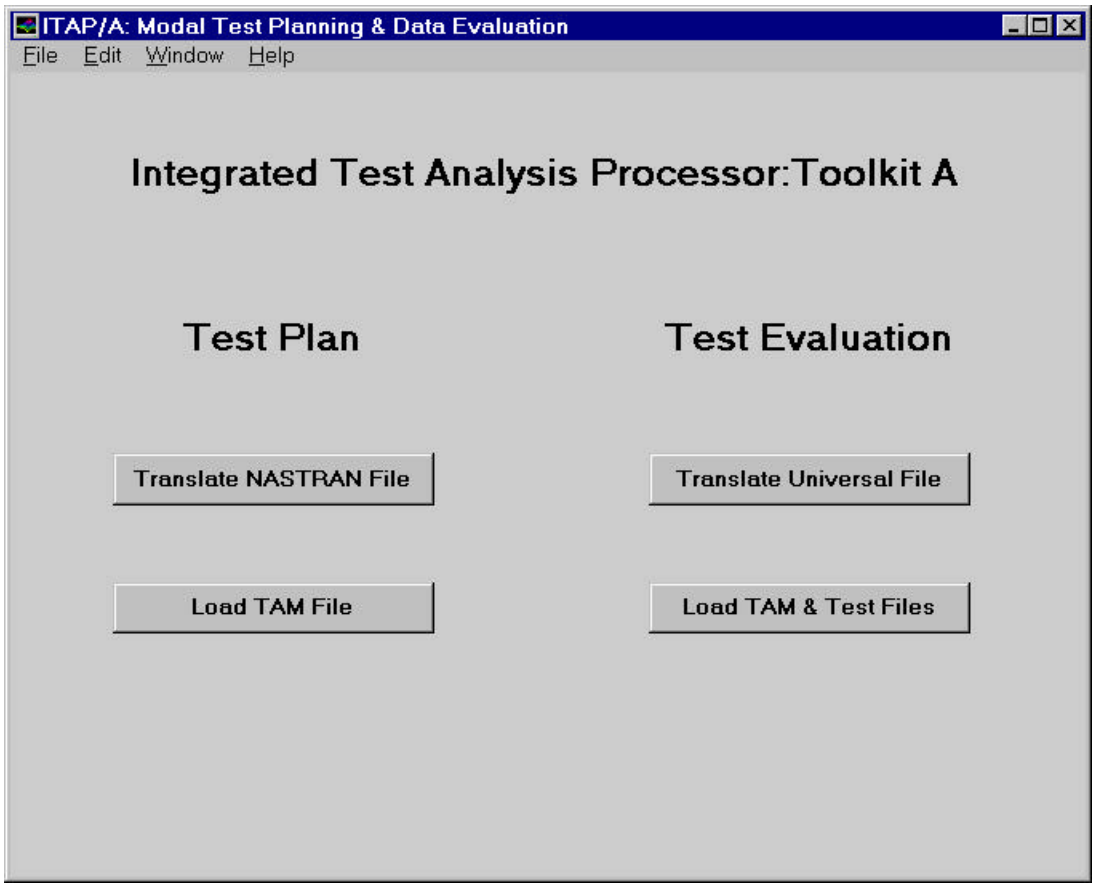
3.0 Modal Test Planning

3.1 ITAP-A Implementation Summary

3.2 Illustrative Test Planning Sessions

3.1 ITAP-A Implementation Summary

From the MATLAB command window, the user types "**itapa**" to initiate a modal test planning session. A screen with four push-button choices appears as shown below:



If the upper left button (Translate NASTRAN File) is pressed, the module **makmodl** is invoked, an OUTPUT4 TAM file is translated and modal test planning operations (driven by the function module, **modprep2**) are performed. If a TAM file has already been translated, the TAM file is accessed and modal test planning operations (**modprep2**) are initiated by pressing the lower left button (Load TAM File).

The major operational steps performed by **modprep2** are as follows:

1. Either (a) translate UAI/NASTRAN OUTPUT4 data to define an ITAP-A TAM file, or (b) access an existing TAM file.

2. A model geometry display is generated. The user may elect to perform display and analysis operations by selecting one of the following menu options (which may be accessed in any user desired sequence):

    (a) **Views** (choices are - 3D-1, 3D-2, Side, Top, Front

    (b) **Hardcopy**

    (c) **View Triads** (This option allows the user to review local displacement coordinate system triads)

    (d) **Operation** (choices are – "**Response MAP**", "**Excitation MAP**", "View Modes", "**End Session**")

3. Evaluate adequacy of ITAP-A TAM response "measurement" dofs by selecting the **Operation - Response MAP**" option on the model display window. The theoretical considerations described in Sections 1.3, 1.5, and 1.6 form the basis of this operation.

4. Evaluate adequacy of user specified "excitations" by selecting the **Operation – Excitation MAP**" option on the model display window. Steady-state response to harmonic excitation loading described in Subsection 1.4.3 (**mfresp**) forms the basis of this operation.

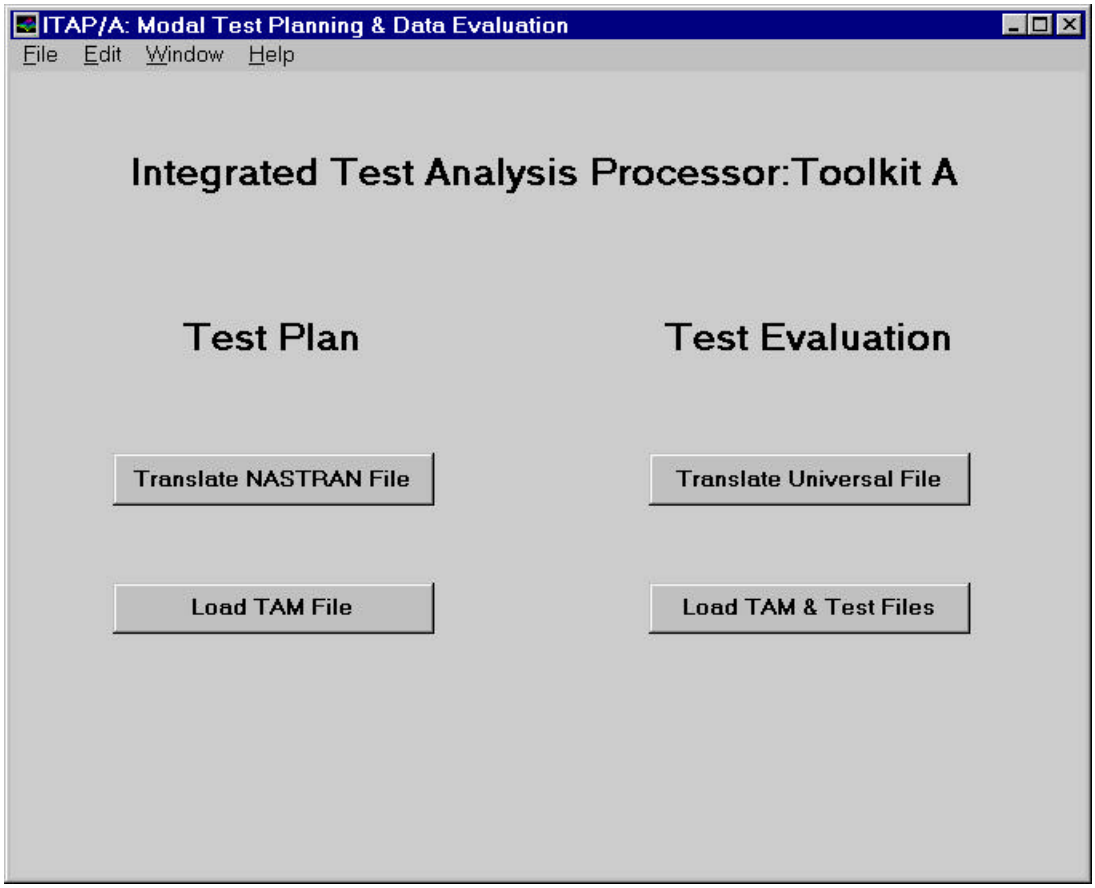
5. Study the character and content of each TAM normal mode by selecting the **Operation – View Modes**" option on the model display window. Various display options, based on theoretical considerations described in Section 1.3, may be accessed by the user.

The interactive operations performed by **modprep2** are demonstrated, with a simple case structure, in the following sections.

3.2 Illustrative Test Planning Sessions

3.2.1 NASTRAN OUTPUT4 File Translation or ITAP-A TAM File Access

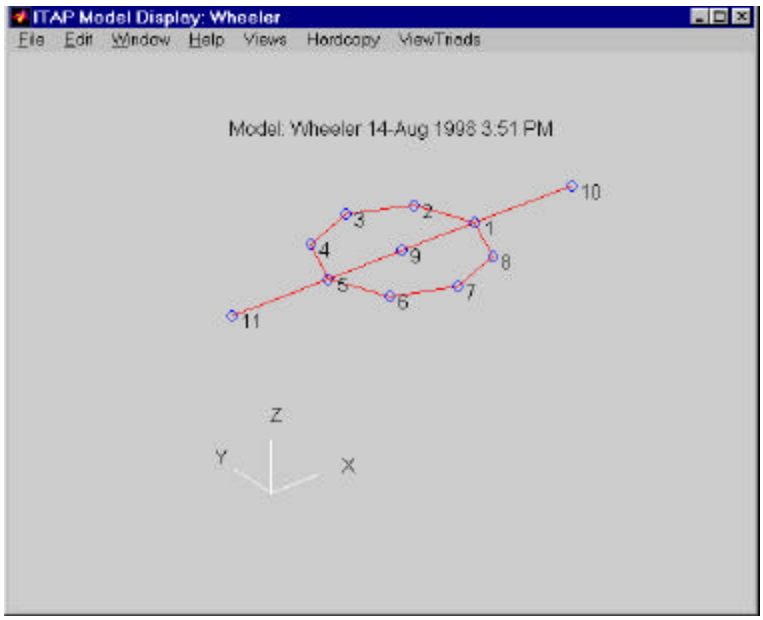
From the MATLAB command window, the user types "**itapa**" to initiate the test planning session. A screen with four push-button choices appears as shown below:



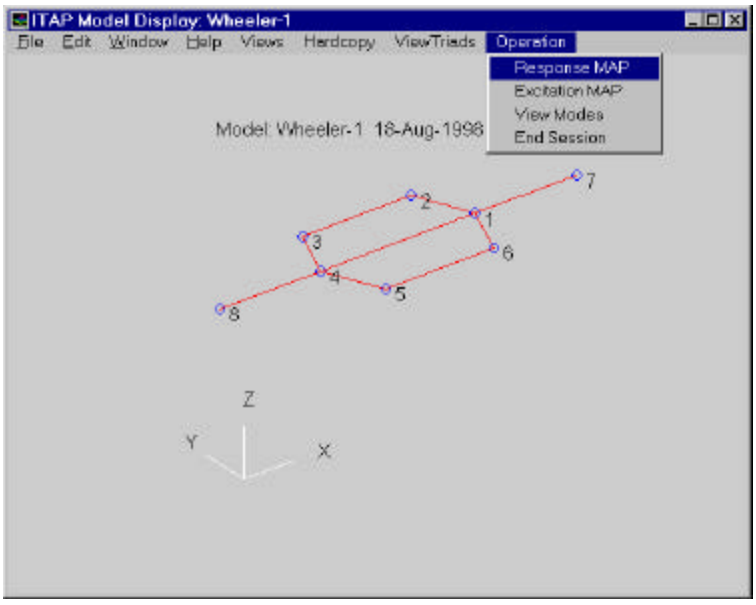
If the upper left button (Translate NASTRAN File) is pressed, the module **makmodl** is invoked, an OUTPUT4 TAM file is translated (see Chapter 2) and remains accessed for further evaluations. If the lower left button (Load TAM File) is pressed, an existing TAM file is accessed.

3.2.2 Adequacy of TAM Response "Measurement" Dofs (First Try)

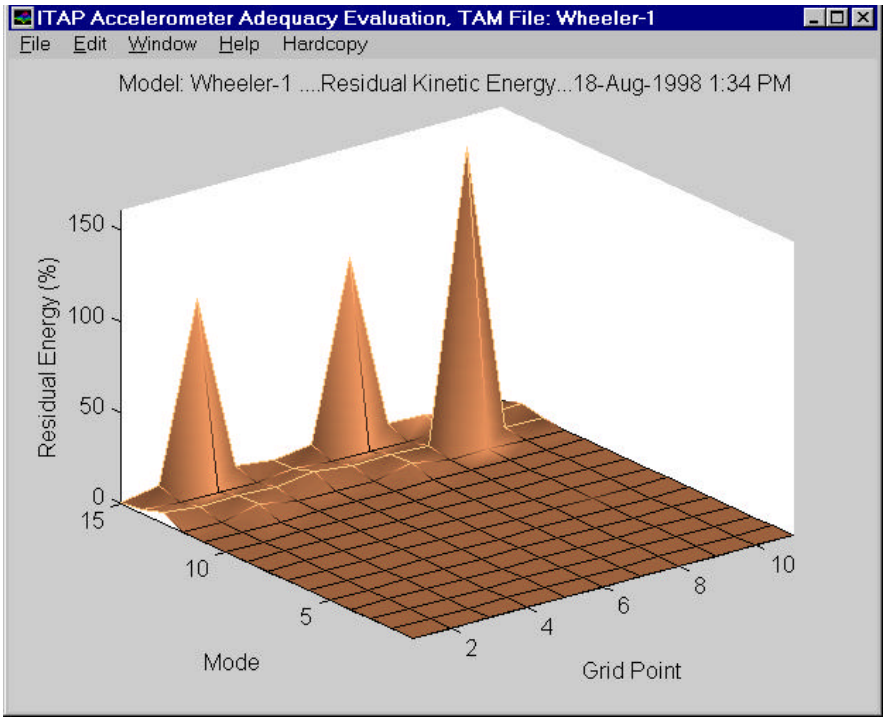
The (11 grid point) finite element model, which was translated in Chapter 2, is evaluated in this section. This model consists of a ring with a boom, which attaches to diametrically opposing points on the ring, as shown below.

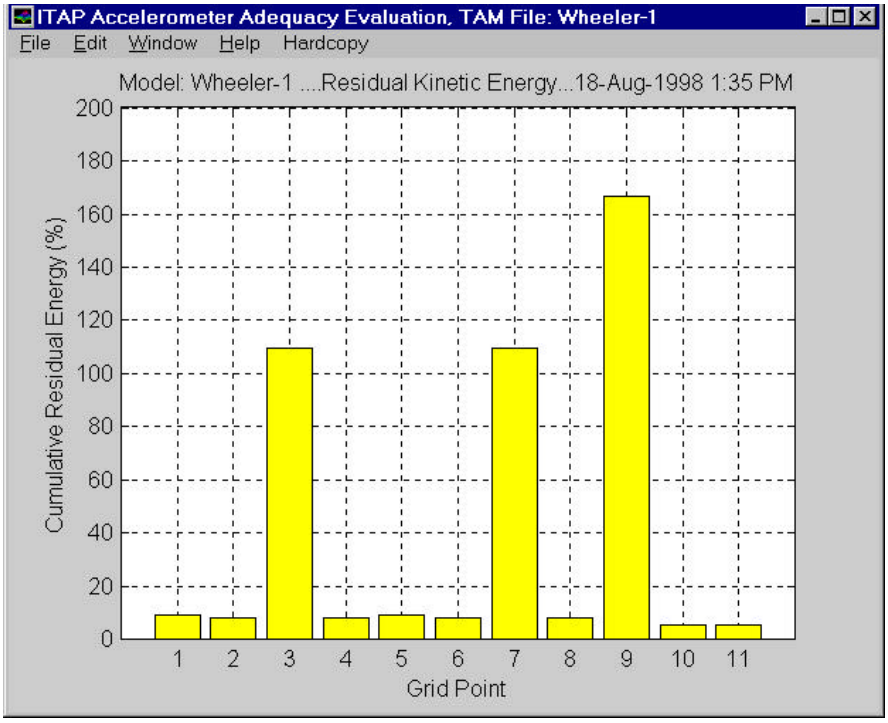


In a "first try" allocation of response dofs, grid points 1, 2, 4, 5, 6, 8, 10, 11 are selected (as "Y" set points in the UAI/NASTRAN Rigid Format Alter analysis). The OUTPUT4 file is translated and **Operation – Response MAP**" is selected (see below)



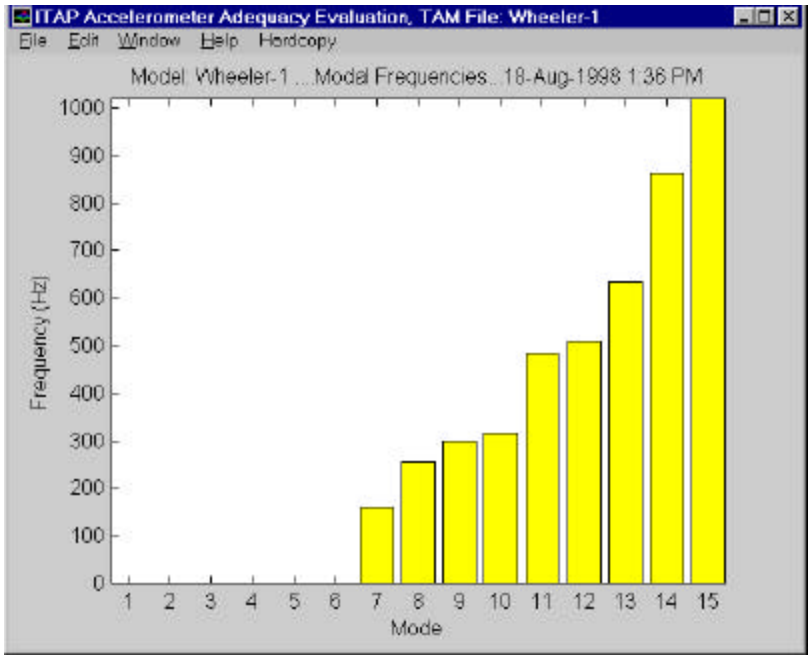
Upon clicking this option, a series of residual kinetic energy displays appear (successive displays are accessed by pressing the “return” or “enter” key) as shown below:



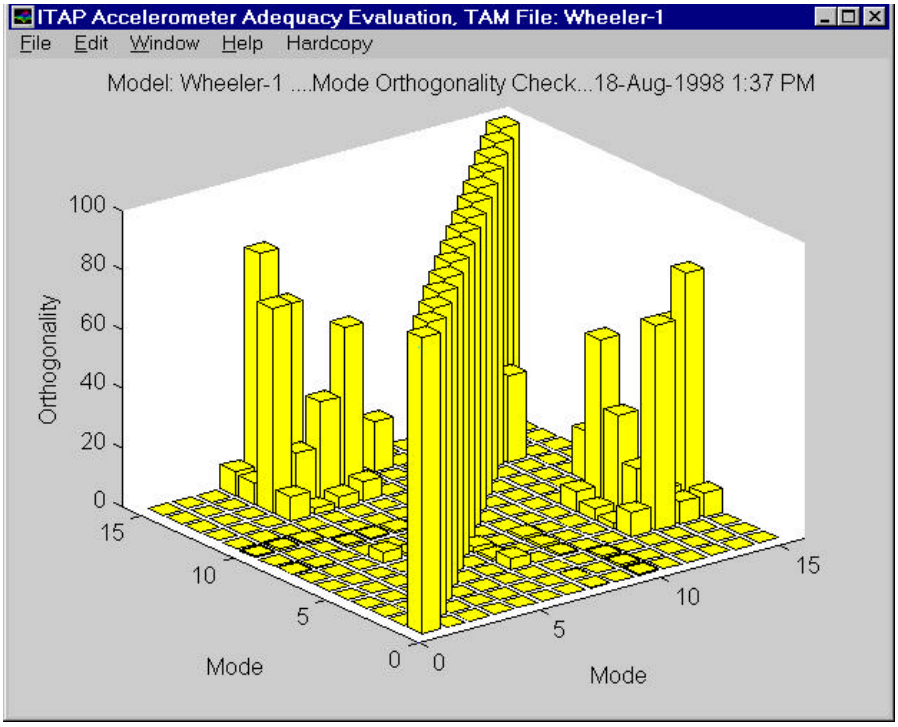


The first “residual kinetic energy display indicates that the “first try” allocation of response dofs is adequate for mapping of modes 1-12. A pronounced residual peak in mode 13 at the FEM grid point 9 indicates that it is required for mapping of this mode. The pronounced peaks in mode 15 at FEM grid points 3 and 7 indicate their significance for mapping of mode 15. The second “cumulative” residual energy display clearly indicates the need to add grid points 3,7 and 9 to the response dof set.

The next clicked display provides a bar graph of the lowest 15 modal frequencies (the first 6 are associated with rigid body modes), shown below:



The next clicked display is the TAM orthogonality check shown below as a 3-D bar graph:

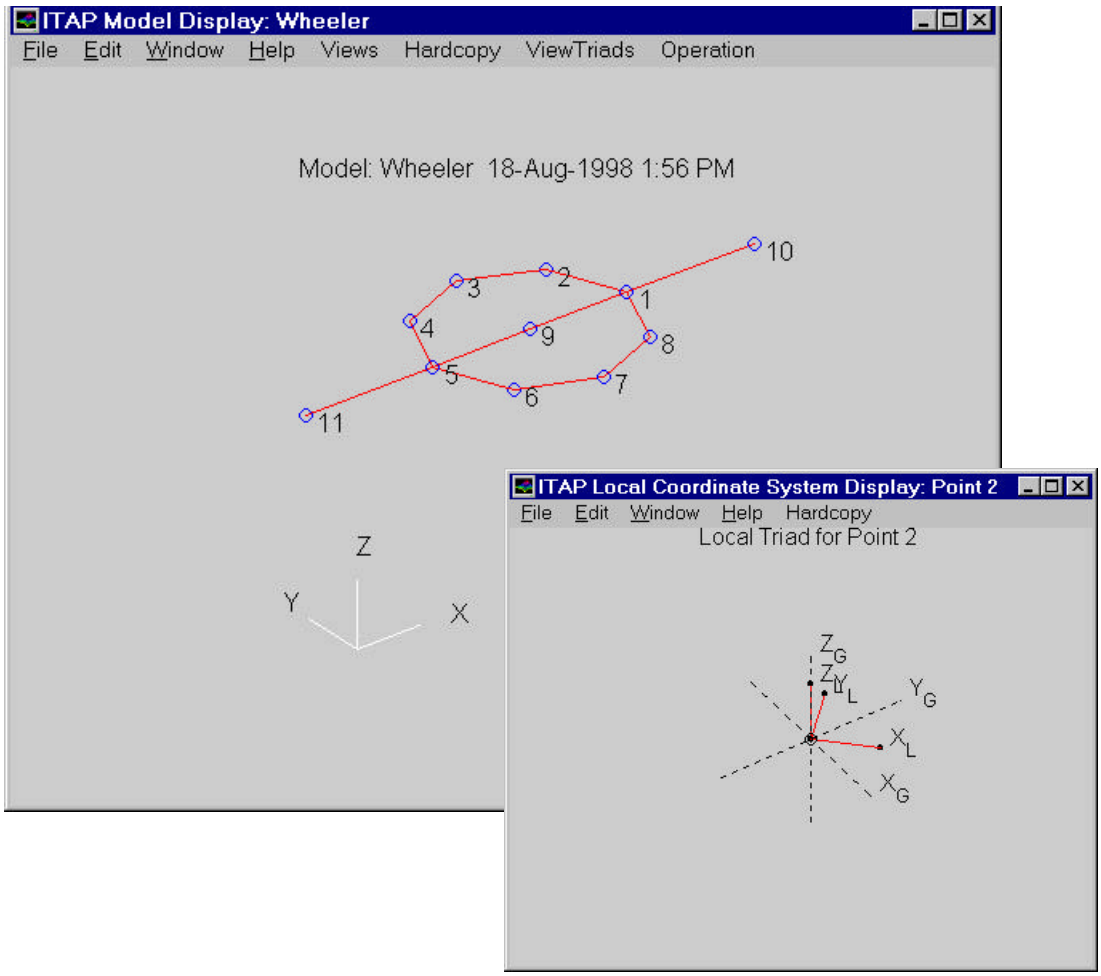


Note that orthogonality among modes 1-12 is excellent with mode 12 exhibiting somewhat reduced, yet satisfactory orthogonality. Orthogonality of modes 13-15, however, is poor due to inadequate instrumentation as indicated by the residual energy plot.

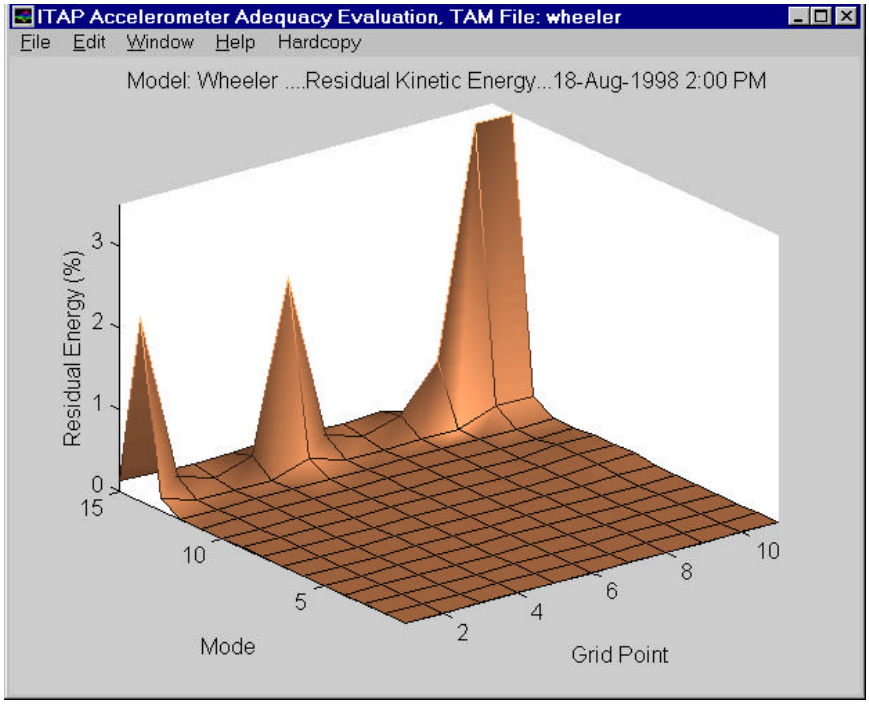
3.2.3 Adequacy of TAM Response "Measurement" Dofs (Revised Set)

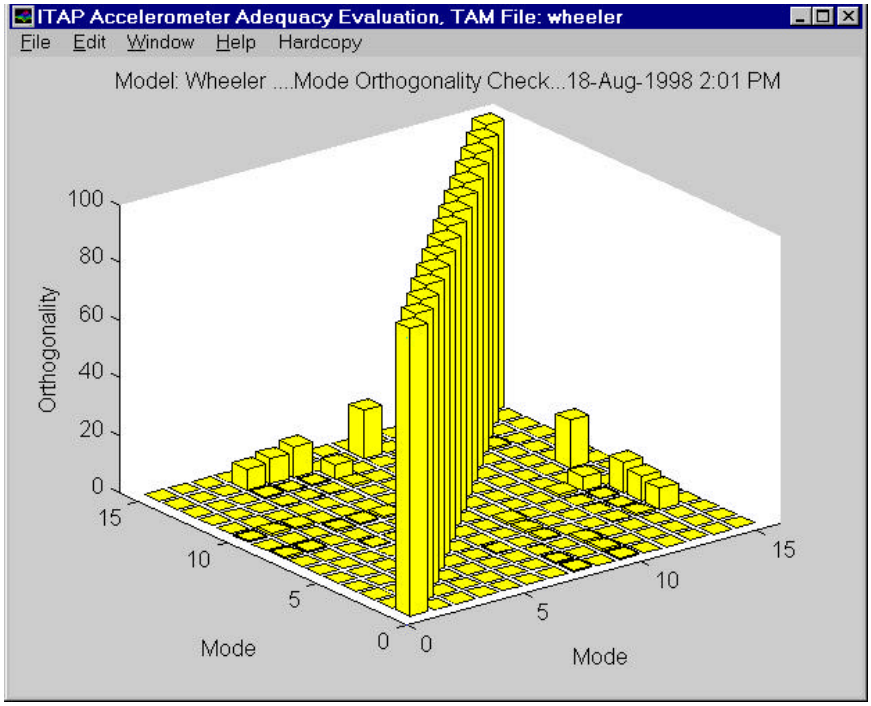
A revised TAM response measurement set, which includes FEM grid points 1,7, and 9, was defined and a second UAI/NASTRAN Rigid Format Alter analysis was completed. Details of the **makmodl** file translation process for this particular example are found in Section 2.5. The reader is reminded that the TAM has been segmented into four "substructures" composed of TAM grid points [1-8], [9],[10], and [11], respectively.

The TAM display shown below illustrates the "**ViewTriads**" feature, which allows the user to review local TAM grid point coordinate systems. Note the local cylindrical coordinate system triad for grid point 2. In order to review all local coordinate systems, the user clicks on that display window until all TAM grid points have been reviewed.

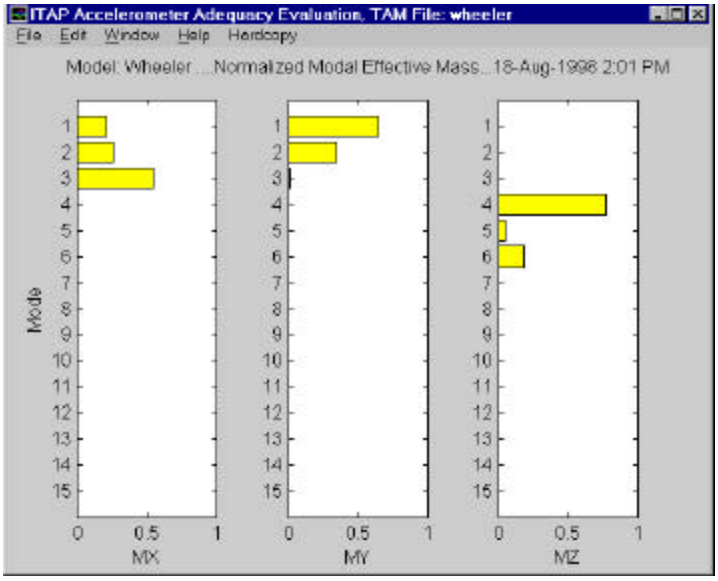


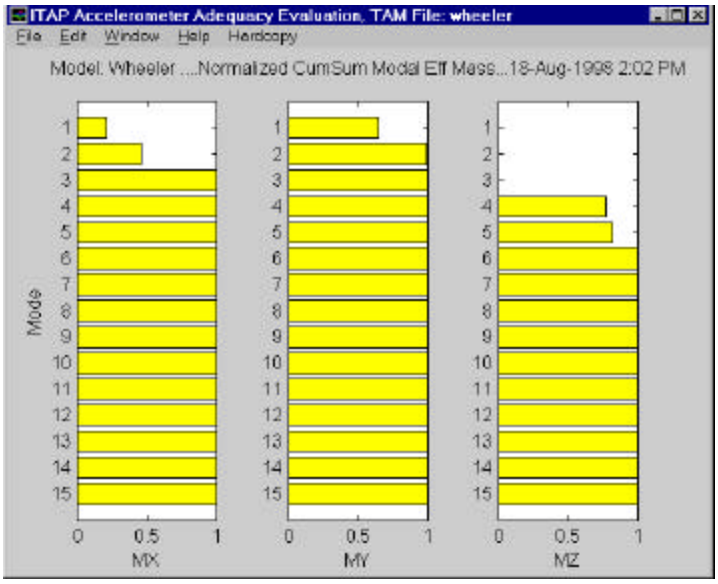
Selection of **Operation – Response MAP**" initiates the review of TAM response dof adequacy. The displays shown below confirm adequacy of the revised response dof set:





The operation is completed after clicking the following modal effective mass displays:





These displays are the result of a particular type of modal effective mass calculation which uses the TAM modes, $[\Phi_{TAM}]$, mass matrix, $[M_{TAM}]$, and rigid body vectors (for global x, y,z, θ_x, θ_y, θ_z motions) $[\Psi_{RY}]$ =[PSIRY], where

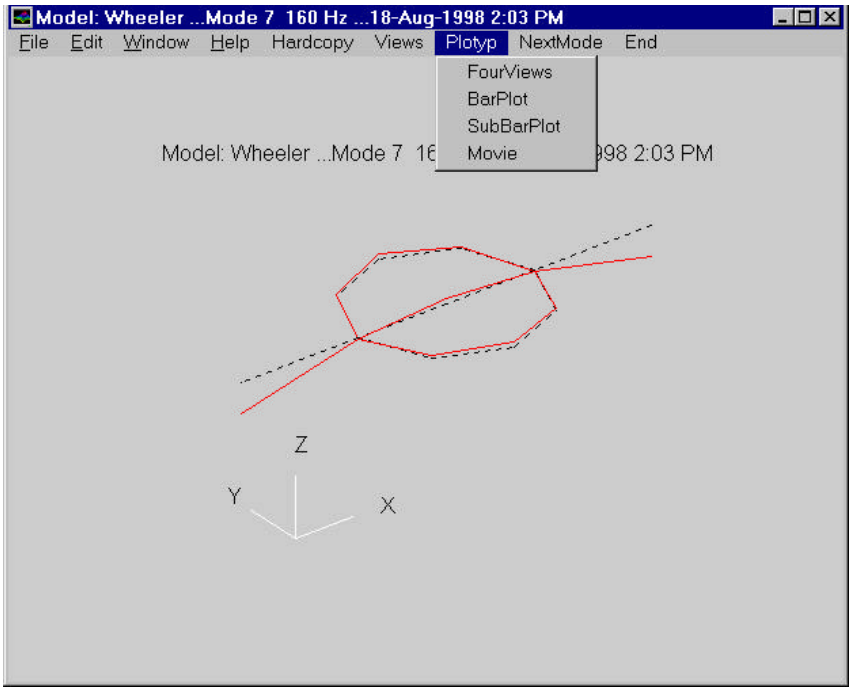
$$[M_{eff}] = ([\Phi_{TAM}]^T [M_{TAM}][\Psi_{RY}]) \otimes ([\Phi_{TAM}]^T [M_{TAM}][\Psi_{RY}])$$

This definition of modal effective mass is a useful indicator of modal "purity" and mode set "completeness". For a free-free TAM, all of the

modal effective mass contributions must be in the first six rigid body modes; no modal effective mass contributions occur in the flexible body modes since they are orthogonal to rigid body vectors (and rigid body modes). Mode set “completeness” is evaluated on the basis of cumulative column summation of modal effective mass.

3.2.4 Review of TAM Modes

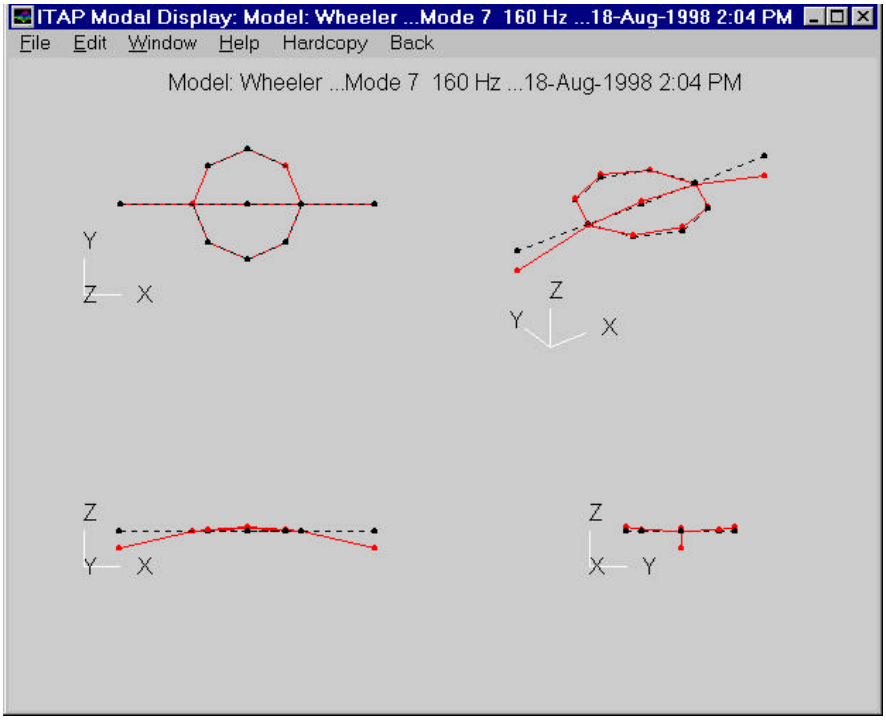
Since adequacy of the TAM response dof set has been established, the modal test planning session continues by selecting **Operation – View Modes**”. By clicking the orthographic modal display, the user scrolls through successive modes. Shown below is the primary display for mode 7 (the first flexible body mode):



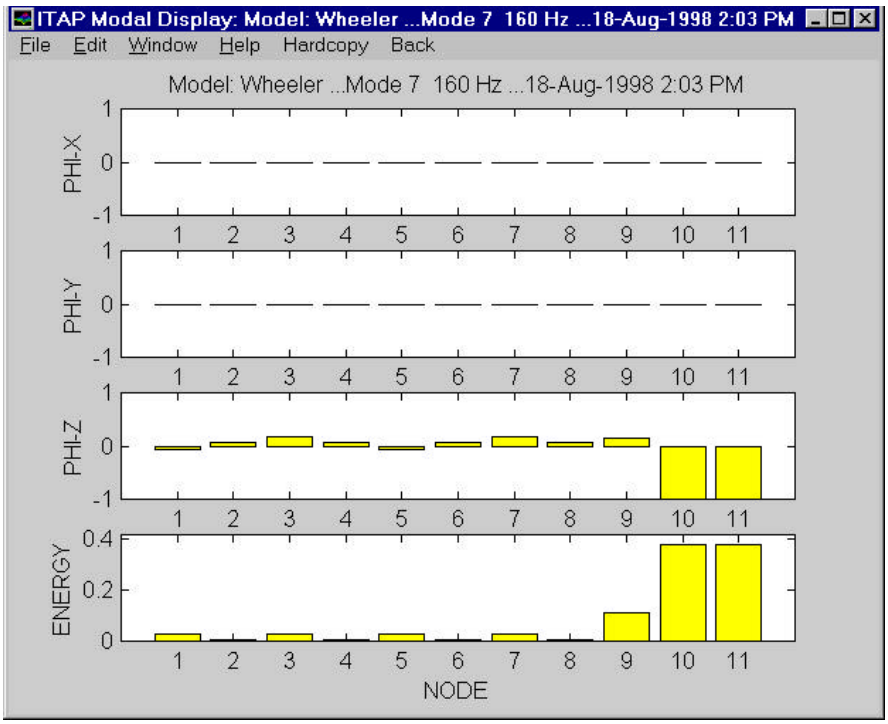
By selecting the “**Plotyp**” menu pick, the user may access various mode descriptions.

Shown below are the various display options for mode 7:

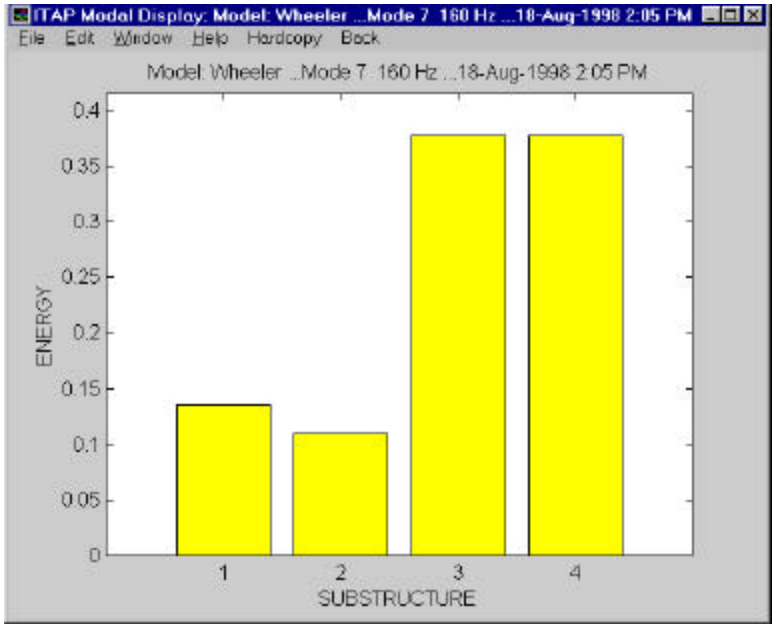
"Four Views"



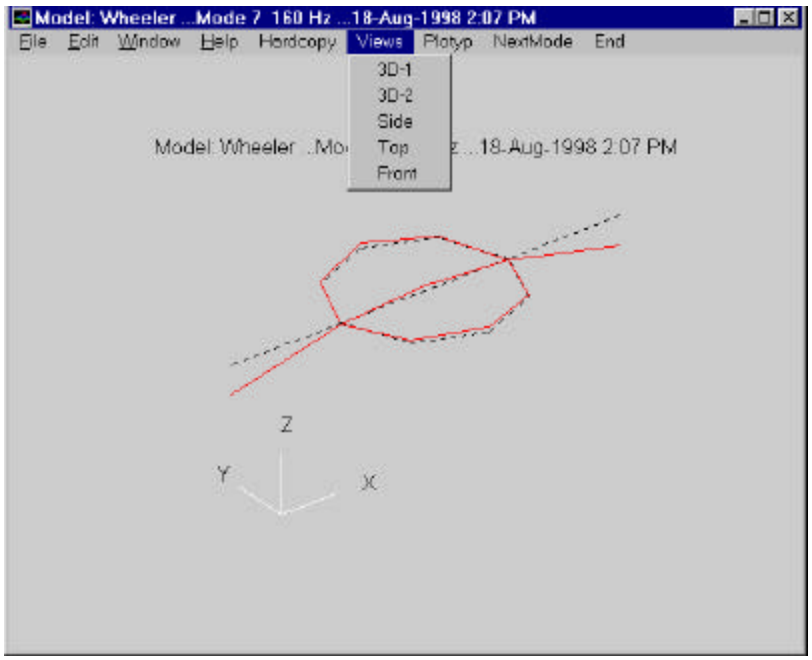
"Bar Plot"



"Sub Bar Plot" (Substructure Breakdown of Kinetic Energy)

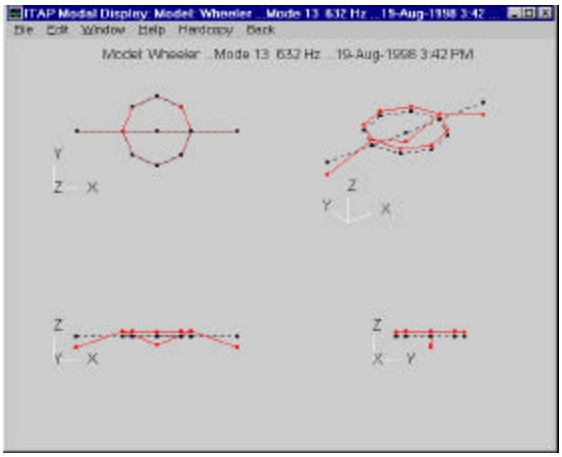


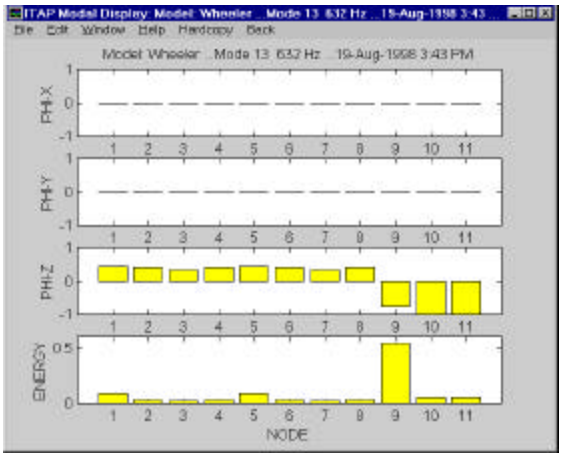
In order to go back to the primary mode display menu, the user clicks on the "**Back**" menu pick. Finally, the user may select different views of the basic modal display by clicking the "**Views**" menu pick as shown below:



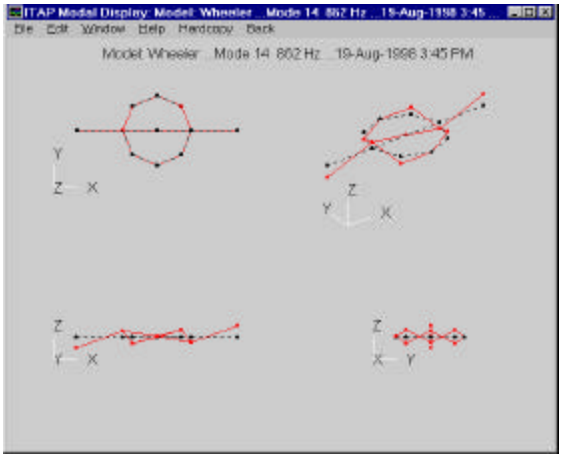
It is of particular interest to review the characteristics of modes 13-15 in order to understand why grid points 3, 7 and 9 need to be included in the TAM. Relevant displays of those modes are provided below:

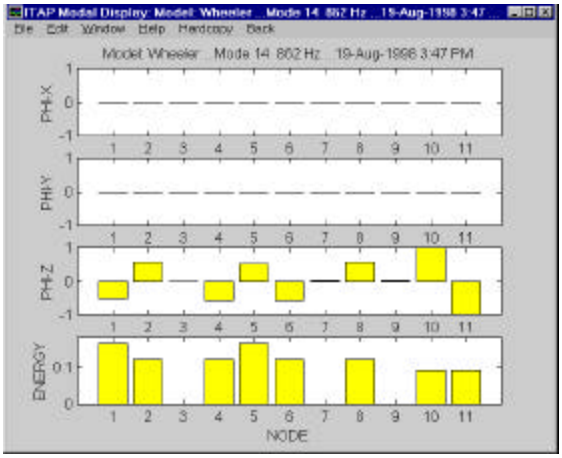
Mode 13 – Grid Point 9 is a Major Contributor to Kinetic Energy



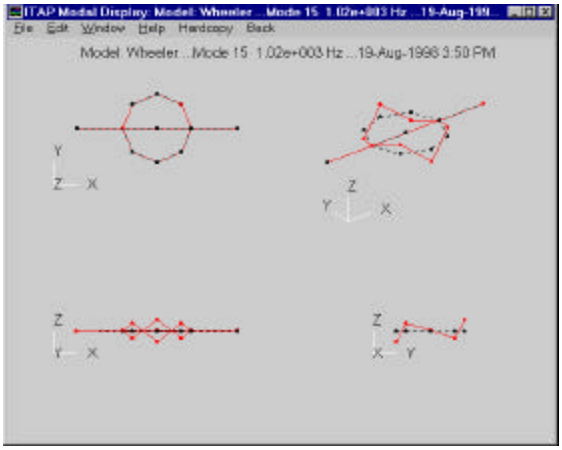


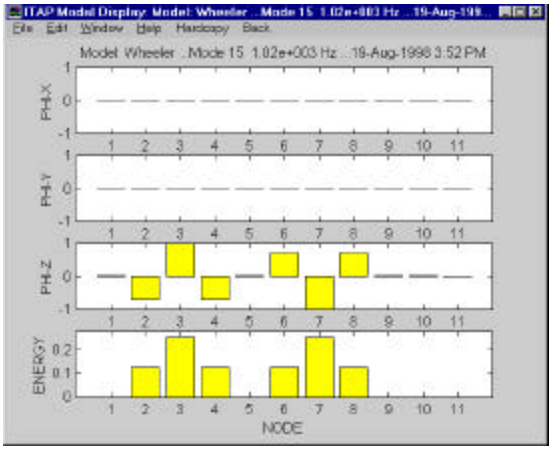
Mode 14 – Grid Points 3, 7, and 9 do not Contribute to Kinetic Energy





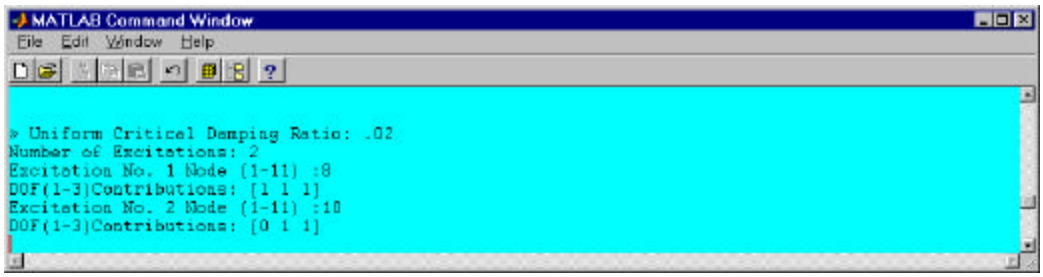
Mode 15 – Grid Points 3 and 7 are Major Contributors to Kinetic Energy





3.2.5 Allocation of Excitation Resources

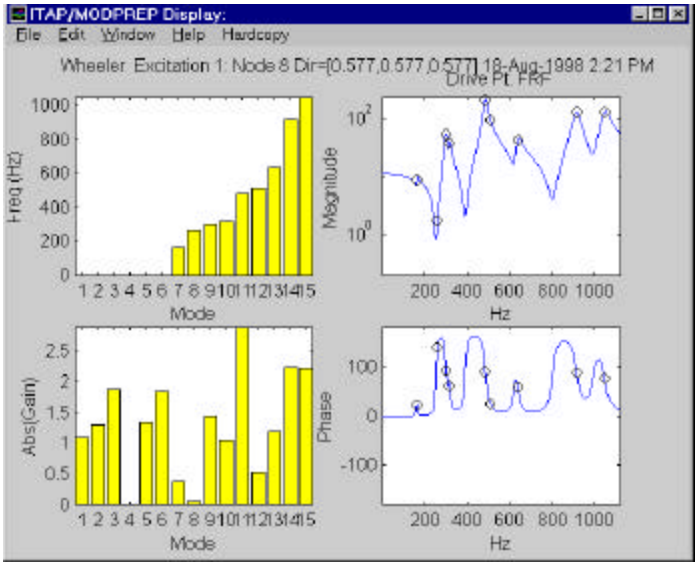
Review of each TAM mode, on an individual basis, provides an indication on how to best excite the particular mode, i.e., at locations, which have significant modal displacement. In order to effectively and efficiently measure system frequency responses, from which all modes of interest may be estimated, accessible excitation locations and directions must be established and verified as part of the modal test plan. To perform this test planning task, the user clicks on the **Operation – Excitation MAP**" button and responds to requests in the MATLAB command window as shown below:



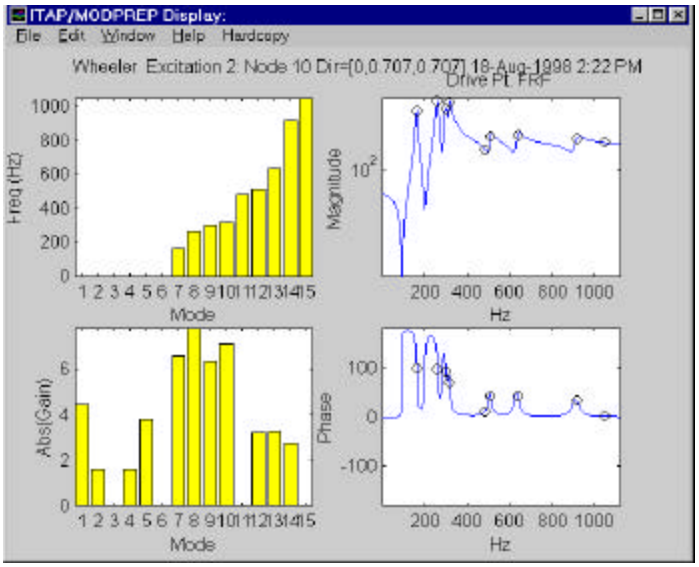
The user specified uniform critical modal damping ratio (.02) and defined two exciter locations, namely at (1) TAM grid point 8 (directed with equal contributions in the local x, y, and z axis), and (2) TAM grid point 10 (directed with equal components in the local y and z axis).

TAM modal frequency response (including all TAM modes) is then calculated for each excitation location. Attributes of modal gain, $[\Phi^T\Gamma]$, and drive point frequency response are then displayed for each location (invoked by thew user depressing the "return" or enter "key"). Displays associated with the two specified excitation locations are shown below:

Excitation Location 1

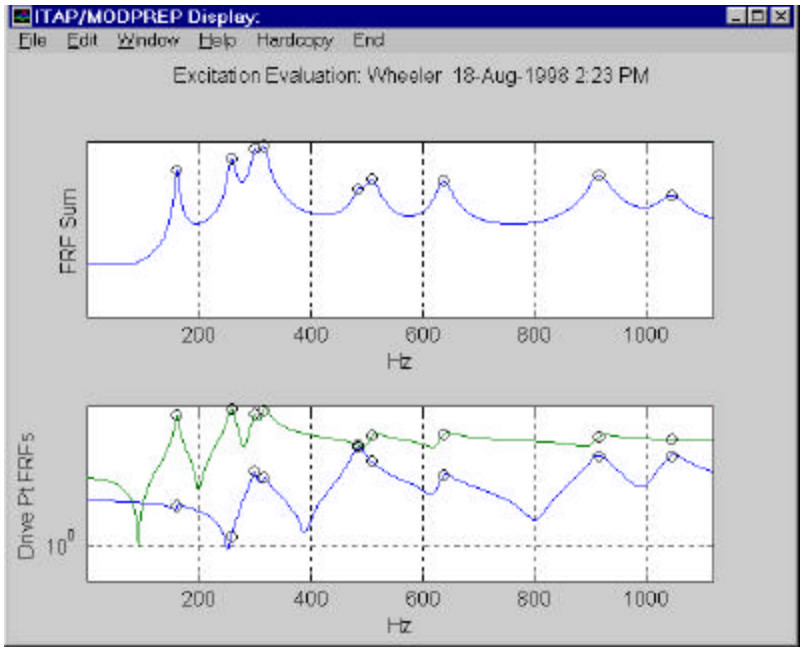


Excitation Location 2



Each of the above displays indicates modal frequencies and modal gains in bar graph format as well as drive point frequency response magnitude and phase. TAM natural frequency points are noted in the frequency response plots with circle (o) symbols. Review of these displays helps the user to evaluate effectiveness of the selected excitations. Note that Excitation location 1 produces clear response in modes 9-11 and 13-15 while excitation location 2 produces clear response in modes 7-10 and 12-14.

The final summary display provided by the “Response MAP” task is shown below:



The upper graph represents the sum of all frequency response function magnitudes associated with both excitations. The lower graph illustrates the two driv point frequency response magnitudes. Review of this display indicates that all TAM flexible body modes (mode 7-15) are sufficiently excited, thus completing the modal test planning process.

Note that the user may repeat any desired “**Operation**” options during a **modprep2** session in order to complete a test plan.

4.0 Modal Test Data Evaluation

4.1 Accessing Modal Test Data

4.2 ITAP-A Implementation Summary

4.3 Illustrative Modal Test Data Evaluation Sessions

4.1 Accessing Modal Test Data

Modal test data may come from a variety of sources. A complete test data file may be the result of an ITAP-T session (See Chapter 7) or it may be translated from an external source, which is written in SDRC Universal File (or other) format. It is preferred that modal test data from ITAP-T be utilized, whenever possible, since that approach employs extensive quality checks on measured data throughout the data analysis process.

If modal test data comes from an external source, in SDRC Universal File (or other) format, it is translated to a MATLAB .mat file with the ITAP-A routine, **unvmodal**. The ITAP-A routine, **unvmodal**, is a "place holder" module. It is the user's option to incorporate a translation routine (commercially available or custom written) which satisfies the needs and/or nuances of the supplied "SDRC Universal File" data. See Chapter 5 for further details.

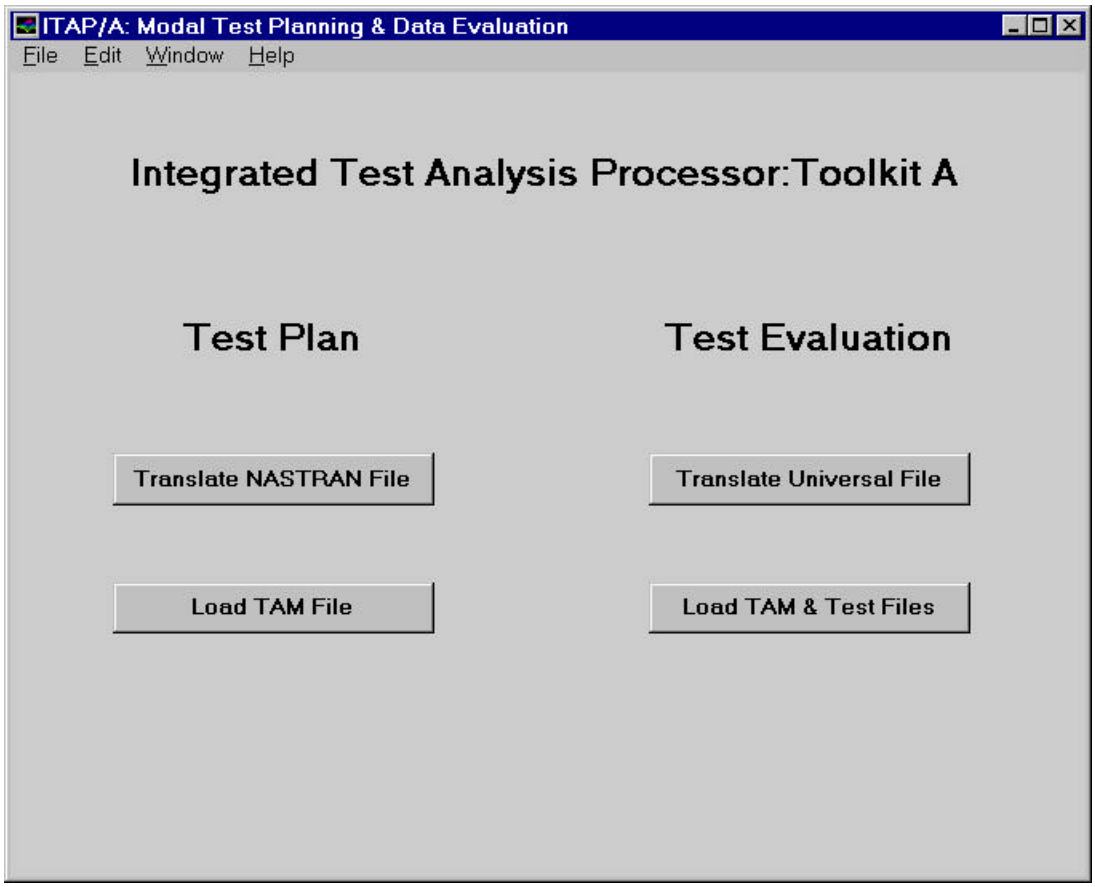
If modal test data comes from ITAP-T, the test data file is already in MATLAB .mat format and ready for modal test data evaluation in ITAP-A.

The following matrices comprise the minimum necessary data set for ITAP-A modal test data evaluation:

ITAP Matrix Name	Description	Rows	Columns
FREQ	Modal Frequencies	modes	1
ZETA	Modal Damping (ζ)	modes	1
PHI	Mode Matrix	channels	modes

4.2 ITAP-A Implementation Summary

From the MATLAB command window, the user types "**itapa**" to initiate a modal test data evaluation session. A screen with four push-button choices appears as shown below:



If the upper right button (Translate Universal File) is pressed, the module **unvmodal** is invoked to translate modal test data from SDRC Universal File (or other) format to the appropriate ITAP-A format. If the modal test file is already in the appropriate ITAP format, the lower right button may be pressed to initiate modal test data evaluation (**modtest2**).

The major operational steps performed by **modtest2** are as follows:

1. Either (a) translate SDRC Universal File (or other format) data to define an ITAP Modal Test Data file, or (b) access an existing Modal Test Data file.

2. A model geometry display is generated. The user may elect to perform display and analysis operations by selecting one of the following menu options (which may be accessed in any user desired sequence):

   (a) **Views** (choices are - 3D-1, 3D-2, Side, Top, Front)

   (b) **Hardcopy**

   (c) **View Triads** (This option allows the user to review local displacement coordinate system triads)

   (d) **Operation** (choices are – "**Test Quality**", "**View Modes**", "**Correlation**", "**End Session**")

3. Evaluate overall quality and content of modal test data by selecting the **Operation - Test Quality**" option on the model display window. The theoretical considerations described in Section 1.5 form the basis of this operation.

4. Study the character and content of each Modal Test Data file mode by selecting the **Operation – View Modes**" option. Various display options, based on the theoretical considerations, which are described in Section 1.3, may be accessed by the user.

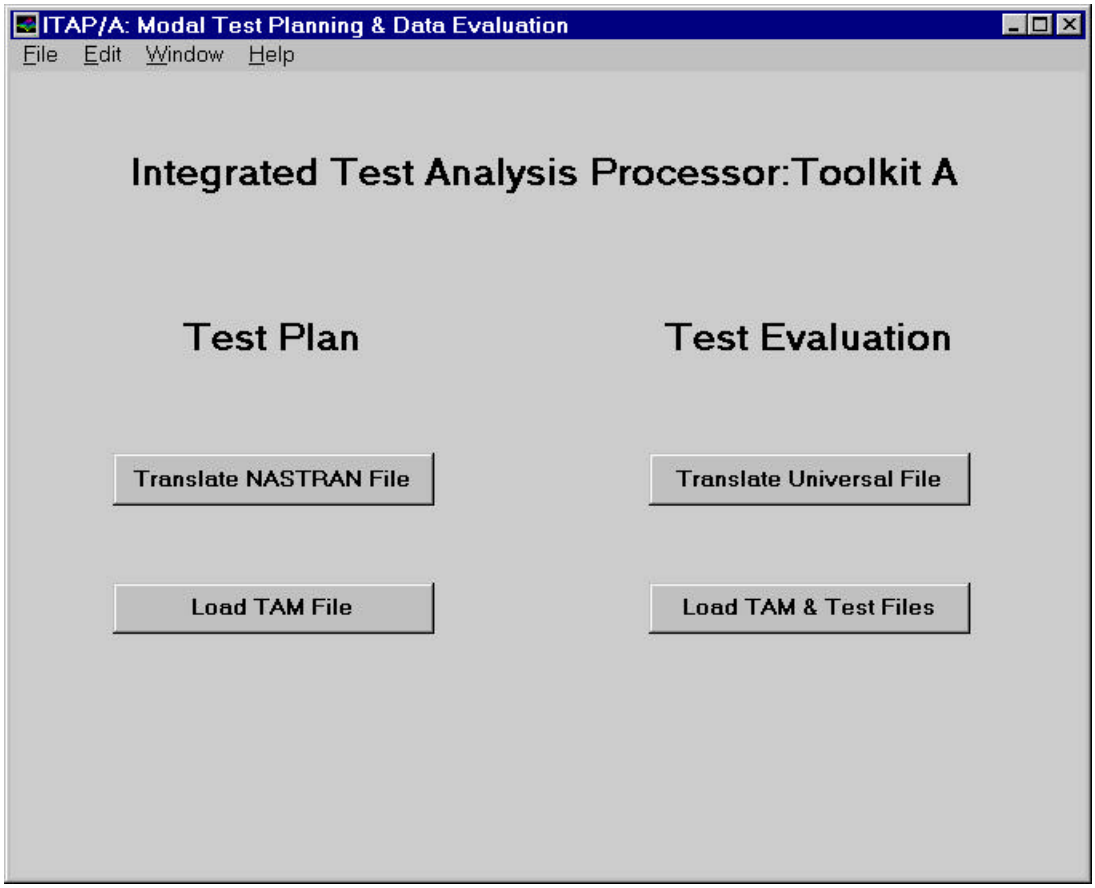
5. Review of corresponding test and TAM modes by selecting the "**Operation – Correlation**" option. Corresponding test and TAM modal pairs are automatically grouped based on the maximum absolute value along each column of the cross-orthogonality matrix.

The interactive operations performed by **modtest2** are demonstrated, with a simple case structure, in the following section.

4.3 Illustrative Modal Test Data Evaluation Sessions

4.3.1 SDRC Universal File Translation or ITAP-A Modal Test File Access

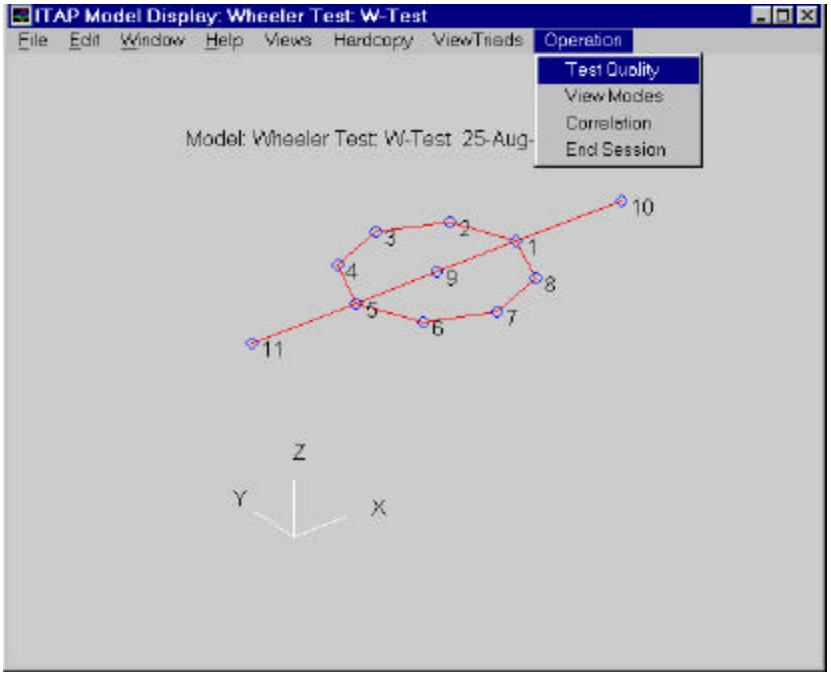
From the MATLAB command window, the user types "**itapa**" to initiate the modal test data evaluation session. A screen with four push-button choices appears as shown below:



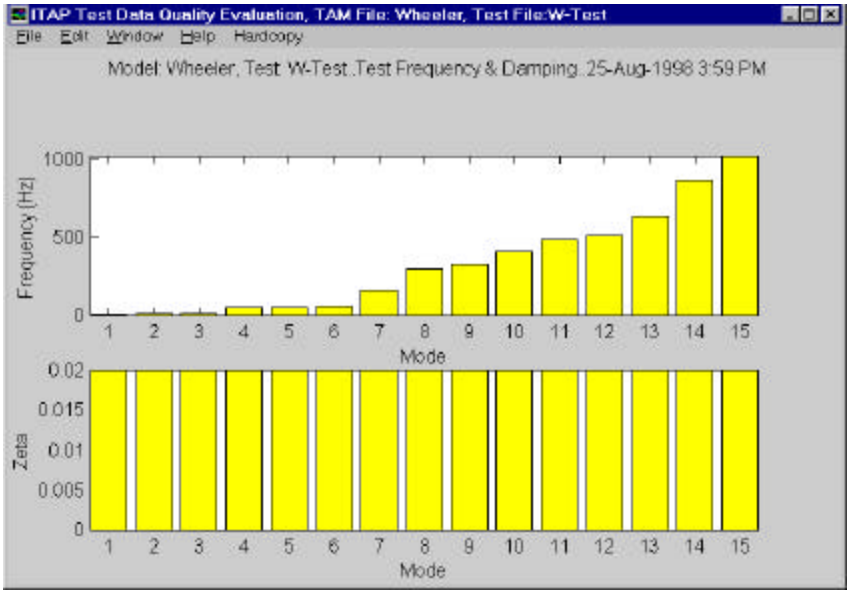
If the upper right button (Translate Universal File) is pressed, the module **unvmodal** is invoked, and an ITAP-A Modal Test Data file is created. The user is then prompted to proceed with further **modtest** operations. If the lower right button (Load TAM & Test Files) is pressed, TAM and ITAP-A Modal Test Data files are accessed followed by **modtest** operations

4.3.2 Modal Test Data Quality and Content

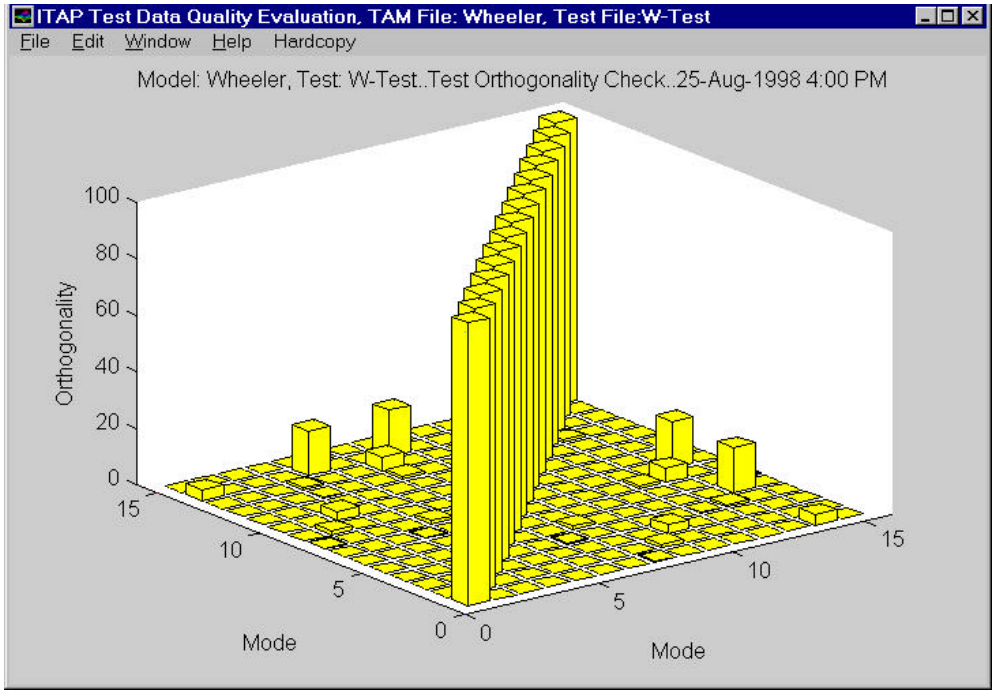
After the TAM and ITAP-A Modal Test Data files have been accessed, the model geometry display, shown below, is generated:



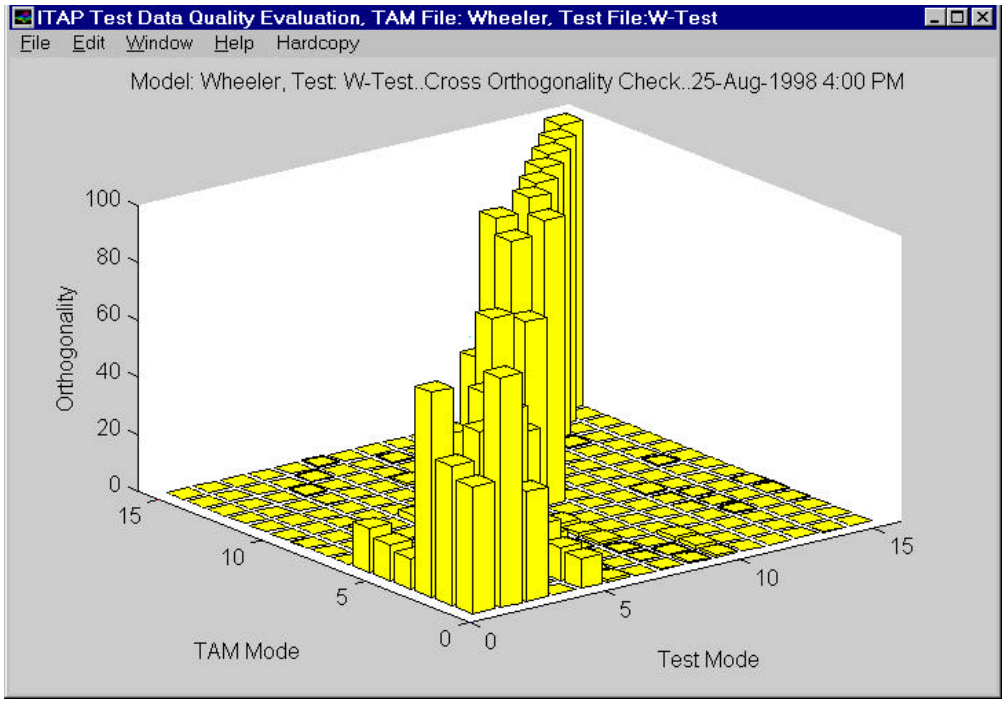
Upon clicking the “Test Quality” option, the modal test natural frequencies and critical damping ratios are displayed as shown below:



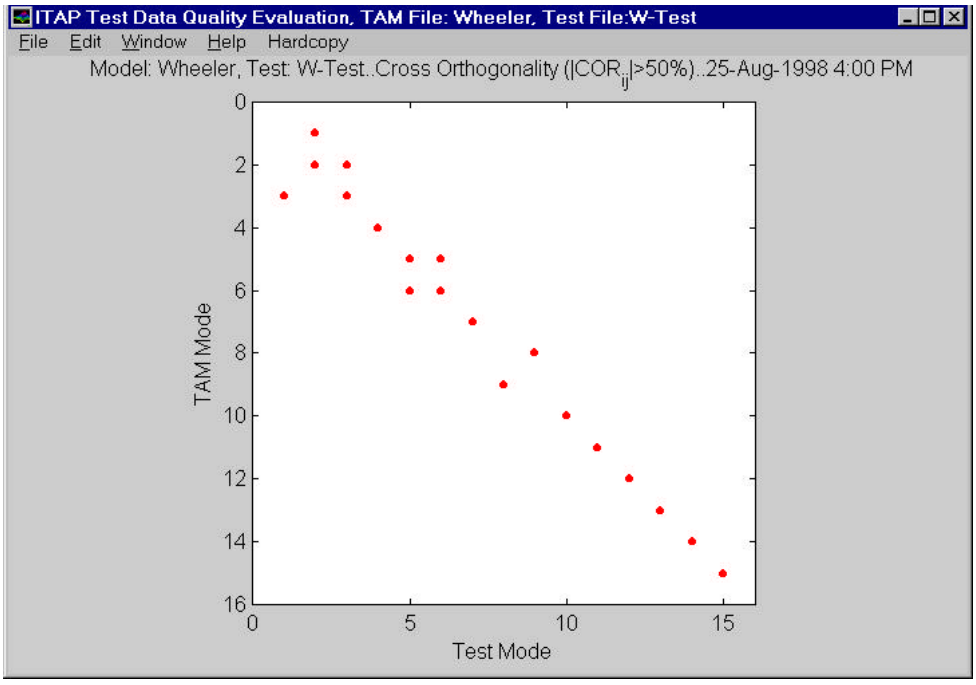
Upon pressing the return key, the test mode orthogonality check (which uses the TAM mass matrix) is displayed in 3-D bar graph format, as shown below:



After pressing the return key again, the TAM, test mode orthogonality matrix is displayed as shown below:

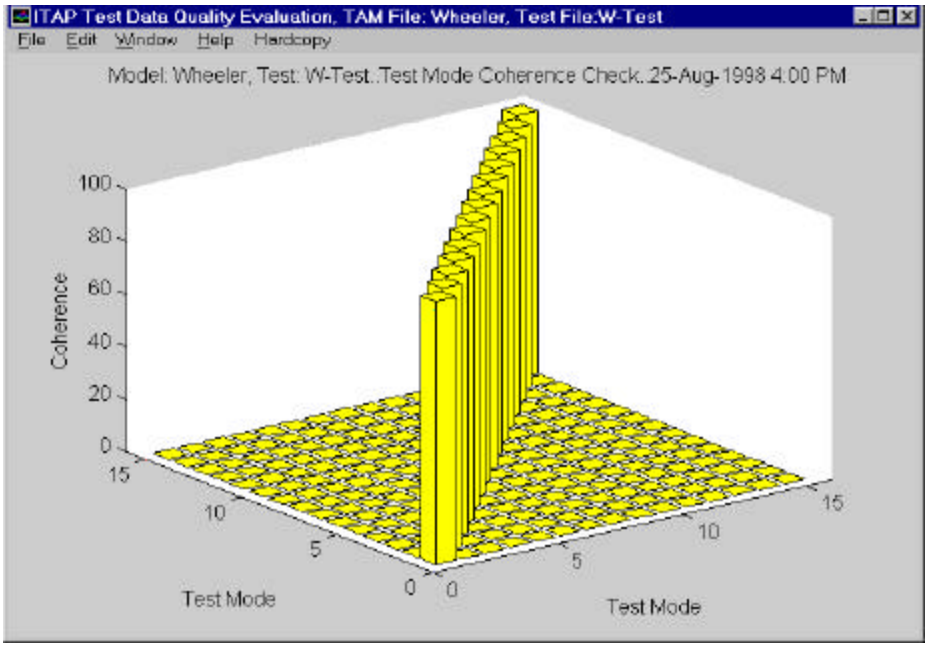


In order to clarify and highlight key cross-orthogonality information, the following MATLAB “spy” format display is generated (by pressing the return key):

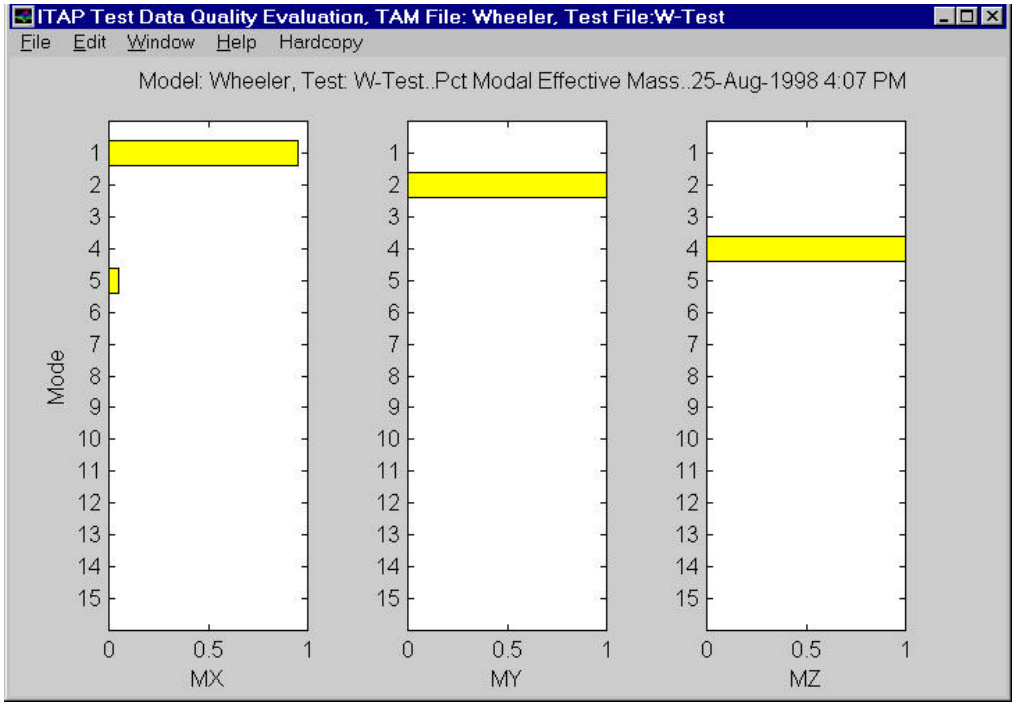


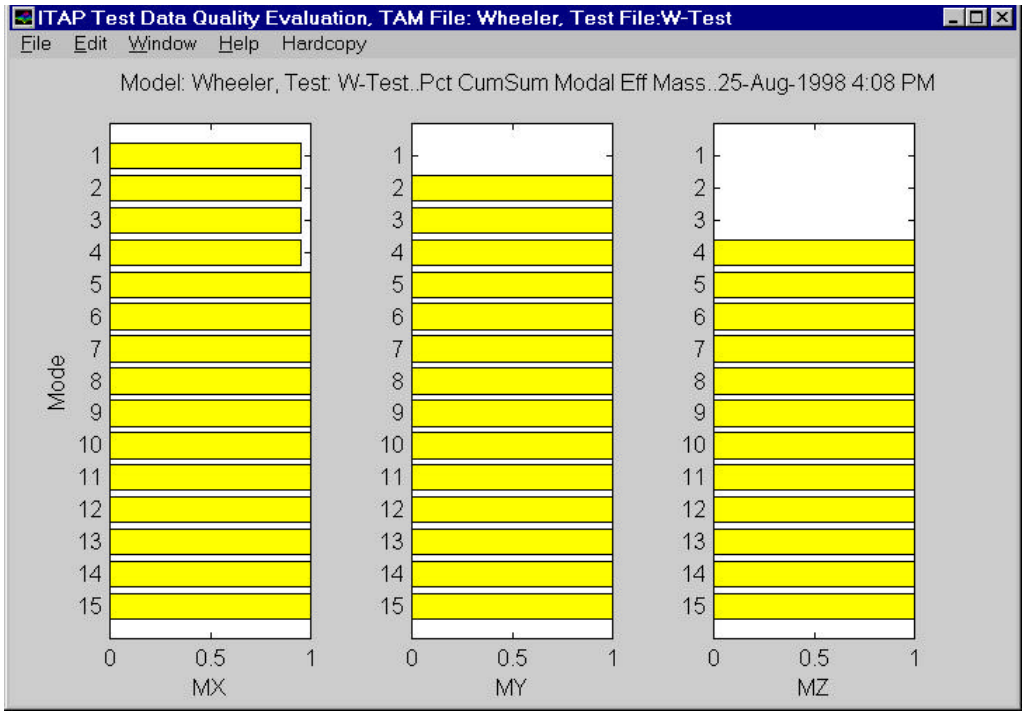
The above display indicates terms in the cross-orthogonality matrix with magnitude greater than 50%.

Again pressing the return key, the modal coherence matrix is displayed as shown below:



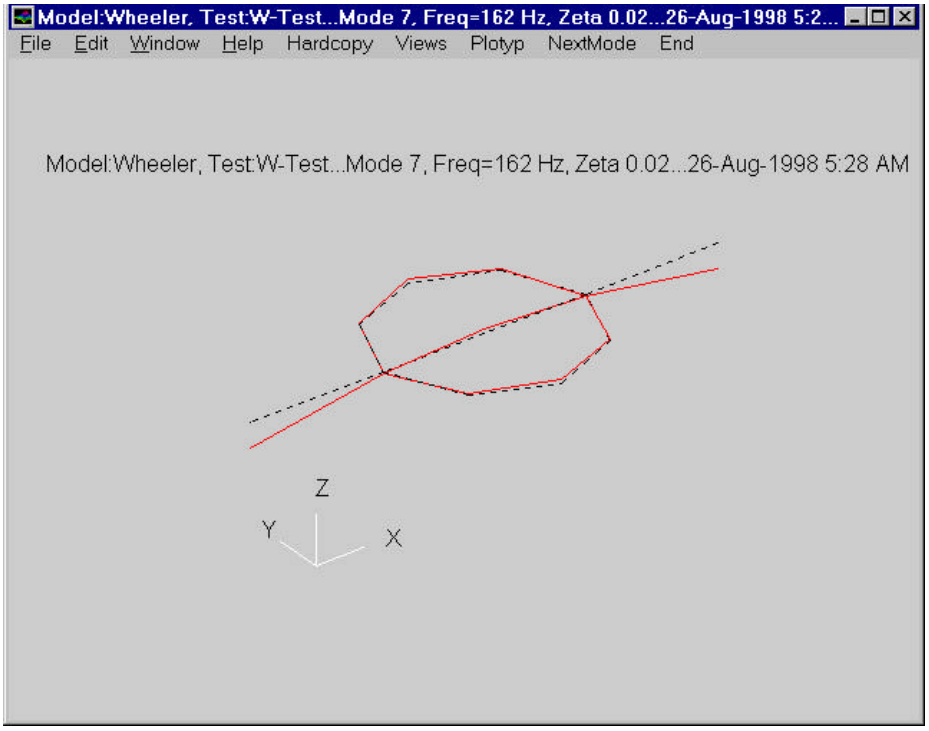
Finally, the modal effective mass and cumulative sum modal effective mass, respectively, are displayed (as shown below) by successively pressing the return key.





4.3.3 Review of Test Modes

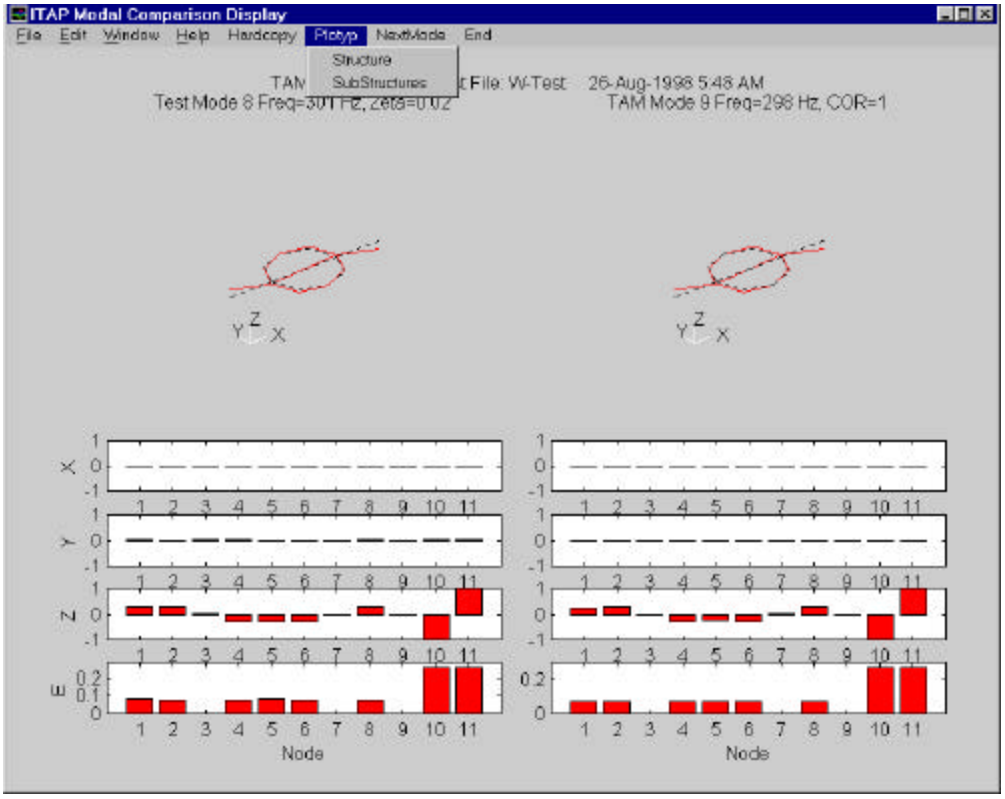
The **modtest2** session continues by selecting "**Operation – View Modes**" on the model geometry display. Successive modes are then displayed by repeatedly clicking the "**NextMode**" menu pick. Shown below is a typical orthographic modal display.



The modal display options for test modes are identical to those offered in modal test planning, **modprep2**, which are described in Subsection 3.2.4.

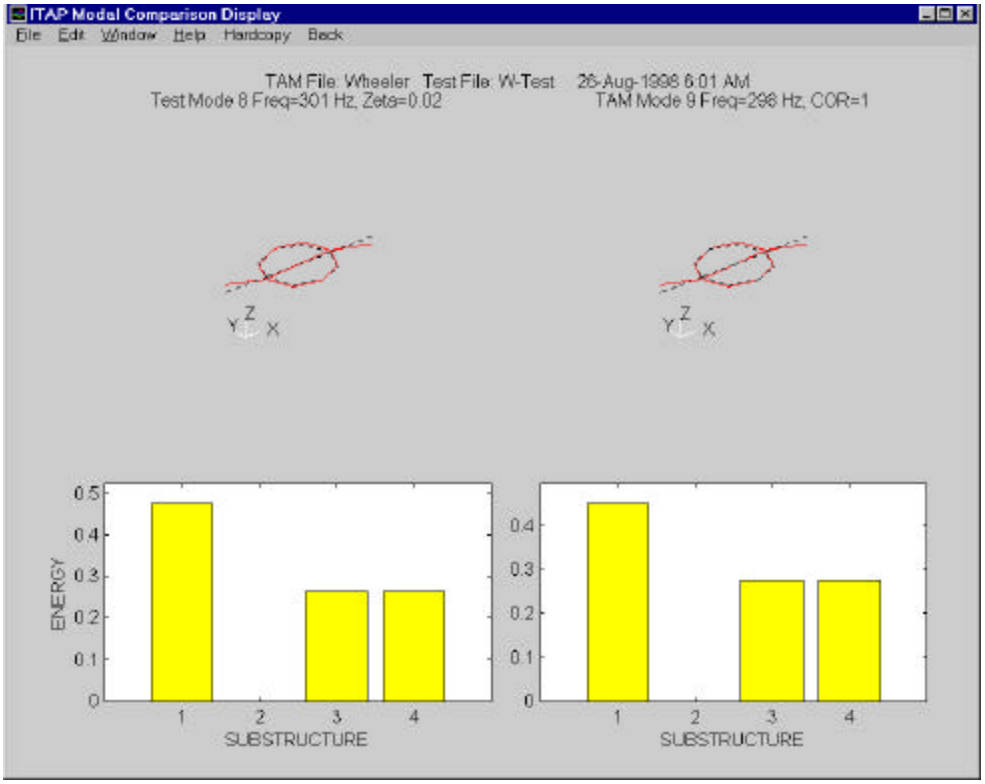
4.3.4 Correlation of Test and TAM Modes

A detailed review of corresponding test and TAM modes may be initiated by selecting the "**Operation – Correlation**" menu pick on the model geometry display. Corresponding test and TAM modal pairs are automatically grouped based on the maximum absolute value along each column of the cross-orthogonality matrix. Successive test and TAM modal pairs are accessed by repeatedly clicking the "**NextMode**" menu pick. Shown below is a typical basic mode comparison display:



Each basic modal comparison display illustrates an orthographic geometry plot and mode bar graph set (local x, y, z displacements and kinetic energy) for the test and TAM mode.

Two choices of mode comparison bar graph plot type are offered, namely, (a) a nodal breakdown ("**Plotyp – Structure**" which is also the default) as shown above, or (b) substructure kinetic energy distribution ("**Plotyp – Substructure**") as shown below:



Note that the user may obtain a hard copy of the display by clicking the "**Hardcopy**" menu pick. In addition, the user may toggle back to the default "**Plotyp – Structure**" display by pressing the "Back" menu pick. The "**Operation – Comparison**" session is completed either when all test modes have been viewed or when the user clicks the "**End**" menu pick.

5.0 ITAP-A Function Modules

ITAP-A serves as (a) a utility for modal test planning and modal test data evaluation and (b) a MATLAB-based dynamic analysis toolbox. In the former capacity, ITAP-A may be employed as a self-sufficient interactive utility for test planning and post-test data evaluation. As a toolbox, ITAP-A modules may be assembled in user defined .m files to perform a wide variety of dynamic simulation and data evaluation tasks.

While the advanced ITAP-A user may develop dynamic models within the MATLAB environment, UAI/NASTRAN (or other NASTRAN versions) offer a powerful means for development of comprehensive dynamic models. Such models are accessed by an ITAP-A based, general purpose (NASTRAN) OUTPUT4 file translator.

Modal test data may be accessed from MATLAB .mat files, most likely developed from ITAP-T data analysis or translated from an external source originally written in SDRC Universal File (or other) format.

This Chapter provides a description of all user accessible ITAP-A function modules.

5.1 Overview of ITAP-A Function Modules by Category

The ITAP-A toolbox is composed of function modules, which fall into four distinct categories as grouped in the tables provided below:

Modal Test Planning and Modal Test Data Evaluation

Module	Description
accrevu	Evaluate adequacy of an accelerometer array
cmodplot	Modal comparison display menu
conmap	Generate a TAM nodal connection map
conplot	Generate a TAM geometry display
itapa	ITAP-A modal test plan and test data evaluation menu
makmodl	TAM file assembly from NASTRAN output data
modbar	Generate a modal bar graph display
modplots	Modal plot generation menu (for test planning)
modpl12	Generate a modal comparison plot display
modprep2	Modal test planning operation selection menu
modtest2	Modal test data evaluation operation selection menu
mprepfrf	Evaluate adequacy of excitation resources
smodbar	Generate a “substructure” modal bar graph display
smodpl12	Generate a “substructure” modal comparison plot display
submap	Allocate TAM substructure nodal groups
tmodplot	Modal plot generation menu (for modal test data)

General File Translation and Display Utilities

Module	Description
frfpall	Generate a frequency response plot display
pmenu	Generate a graphic hard copy
out4read	Translate a NASTRAN OUTPUT4 file to a MATLAB .mat file
unvmodal	Translate SDRC Universal File (or other format) modal data

Modal Data Conditioning Modules

Module	Description
berman	TAM mass and stiffness matrix updating (Berman method)
modcoh	Mode set comparisons (orthogonality, coherence, cross-orthogonality)
modnorm	Modal matrix normalization to unit modal mass
modorth	Modal matrix orthogonalization (Cholesky or spectral method)
modsweep	Modal matrix block-sweep orthogonalization

System Dynamic Analysis Modules

Module	Description
fresp	Direct frequency response for a [M], [B], [K] system
mfresp	Modal frequency response (real mode vectors)
modalr	Real modal solution for a [M], [K] system
modalc	Complex modal solution for a [M], [B], [K] system
modres	Calculate orthonormal residual vectors for a [M], [K], [Γ] system
mtresp	Modal transient response (real modes)
mtrespc	Modal transient response (complex modes)
rresp	MDOF response to a single random excitation autospectrum
sdof	Transient response for a SDOF 2nd order oscillator
sdof0	Transient response for a SDOF “mass” only system
sdof1	Transient response for a SDOF 1st order oscillator
shock	Normalized shock or response spectrum

5.2 ITAP-A Function Module Descriptions

This section provides descriptions of user accessible ITAP-A function modules.

Purpose

Evaluate adequacy of a selected accelerometer array

Syntax

accrevu

Description

accrevu displays results of test-analysis model (TAM) calculations performed in the **makmodl** function module. The theoretical basis for evaluation of a selected accelerometer array is presented in Section 1.6 of this document. Example displays are described in Subsections 3.2.2 and 3.2.3.

The following displays are generated by **accrevu**:

1. residual kinetic energy versus mode and FEM node indices
2. cumulative sum residual kinetic energy versus FEM node index
3. Modal frequency bar graph
4. FEM mode set - TAM mass orthogonality check matrix
5. Normalized modal effective mass
6. Cumulative sum modal effective mass

Example

see Subsections 3.2.2, 3.2.3

Purpose

Optimal Mass and Stiffness Matrix Updates

Syntax

[Mp,Kp]=berman(M,K,Phi,Freq);

Description

berman performs computations on mathematical model mass, **[M]**, and stiffness, **[K]**, matrices for a structural dynamic system utilizing test deduced mode shapes, **[Phi]**, and natural frequencies **{Freq}**. The updated mass, **[Mp],** and stiffness, **[Kp],** matrices are calculated using relationships developed by Berman and Wei.
The updated mass and stiffness matrices are formed such that the test deduced mode shapes and natural frequencies are eigensolutions of the revised system.

The following operations are performed by **berman**:

1. mode normalization to unit modal mass
2. calculate optimally adjusted mass matrix, **[Mp]**
3. calculate optimally adjusted stiffness matrix, **[Kp]**

Note: If the test mode shapes are exactly orthogonal, adjustments will not be made to the mass matrix.

Purpose

Generate a modal comparison plot and display menu

Syntax

cmodplot

Description

cmodplot generates a modal display graphic, which compares two corresponding mode shapes. Each mode is displayed in terms a deformed geometry plot and a bar graph indicating TAM nodal displacement and kinetic energy distribution. In addition, a menu pick is provided to allow the user to display an alternative bar graph indicating TAM substructure kinetic energy distribution. **cmodplot** operates on ITAP-A standard matrix data. The graphical displays are generated with the **modpl12** and **smodpl12** function modules, which are accessed by **cmodplot**.

Example

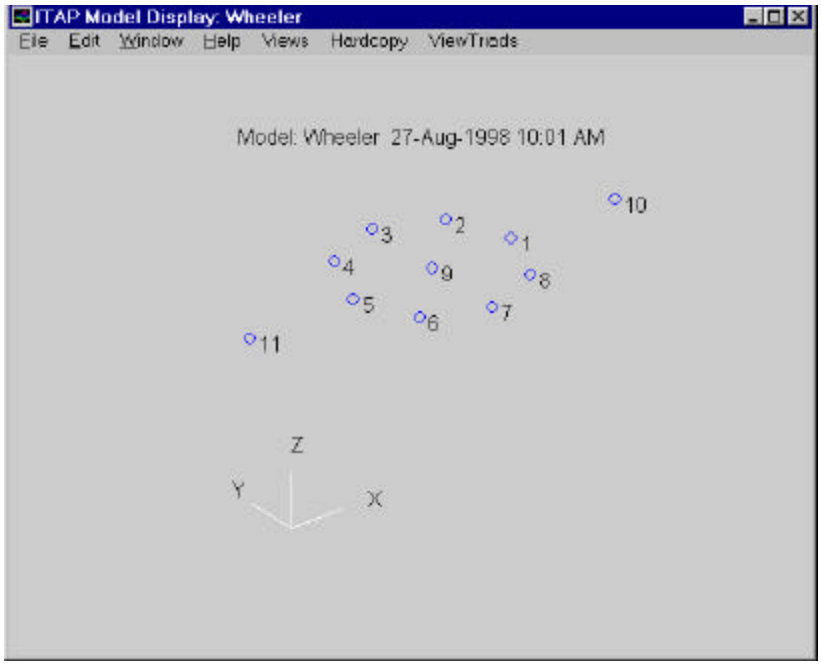
See Subsection 4.3.5

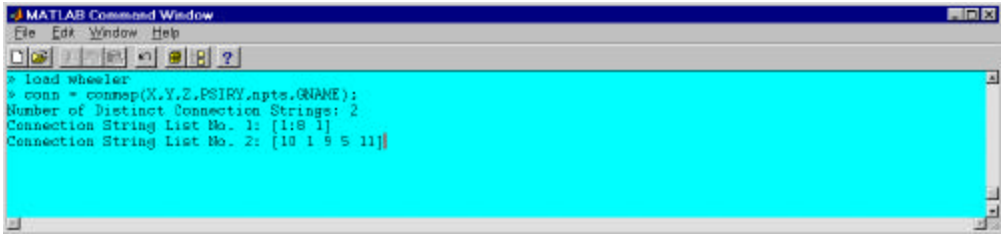
Purpose Generate test-analysis model (TAM) nodal connection map

Syntax **conn = conmap(X,Y,Z,PSIRY,npts,GNAME);**

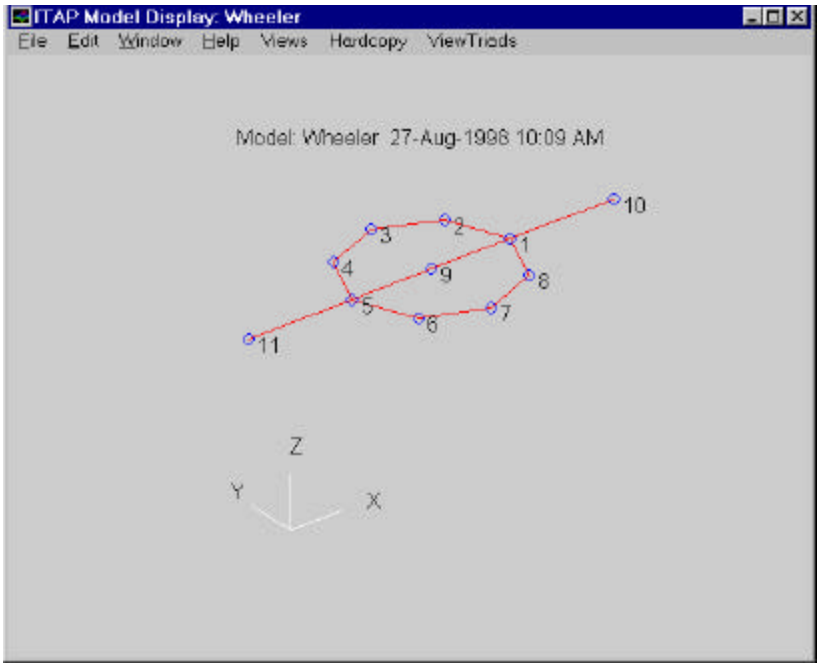
Description **conmap** generates a TAM model geometry display nodal connection string, **conn** from TAM node locations **(X,Y,Z)**, rigid body displacement matrix **[PSIRY],** and number of TAM nodes **(npts)**. GNAME is the TAM file name, which is used in generation of display labels. This routine is called by function module, **makmodl**. It may optionally be invoked, from the MATLAB command window, to redefine TAM geometry display connectivity.

Example Consider the illustrative example, “Wheeler” described in Chapter 3. By invoking the command syntax, after loading the model file, the display, shown below, appears and the user responds to the command prompts as indicated.





The connected geometry display, shown below, is generated:



The generated connection string, conn is as follows:

conn =

Columns 1 through 12

1 2 3 4 5 6 7 8 1 0 10 1

Columns 13 through 16

9 5 11 0

Note that the “zero” is placed at the end of each connection string segment.

Purpose

Generate a test-analysis model (TAM) geometry display

Syntax

conplot(X,Y,Z,PSIRY,conn,npts,GNAME);

Description

conplot generates a TAM model geometry from TAM node locations **(X,Y,Z),** rigid body displacement matrix **[PSIRY**], nodal connection string, **conn**, and number of TAM nodes **(npts). GNAME** is the TAM file name, which is used in generation of display labels. If **conn** is specified as the number, "0", nodes are displayed without connectivity lines. The user may select different view from the "**Views**" menu pick. In addition, the user may review local nodal coordinate system triads by clicking the "**ViewTriads**" menu pick.

Example

See Subection 3.2.3 and the description of **conmap**.

Purpose

Calculate structural dynamic system frequency response.

Syntax

[A,V,U] = fresp(K,M,B,Gam ,f);

Description

fresp calculates the complex frequency response by a direct matrix inversion method. **K, M** and **B** are the system mass, stiffness and damping matrices, respectively. **Gam** is the applied load allocation. **f** is the array of frequencies for which the frequency response solution is to be calculated. **A,V,** and **U** are the solution matrices for acceleration, velocity and displacement, respectively, with each column corresponding to the solution evaluated at a specific frequency, $\omega=2\pi f$.

Steady-state response of a MDOF system subjected to a unit amplitude harmonic force excitation, $F(t) = e^{i\omega t}$, is of the form $\{U(t)\} = \{U(\omega)\}e^{i\omega t}$. Direct substitution of these expressions into the conventional linear structural dynamic formulation results in:

$$[K + i\omega B - \omega^2 M]\{U(\omega)\} = [\Gamma]$$

or

$$\{U(\omega)\} = [K + i\omega B - \omega^2 M]^{-1}[\Gamma]$$

The complex velocity and acceleration amplitude solutions are

$$\{\dot{U}(\omega)\} = i\omega\{U(\omega)\} \qquad \{\ddot{U}(\omega)\} = -\omega^2\{U(\omega)\}$$

Example

Consider the following two dof system:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 50 & -10 \\ -10 & 10 \end{bmatrix},\ K = \begin{bmatrix} 200000 & -100000 \\ -100000 & 100000 \end{bmatrix}$$

$$Gam = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ frq=[1:.25:100]$$

invoke the command

[A,V,U] = fresp(K,M,B,Gam,frq);

Results of this function calculation are displayed in the example discussion for **frfpall**.

References

See Section 1.4.3

Purpose

Generate a frequency response function plot display

Syntax

f12=frfpall(frq,H,comment,f12,opt);

Description

frfpall generates a plot display of a complex frequency response function, **H**, defined over the frequency band, **f**, where frequency is defined in terms of Hz units. The user specifies a two parameter plot frequency range, for example, **f12**=[0 100]. In addition, several plot format options (**opt**) may be specified. In particular,

if **opt=0** display all plot types & select frequency band
if **opt=1** display magnitude & phase
if **opt=2** display real & imaginary parts
if **opt=3** display real versus imaginary parts

When **opt=0** is selected, the output function, **f12**, is the selected frequency band limits.

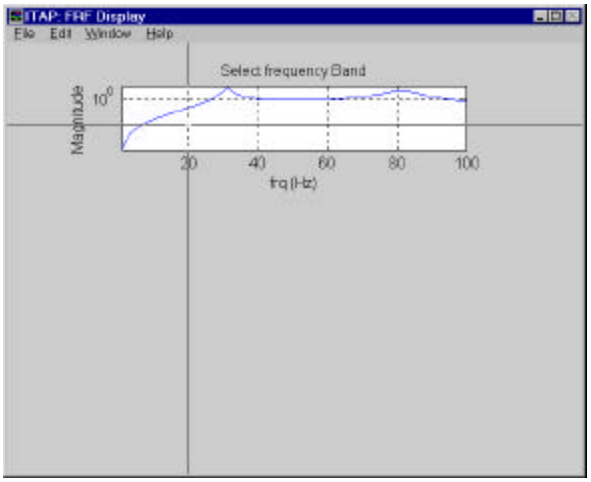
Example

Results of the example analysis from the **fresp** discussion are used here.

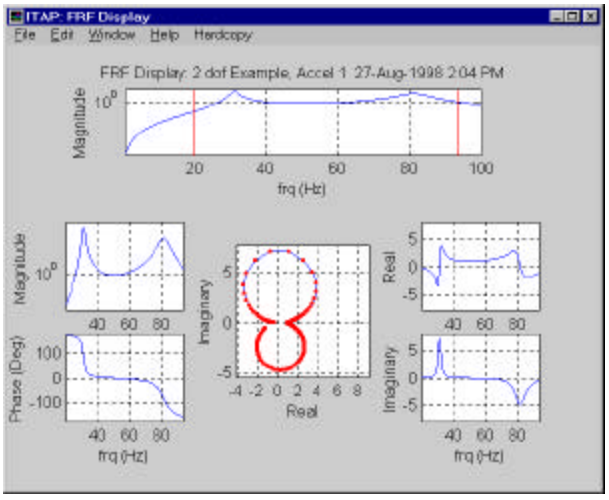
The MATLAB command

f12=frfpall(frq,A(1,:),'2 dof Example, Accel 1',[1 100],0);

is invoked, resulting in the display



The user clicks the cursor twice to define a selected frequency band, and the multiple plot window, shown below Is displayed:

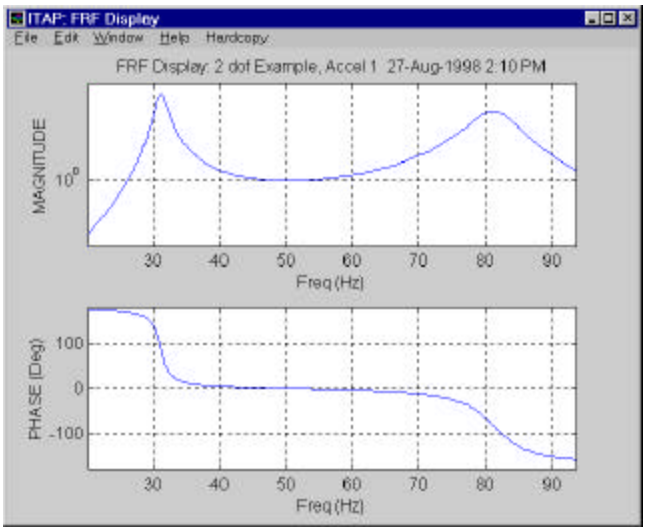


In addition, the selected frequency band, f12 = [20.25 95.50] is output for further processing.

The MATLAB command (using the selected frequency band)

frfpall(frq,A(1,:),'2 dof Example, Accel 1',f12,1);

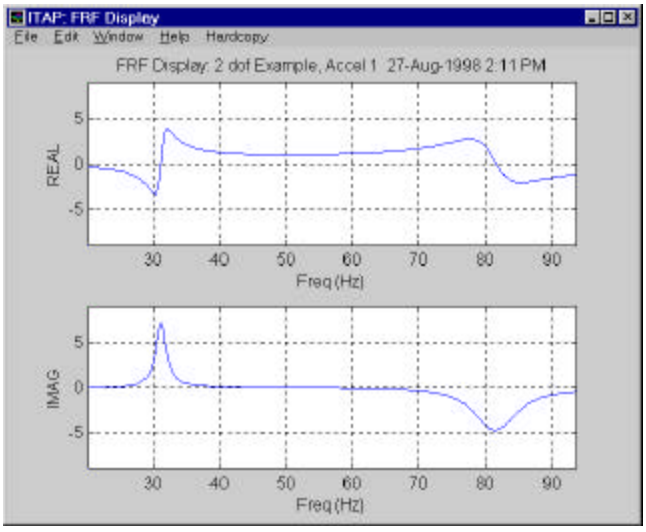
is invoked, resulting in the display



The MATLAB command (using the selected frequency band)

frfpall(frq,A(1,:),'2 dof Example, Accel 1',f12,2);

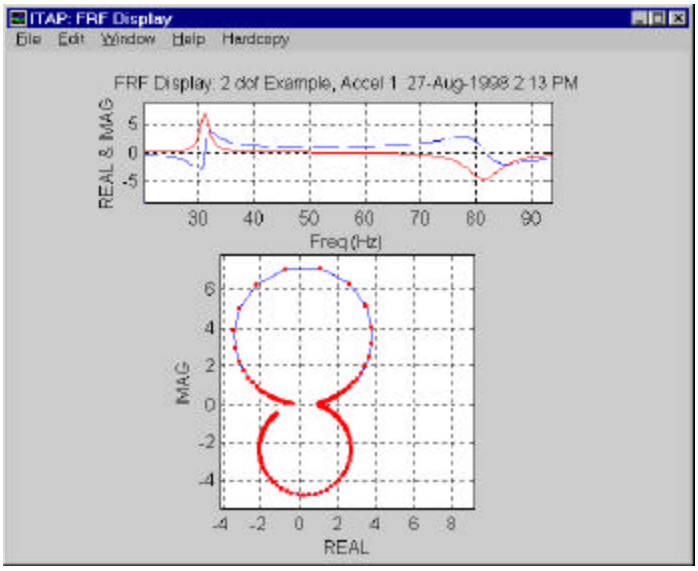
is invoked, resulting in the display



The MATLAB command (using the selected frequency band)

frfpall(frq,A(1,:),'2 dof Example, Accel 1',f12,3);

is invoked, resulting in the display



Purpose

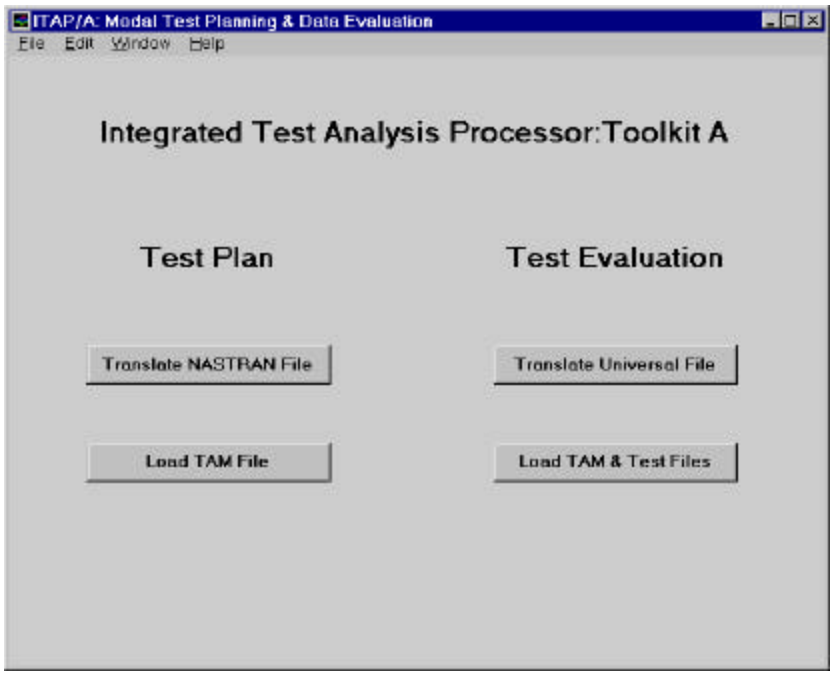
Initiate an ITAP-A session

Syntax

itapa

Description

itapa enables the main ITAP-A display for initiating a modal test planning or modal test data evaluation session. The user selects from one of four options on the display shown below:



Example

See Chapter 3

Purpose

Translate a NASTRAN TAM file and assemble an ITAP-A TAM file

Syntax

makmodl

Description

The ITAP-A routine **makmodl** performs a variety of calculations aimed at definition of the ITAP based TAM for a structural or mechanical dynamic system.

The major operational steps performed by **makmodl** are as follows:

7. Access the OUTPUT4 file “ITAPS” and translate the matrix files to a MATLAB compatible format. Select a name for the ITAP-A TAM file.

8. Using information contained in the rigid body displacement matrix, **PSIRY**, define (a) the global “x,y,z” position matrix, **XYZ**, for all TAM grid points, (b) the local-to-global triaxial dof displacement transformation matrix, **TMAT**, and (c) the rigid body displacement matrix (in global coordinates), **RBMAT**.

9. Generate a set of descriptive grid point names for the TAM and a dof “roadmap”, **TRP**, for subsequent data processing.

10. Define a TAM model connectivity file, **conn**, containing a grid string to be used in generation of model geometry displays.

11. Optionally define a TAM substructure breakdown in terms of grid point groupings (**sub1, sub2,** …), for use in generation of substructure tabular mode shape displays.

12. Save the ITAP-A TAM file.

Example

See Chapter 2

Purpose

Calculate structural dynamic frequency response from orthonormal mode parameters

Syntax

[A,V,U]= mfresp(Phi,frq,zeta,Gam,f);

Description

mfresp calculates complex frequency response by the modal method (i.e., superposition of orthonormal mode responses. **Phi, frq, and zeta** are the system modal matrix, undamped modal frequencies and modal critical damping ratios, respectively. **GAM** is the applied load allocation (column) matrix. **f** is the array of frequencies for which the frequency response solution is to be calculated. **A, V,** and **U** are the solution matrices for acceleration, velocity and displacement, respectively, with each column corresponding to the solution evaluated at a specific frequency, $\omega=2\pi f$.

Modal steady-state response of a mdof system subjected to a unit amplitude harmonic force excitation, $F(t) = e^{i\omega t}$, is of the form $\{q(t)\} = \{q(\omega)\}e^{i\omega t}$, where the uncoupled modal dynamic equations are:

$$q_n(\omega) = (\Phi_n^T \Gamma)/(\omega_n^2 + 2i\zeta_n \omega_n \omega - \omega^2)$$

and physical displacements are recovered using the modal transformation,

$$\{U(\omega)\} = [\Phi]\{q(\omega)\}$$

The complex velocity and acceleration amplitude solutions are

$$\{\dot{U}(\omega)\} = i\omega\{U(\omega)\} \qquad \{\ddot{U}(\omega)\} = -\omega^2\{U(\omega)\}$$

Example

Consider the following two dof system:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ K = \begin{bmatrix} 200000 & -100000 \\ -100000 & 100000 \end{bmatrix},\ Gam = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Orthonormal modal parameters are first calculated with **modalr** by invoking the following command:

[Phi,frq,orth,ke,gain]=modalr(K,M,GAM,eye(2,2));

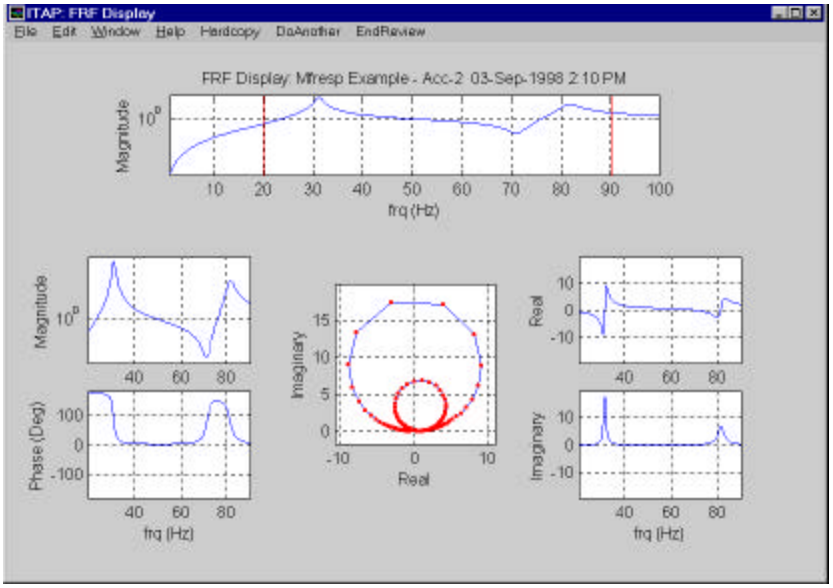
Defining the forcing frequency array and damping as

frq=[1:.25:100] , zeta=[.02 .02]'

invoke the **mfresp** command

[A,V,U]=mfresp(Phi,frq,zeta,GAM,f);

Results of this analysis (drive point acceleration at dof 2) are displayed below using the **frfpall** routine:



References

See Section 1.4.3

Purpose

Calculate complex eigenvalues and eigenvectors of a damped, dynamic system described by real matrices

Syntax

[Phi,Phil,lam,gain]=modalc(K,M,B,gam);

Description

modalc calculates complex eigenvectors, **Phi**, and eigenvalues, **lam**, for the damped dynamic system described in first order form as,

$$\begin{Bmatrix} \dot{V} \\ \dot{U} \end{Bmatrix} + \begin{bmatrix} M^{-1}B & M^{-1}K \\ -I & 0 \end{bmatrix} \begin{Bmatrix} V \\ U \end{Bmatrix} = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} \{F(t)\}$$

The complex eigenvalue problem associated with this system is,

[A] [Phi] = [Phi][lam] where,

$$[A] = \begin{bmatrix} -M^{-1}B & -M^{-1}K \\ I & 0 \end{bmatrix}$$

Left hand eigenvectors, calculated as

[Phil] = $[Phi]^{-1}$ produce an uncoupled dynamic matrix,

[lam] = [Phil] [A] [Phi]

This routine calculates modal excitation gains,

$$[gain] = [Phil] \begin{bmatrix} gam \\ 0 \end{bmatrix}$$

The eigenvalues and eigenvectors are sequenced according to increasing magnitude of eigenvalues. The eigenvectors are normalized to unit length.

Example

Consider the following two dof system:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 500 & -100 \\ -100 & 100 \end{bmatrix}$$

$$K = \begin{bmatrix} 200000 & -100000 \\ -100000 & 100000 \end{bmatrix}, \quad Gam = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Invoke the command

[phi,phil,lam,gain]=modalc(K,M,B,Gam);

The following numerical results are obtained:

phi =

```
  0.5996 - 0.1798i   0.5996  + 0.1798i   1.0000                1.0000
  1.0000             1.0000               -0.3800 + 0.3604i  -0.3800 - 0.3604i
 -0.0017 - 0.0025i  -0.0017  + 0.0025I  -0.0010 - 0.0018i   -0.0010 + 0.0018i
 -0.0015 - 0.0046i  -0.0015  + 0.0046i   0.0010 + 0.0003i    0.0010 - 0.0003i
```

phil (is not shown here)

lam =

```
-6.3232e+001 +1.9493e+002i
-6.3232e+001  -1.9493e+002i
-2.3677e+002 +4.2668e+002i
-2.3677e+002  -4.2668e+002i
```

gain =

```
 3.8906e-001 +1.8304e-001i
 3.8906e-001  -1.8304e-001i
-2.6621e-001  -2.7086e-002i
-2.6621e-001 +2.7086e-002i
```

References

See Subsections 1.3.8-1.3.9

Purpose

Calculate real modes and natural frequencies of an undamped, symmetric dynamic system.

Syntax

[Phi,frq,Orth,ke,gain,Phis]=modalr(K,M,Gam,Ks)

Description

modalr calculates orthonormal modes, **Phi**, and natural frequencies, **frq**, for the symmetric, real eigenvalue problem described by,

[K] [phi] = [M] [phi] [λ]

where the eigenvalue matrix, [λ], is diagonal.

In addition, this routine calculates (a) modal excitation gains, **gain**, based on applied external loading distributions, **Gam**, and (b) specific modal recovery data, **Phis**, based on the data recovery matrix, **Ks**. The modes (eigenvectors) are sequenced according to increasing magnitude of eigenvalues. The individual natural frequencies are,

$$frq_i = \sqrt{\lambda_i} \ / (2\pi)$$

The modal matrix is normalized such that modal generalized mass is unity. An optioal output quantity is the orthogonality check matrix, **Orth**, which is ideally an identity matrix.

Modal kinetic energy distribution, **ke**, is calculated as,

[ke] = ([M] [phi]) $\otimes$ [phi]

The total modal kinetic energy for an individual mode is 1.0 due to the mode normalization strategy.

Modal gains are calculated as,

$[gain] = [phi]^T [Gam]$

Finally, a user selected linear combination of system displacements (e.g. stresses, internal loads) is calculated as,

[phis] = [Ks] [phi]

Example

Consider the following two dof system:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ K = \begin{bmatrix} 200000 & -100000 \\ -100000 & 100000 \end{bmatrix},$$

$$Gam = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ [Ks] = \begin{bmatrix} 100000 & 0 \\ -100000 & 100000 \end{bmatrix}$$

Invoke the command

[phi,frq,orth,ke,gain,phis]=modalr(K,M,Gam,Ks)

The following numerical results are obtained:

```
phi =   5.2573e-001  8.5065e-001
        8.5065e-001 -5.2573e-001

frq =   3.1105e+001
        8.1434e+001

orth =  1.0    0.0
        0.0    1.0

ke =  2.7639e-001  7.2361e-001
      7.2361e-001  2.7639e-001

gain =  8.5065e-001
       -5.2573e-001

phis =  5.2573e+004  8.5065e+004
        3.2492e+004 -1.3764e+005
```

References

See Subsections 1.3.5-1.3.7

Purpose

Generate a TAM modal bar graph display

Syntax

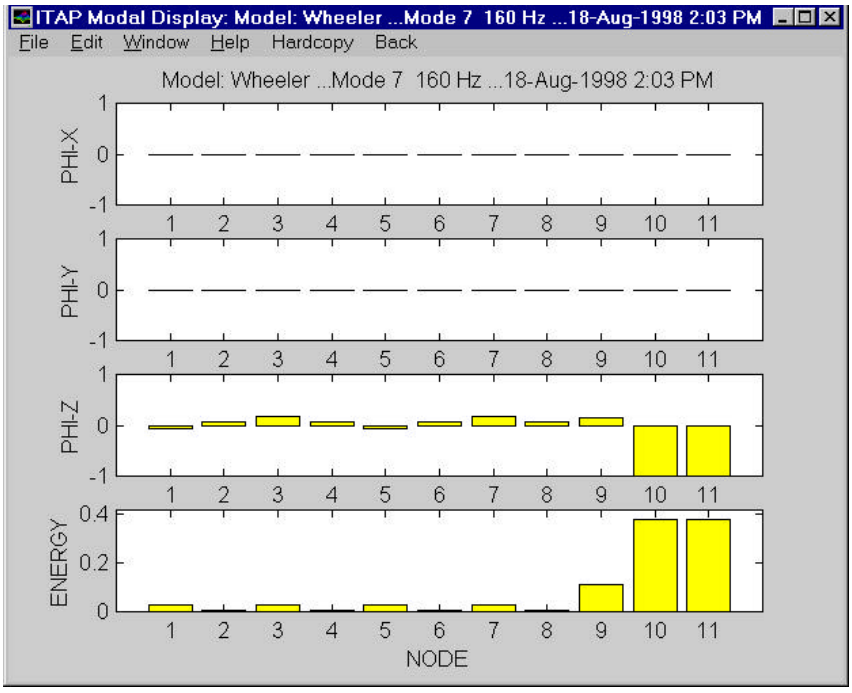
modbar(VEC,EVEC,titl)

Description

modbar generates a bar graph display for a specific ITAP-A TAM normalized modal vector, **VEC**, and associated modal kinetic energy distribution vector, **EVEC**. A descriptive alphanumeric title, "**titl**", is specified as an input variable. This routine is called within the ITAP-A routine, **modplots**, as one of the menu pick options. It is implicitly assumed in **modbar** that the modal vector quantities describe three displacement components per TAM node in the global coordinate system. While the user may invoke this function module on its own, it is recommended that **modplots** be employed to maximize user plot option versatility.

Example

Shown below is a typical modal bar graph display



Reference

See Subsection 3.2.4

Purpose

Calculate mode set comparison matrices

Syntax

[COH,ORX,ORY,COR] = modcoh(Y,X,M);

Description

modnorm performs mode (or more generally, vector) set comparison calculations following theoretical considerations described in Sections 1.5.3 and 1.5.4 of this document. The input data for this function routine consists of:

X = Reference mode matrix (not necessarily normalized)

Y = "Test" mode matrix (not necessarily normalized)

M = Reference mass matrix

The following output quantities are calculated:

$$[ORX] = [X_N]^T [M][X_N] \ , \ [ORY] = [Y_N]^T [M][Y_N]$$

(where $[X_N]$ and $[Y_N]$ are unit mass normalized modal matrices, see **modnorm**),

$$[COR] = [ORX]^{-1}\left([X_N]^T [M][Y_N]\right) ,$$

$$[COH] = [ORY]^{-1/2}[COR]^T [OR][COR][ORY]^{-1/2}$$

Example Consider the following input data:

```
M =   2    0    0
      0    1    0
      0    0    1

X =   1.0000e+000 -6.4039e-001
      1.0000e+000  2.8078e-001
      1.0000e+000  1.0000e+000

Y =   1.1893e+000 -6.4194e-001
      1.1141e+000  2.9285e-001
      1.2190e+000  1.0647e+000
```

Invoking the following statement in the MATLAB command window:

[COH,ORX,ORY,COR]=modcoh(Y,X,M);

the resulting output matrices are

```
COH =  9.9894e-001 -4.3732e-004
      -4.3732e-004  9.9982e-001

ORX =  1.0000e+000  1.6653e-016
       1.6653e-016   1.0000e+000

ORY =  1.0000e+000  2.8869e-002
       2.8869e-002  1.0000e+000

COR =  9.9946e-001  2.5759e-002
       2.6699e-003  9.9957e-001
```

References

Subections 1.5.3, 1.5.4

Purpose

Normalize a modal matrix with respect to a mass matrix

Syntax

[Xn,ORX] = Modnorm(M,X);

Description

modnorm normalizes a modal matrix, **X,** such that each modal vector (column) produces unit modal mass, i.e.,

$\{Xn\}_i^T [M] \{Xn\}_i = 1.0$

In addition, modnorm produces an orthogonality check matrix, **ORX**, which expresses the level of orthogonality of the normalized modal matrix, **Xn**, with respect to the mass matrix, **M**, i.e.,

$[ORX] = [Xn]^T [M] [Xn]$

where the on-diagonal terms are unity.

Example

Consider the following input data:

```
M =    2    0    0
       0    1    0
       0    0    1

X  =   1.1893e+000 -6.4194e-001
       1.1141e+000  2.9285e-001
       1.2190e+000  1.0647e+000
```

Invoking the following statement in the MATLAB command window:

[Xn,ORX] = Modnorm(M,X);

the resulting output matrices are

```
Xn =    4.9506e-001 -4.6265e-001
        5.4082e-001  1.9190e-001
        4.6620e-001  7.3150e-001

ORX =  1.0000e+000 -1.3269e-002
      -1.3269e-002   1.0000e+000
```

Reference

Subsection 1.5.3

Purpose

Orthogonalize a modal matrix with respect to a mass matrix

Syntax

[Xo,ORX] = modorth(M,X,Typ);

Description

modorth orthogonalizes a modal matrix, **X**, such that the modes are normalized and optimally adjusted (and stored in the output matrix, **Xo**) by a selected strategy to produce a unit diagonal orthogonality matrix,

$$[ORX] = [Xo]^T [M] [Xo]$$

If **Typ=1**, the Gram-Schmitt (or Cholesky) approach is employed. This method orthogonalizes each successive mode with respect to the prior modes. This strategy is typically employed on a set of measured modes if it is assumed that the lowest frequency modes are assumed more accurate than the higher frequency modes.

If **Typ=2**, the Targoff (or Spectral Decomposition) approach is employed. This method orthogonalizes the mode set in a manner which assumes that all modes have been measured with a similar degree of accuracy.

Example

Consider the following input data:

```
M =    2    0    0
       0    1    0
       0    0    1

X =    1.0902e+000 -6.4286e-001
       1.1910e+000  2.6666e-001
       1.0266e+000  1.0164e+000
```

The orthogonality check matrix for this data (from **modnorm**) is

```
OR =   1.0000e+000 -1.3269e-002
      -1.3269e-002  1.0000e+000
```

Invoking the MATLAB command window statement:

```
[X1,OR1]=modorth(M,X,1)
```

the following Gram-Schmitt (Cholesky) orthogonalization results are obtained

```
X1 =    4.9506e-001 -4.5612e-001
        5.4082e-001  1.9910e-001
        4.6620e-001  7.3775e-001

OR1 =  1.0000e+000  5.5511e-017
       5.5511e-017  1.0000e+000
```

Invoking the MATLAB command window statement:

```
[X2,OR2]=modorth(M,X,2)
```

the following Targoff (or Spectral Decomposition) orthogonalization results are obtained

```
X2 =    4.9271e-001 -4.5865e-001
        5.4183e-001  1.9632e-001
        4.6998e-001  7.3535e-001

OR2 =  1.0000e+000  5.5511e-017
       5.5511e-017  1.0000e+000
```

References

See Subsection 1.5.5

Purpose TAM modal plot generation menu

Syntax **i=1;modplots**

Description **modplots** generates an orthographic modal deformation plot and menu picks for selection of additional modal plot displays. This routine is specifically designed for operations on an ITAP-A TAM file. The user initiates a "**modplots**" session by specifying "i=1" to begin at the first TAM mode.

Example See Subsection 3.2.4

Purpose

Modal test data and TAM modal comparison plot generation menu

Syntax

imod=1;modpl12

Description

modpl12 generates a test/TAM mode comparison plot and menu picks for selection of additional modal plot displays. This routine is specifically designed for operations on ITAP-A Modal Test Data and TAM files. The user initiates a "**modpl12**" session by specifying "imod=1" to begin at the first test mode. Modal test and TAM mode pairs are sorted according to the cross-orthogonality matrix, which is calculated by the **modtest2** function module.

Example

See Subsection 4.3.5

Purpose

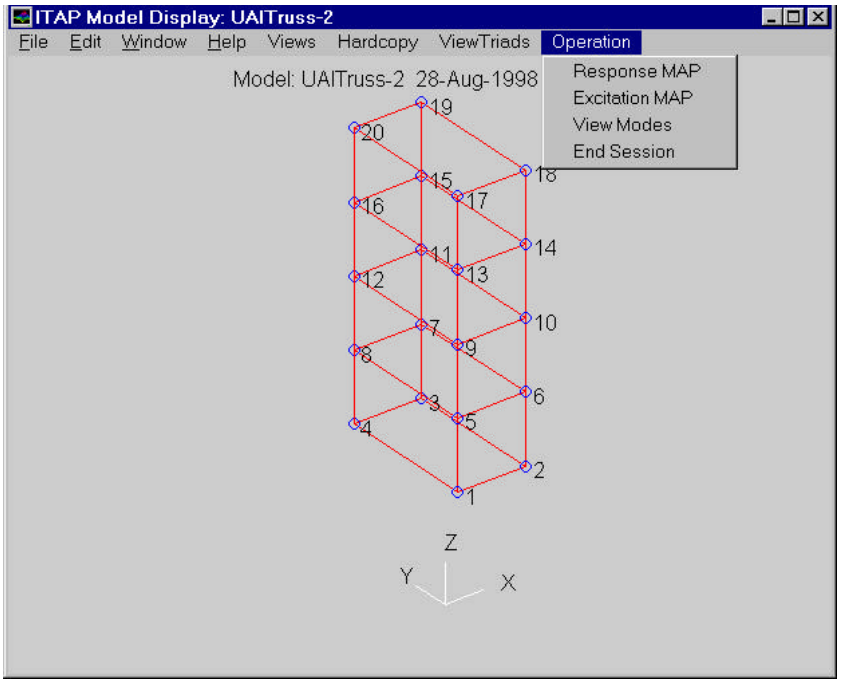
Modal test planning operations selection menu

Syntax

modprep2

Description

modprep2 generates a TAM structural geometry display with menu picks for further user “modal test planning” operations as shown below:



Purpose

Calculate quasi-static residual modes associated with an applied load distribution matrix

Syntax

[Phir,fmodr,Orr] = modres(K,M,Phi,fmod,GAM,lam0)

Description

modres generates a set of orthonormal residual modes, and associated pseudo natural frequencies, which, are appended to a truncated set of modes, **Phi** with natural frequencies, **fmod** for a dynamic system described by a mass, **M** and stiffness, **K**, matrix.. The resulting mode set (modes, **Phir** and natural frequencies **fmodr**) provides a complete description of quasi-static system response to applied loading patterns, **GAM** plus dynamic behavior in the frequency band spanned by **fmod**. When the stiffness matrix is singular, a shift operator, **lam0>0** is employed; otherwise, **lam0** is set to “0”.

Example

Consider the 6 dof dynamic system defined by the following mass and stiffness matrices:

$$M = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 20000 & -10000 & & & & \\ -10000 & 20000 & -10000 & & & \\ & -10000 & 20000 & -10000 & & \\ & & -10000 & 20000 & -10000 & \\ & & & -10000 & 20000 & -10000 \\ & & & & -10000 & 10000 \end{bmatrix}$$

The applied loading distribution is

$$[\mathrm{GAM}] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Normal modes for this system, calculated with **modalr**, have the following frequencies:

frq = [3.8368 11.2874 18.0821 23.8258 28.1849 30.9060]^T

The applied force history, F(t), which is the function used for the illustrative example for the **shock** module, has a "dynamic bandwidth", f^* ~ 10 Hz. Therefore only the first mode alone is required for modal transient analysis (**mtresp**). Quasi-static residual response associated with the remaining modes is captured with a single residual mode calculated with **modres** using the following command:

[phir,fmodr,orr]=modres(K,M,phi(:,1),frq(1),GAM,0);

The "modal" frequencies associated with the "augmented, truncated" system are

fmodr = [3.8368 13.4533]^T

Discussion of this illustrative example continues with the illustrative example for **mtresp**.

Purpose

Orthogonalize modal subsets with respect to a mass matrix

Syntax

Phi2p = modsweep(M,Phi1,Phi2)

Description

modsweep orthogonalizes two modal subsets with respect to one another by a block sweeping process using the mass matrix, **M**, as the weighting matrix. The modal subsets are designated as **Phi1** and **Phi2**. The second modal subset, **Phi2**, is "swept" of contributions from the first modal subset, **Phi1** such that the modified modes, **Phi2p**, are orthogonal to the first modal subset,

$[Phi2p]^T$ [M] [Phi1] = [0].

This type of "sweeping" orthogonalization may be employed, for example, to orthogonalize a modal subset assumed to be less accurately measured than a "primary" modal subset.

Neither the primary or secondary modal subsets need to be orthogonal with respect to the mass matrix.

Example

A six dof system is described by the following mass matrix, M, and exact modal matrix, PHI:

```
M =    0.5000         0         0         0         0         0
            0    2.0000   -1.0000         0         0         0
            0   -1.0000    2.0000         0         0         0
            0         0         0    1.0000         0         0
            0         0         0         0    1.0000         0
            0         0         0         0         0    0.5000
```

```
PHI =
   0.4472   -0.6103    0.1088    0.4459    1.1000    0.0833
   0.4472   -0.5083    0.0632   -0.0536   -0.4427   -0.0742
   0.4472   -0.0997   -0.5504    0.2522   -0.2759   -0.1184
   0.4472    0.2056   -0.1872   -0.6922    0.1971    0.4524
   0.4472    0.4421    0.3331   -0.0857    0.1173   -0.6874
   0.4472    0.5309    0.5739    0.7128   -0.2915    0.7718
```

A set of “test” modes, PHIT, consists of the first three exact modes plus modes 4-6 which are randomly corrupted with contributions from the first three modes.

```
PHIT =
   0.4472   -0.6103    0.1088    0.5295    1.1958    0.1424
   0.4472   -0.5083    0.0632    0.0125   -0.3640   -0.0170
   0.4472   -0.0997   -0.5504    0.2426   -0.3172   -0.0829
   0.4472    0.2056   -0.1872   -0.7482    0.1641    0.4987
   0.4472    0.4421    0.3331   -0.1745    0.1192   -0.6250
   0.4472    0.5309    0.5739    0.6123   -0.2712    0.8418
```

Using **modsweep** as follows produces appropriate corrections to modes 4-6:

PHI2P=**modsweep**(M,PHIT(:,1:3),PHIT(:,4:6))

```
PHI2P =    0.4459    1.1000    0.0833
          -0.0536   -0.4427   -0.0742
           0.2522   -0.2759   -0.1184
          -0.6922    0.1971    0.4524
          -0.0857    0.1173   -0.6874
           0.7128   -0.2915    0.7718
```

Reference

See Subsection 1.5.5

Purpose

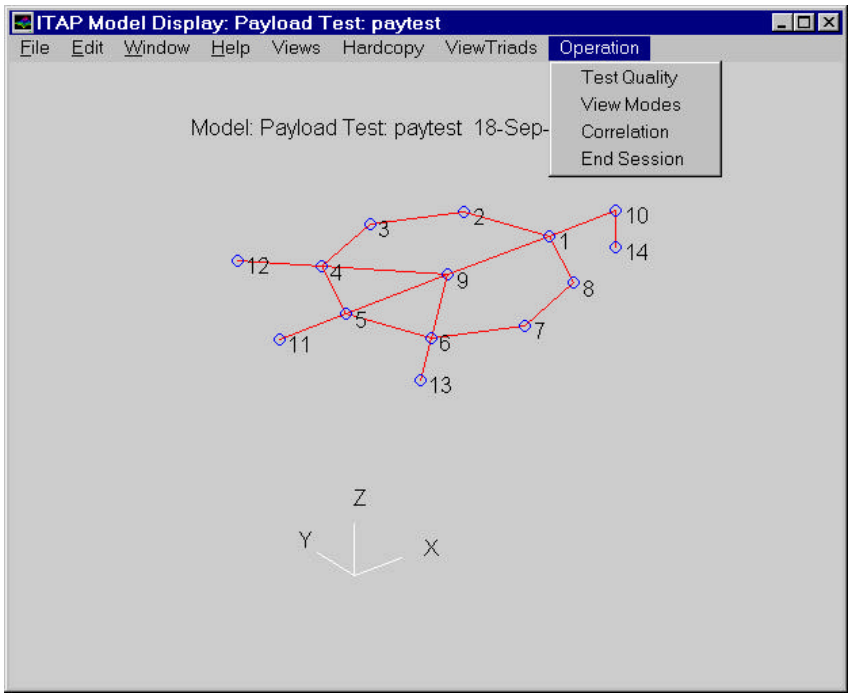
Modal test data evaluation and test-analysis correlation operations selection menu

Syntax

modtest2

Description

modtest2 generates a TAM structural geometry display with menu picks for further user operations as shown below:



Purpose

Evaluate adequacy of candidate excitation resources for modal test planning

Syntax

mprepfrf

Description

mprepfrf generates a TAM structural geometry display and prompts the user for information required to simulate TAM frequency responses associated with candidate excitation resources.

Example

See Subsection 3.2.5

Purpose

Calculate transient response of structural dynamic system described in terms of real, orthonormal modes.

Syntax

[A,V,U] = mtresp(Phi,frq,zeta,Gam,F,dt);

Description

The conventional linear structural dynamic formulation,

$$[M]\{\ddot{U}(t)\}+[B]\{\dot{U}(t)\}+[K]\{U(t)\}=[\Gamma]\{F(t)\},$$

is transformed using, the orthonormal real modes into the series of uncoupled second order dynamic equations,

$$\ddot{q}_n + 2\varsigma_n\omega_n\dot{q}_n + \omega_n^2 q_n = [\Phi_n^T\Gamma]\{F(t)\}.$$

Physical responses, **A**, **V**, and **U**, are recovered using

$$\{A\}=[\Phi]\{\ddot{q}\},\ \{V\}=[\Phi]\{\dot{q}\},\ \{U\}=[\Phi]\{q\}$$

mtresp calculates transient response (using normal mode data calculated with **modalr** and, when desired, with **modres,** and SDOF transient responses calculated with **sdof**). The applied excitation array, **F**, consists of a set of (row) force histories with uniform sampling time, **dt**.

Example

The illustrative example employed in the descriptions of the shock and modres modules is continued here.

The following three response analyses are performed:

Case 1: all 6 modes included

```
[a,v,u] = mtresp(phi,frq,.02*ones(6,1),GAM,F,dt);
```

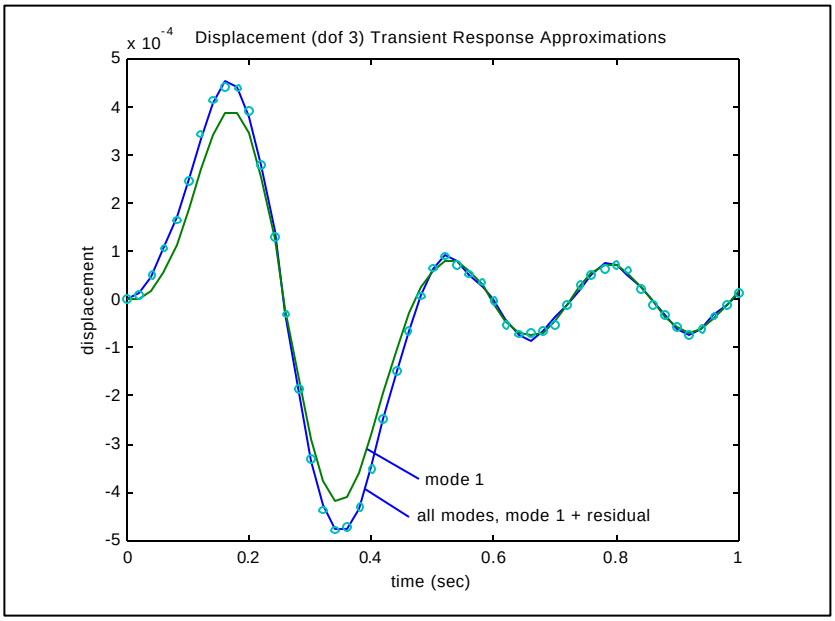
Case 2: only mode 1 included

```
[a2,v2,u2] = mtresp(phi(:,1),frq(1),.02*ones(2,1),GAM,F,dt);
```

Case 3: mode 1 plus single residual mode

```
[a2r,v2r,u2r] = mtresp(phir,fmodr,.02*ones(3,1),GAM,F,dt);
```

Transient response results associated with the third dof for this system are illustrated below:



The above results indicate the benefit of residual vectors when modal truncation is employed in modal transient response analysis.

Reference See Subsection 1.4.5

Purpose

Calculate transient response of structural dynamic system described in terms of complex modes.

Syntax

[dX,X] = mtrespc(Phi,Phil,lam,Gain,F,dt);

Description

The following table relates **mtrespc** syntax variables to the symbols used in the discussion below:

"syntax"	variable
dX	$\begin{Bmatrix}\ddot{U}\\ \dot{U}\end{Bmatrix}$
X	$\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix}$
Phi	$[\Phi]$
Phil	$[\Phi_L]$
lam	$[\lambda]$
Gain	$\begin{bmatrix}\Phi_L \cdot \begin{bmatrix}\Gamma\\ 0\end{bmatrix}\end{bmatrix}$
F	{F(t)}
dt	dt

The conventional linear structural dynamic formulation,

$$[M]\{\ddot{U}(t)\}+[B]\{\dot{U}(t)\}+[K]\{U(t)\}=[\Gamma]\{F(t)\},$$

is expressed in state variable form as

$$\begin{Bmatrix}\ddot{U}\\ \dot{U}\end{Bmatrix}+\begin{bmatrix}M^{-1}B & M^{-1}K\\ -I & 0\end{bmatrix}\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix}=\begin{bmatrix}\Gamma\\ 0\end{bmatrix}\{F(t)\}.$$

The complex mode transformation,

$$\{X\}=\begin{Bmatrix}\dot{U}\\ U\end{Bmatrix}=[\Phi]\{q\}$$

is used to define the uncoupled first order dynamic equations,

$$\dot{q}_n - \lambda_n q_n = \left[\Phi_L \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} \right]_{row-n} \{F(t)\}$$

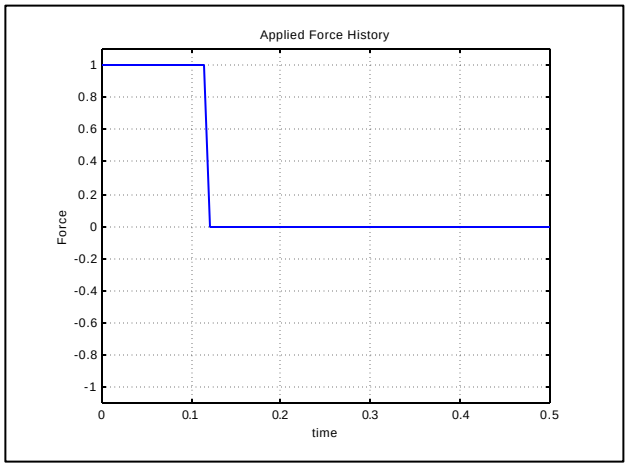
where $[\Phi_L] = [\Phi_n^{-1}]$.

mtrespc calculates transient response (using complex mode data calculated with **modalc** and transient responses calculated with **sdof1**. The applied excitation array, F(t), consists of a set of (row) force histories with uniform sampling time, dt.

Example

The illustrative example employed in the descriptions of the **modalc** module is continued here.

A step-down applied force history, F(t), (sampled at dt=.002), shown below is applied to the system.



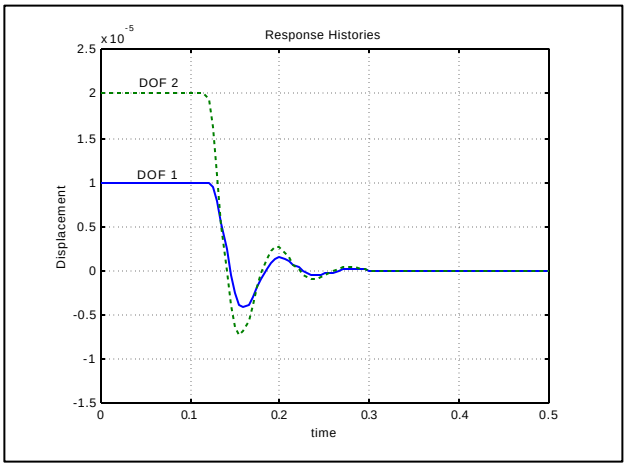
By invoking the command

[DX,X] = mtrespc(phi,phil,lam,gain,F,dt);

the acceleration, velocity and displacement time history response histories, respectively, are extracted as:

ACC=DX([1 2],:); VEL=X([1 2],:); RESP=X([3 4],:);

The displacement time histories are illustrated below:



Reference See Subsection 1.4.6

Purpose

Translate a NASTRAN Output4 (acsii formatted) file to a .mat formatted file.

Syntax

out4read(namfold,namin,namout)

Description

out4read translates a NASTRAN output4 file containing an arbitrary number of matrices into a .mat file which includes the same matrix data. The output4 file, named "**namin**", is located in the folder (or directory) specified by the input name, "**namfold**". The name of the generated ".mat" file is specified by the input name, "**namout**". Names assigned to matrices on the output4 file are preserved in the ".mat" file.

Purpose

Generate a push button switch for a figure which when pushed generates a hard copy.

Syntax

pmenu

Description

The command, **pmenu**, embedded in a .m file or invoked by keystroke command generates the push button with label “hardcopy”. The label remains on the menu bar until the figure is cleared.

Purpose

Calculate MDOF system response to stationary random excitation expressed as an autospectrum.

Syntax

[Y2,X2,GOUT,GIN]=rresp(f,H,ff,GXX);

Description

The ITAP-A routine **rresp** directly calculates the MDOF system response and excitation function variances due to a single stationary random excitation described in terms of an autospectrum, **GXX** (defined at discrete frequencies, **ff**). The MDOF system frequency responses are defined by the matrix, **H** (rows are individual FRFs), at uniformly spaced frequencies, (array) **f**. Within the **rresp** routine, interpolation of the specified input autospectrum to conform with **f** is performed. The interpolation is carried out on the logarithms of frequency and autospectrum yielding interpolated autospectrum values, which are vary linearly on a log-log plot. Computed results consist of the output variance array, **Y2**, the input variance, **X2**, the output autospacetrm, **GOUT**, and the input autospectrum, **GIN**. **GOUT** and **GIN** are defined at frequencies corresponding to **f**.

It is implicitly assumed that the highest frequency in **f** is lower than the maximum value in **ff**.

Example

Consider the following two dof system:

$$M=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ B=\begin{bmatrix}50 & -10\\ -10 & 10\end{bmatrix},\ K=\begin{bmatrix}200000 & -100000\\ -100000 & 100000\end{bmatrix}$$

$$Gam=\begin{bmatrix}0\\ 1\end{bmatrix},\ frq=[10:.25:100]$$

To calculate FRFs, invoke the command

[A,V,U] = fresp(K,M,B,Gam,frq);

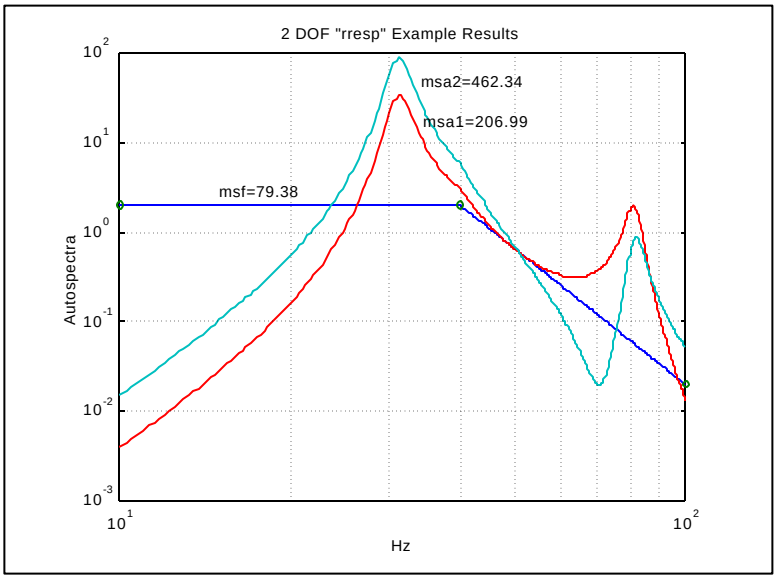
Define the excitation autospectrum and associated frequency array as

GFF=[2 2 .02], ff=[10 40 100]

To calculate random acceleration responses, invoke the commands

[msa1,msf,Ga1,Gf] = rresp(frq,A(1,:):),ff,GFF);
[msa2,msf,Ga2,Gf] = rresp(frq,A(2,:):),ff,GFF);

Results of these calculations are displayed below:



References

See Section 1.4.4

Purpose

Calculate the time history response of a single degree of freedom oscillator.

Syntax

[**x,v,a] = sdof(F,wn,zeta,dt,n1);**

Description

sdof calculates the time history displacement, **x**, velocity, **v**, and acceleration, **a**, responses of a single degree of freedom oscillator excited by an excitation history, **F**. The input system parameters are undamped, non-zero natural frequency, **wn**, and critical damping ratio, **zeta**. The excitation force, **F**, is a column vector having entries at uniform sampling increments, **dt**. The input parameter, **n1**, is used to specify an initial condition which is either, (b) n1=0 for static initial conditions, or (b) n1=1 for zero initial displacement and velocity. If n1=0 is selected, the initial displacement and velocity, respectively, are

$$x(1) = F(1)/wn^2,\ v(1)=0.$$

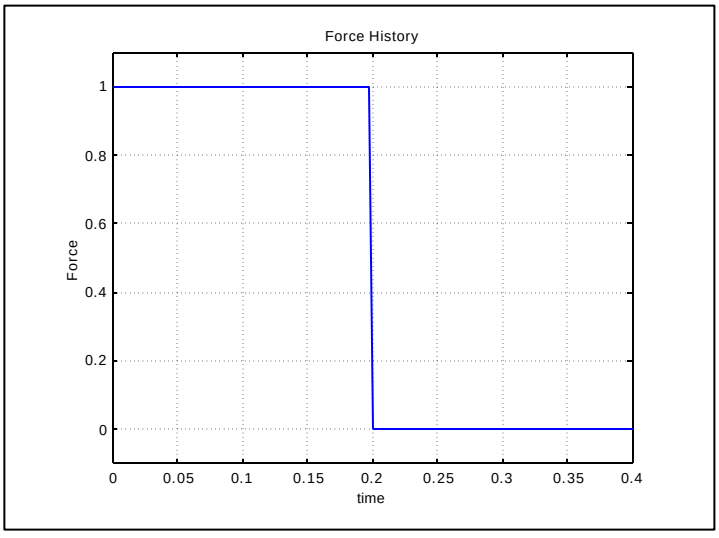
An exact numerical time history solution is calculated based on the assumption that the excitation force has a trapezoidal distribution between sampling points.

Example

Consider the step-down force time history, F, defined as

F=[ones(1,100) zeros(1,101)];

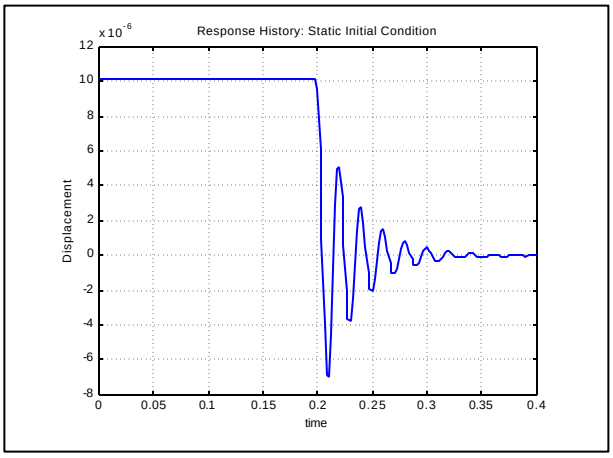
with sampling rate, dt=0.002, as shown below:



The response of a sdof system with undamped natural frequency, f=50 Hz, and critical damping ratio, ζ=0.1, to this excitation with static initial conditions is calculated with the command:

[x,v,a]=sdof(F',2*pi*50,.1,dt,0);

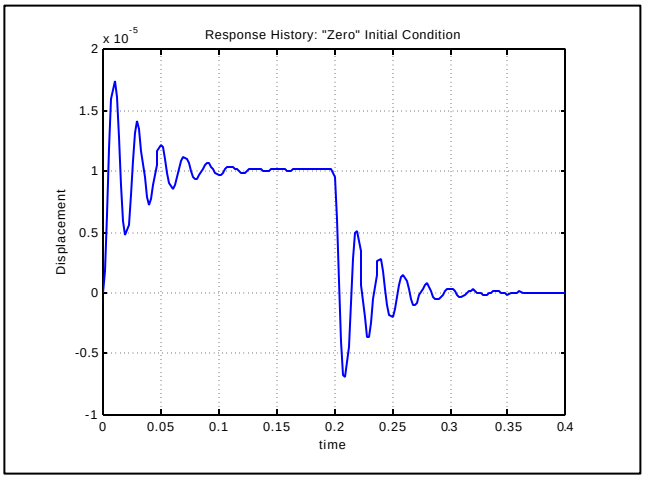
The displacement response is illustrated below:



For the case of zero displacement and velocity initial conditions, the command

[x,v,a]=sdof(F',2*pi*50,.1,dt,1);

results in the displacement response shown below:



References

See Subsection 1.2.3

Purpose

Calculate the time history response of a single degree of freedom unit mass.

Syntax

[x,v,a] = sdof0(F,x0,v0,dt);

Description

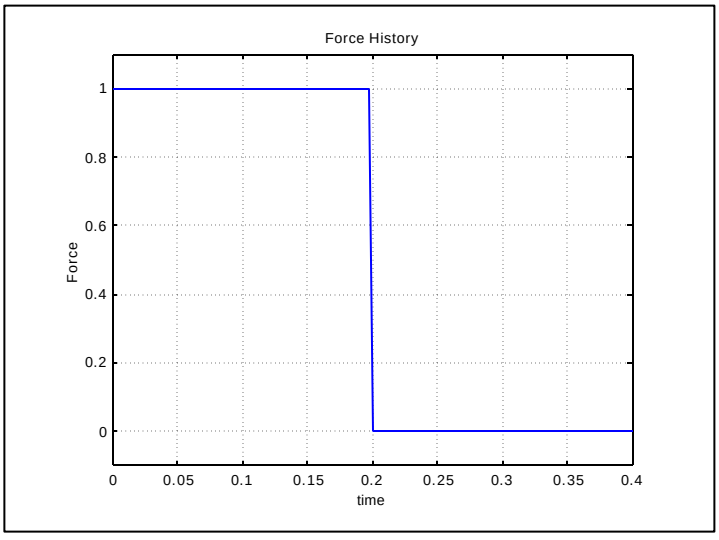
sdof0 calculates the time history displacement, **x**, velocity, **v**, and acceleration, **a**, responses of a single degree of freedom unit mass excited by an excitation history, **F**. The excitation force, **F**, is a column vector having entries at uniform sampling increments, **dt**. The initial displacement and velocity, are specified as **x0** and **v0**, respectively. An exact numerical time history solution is calculated based on the assumption that the excitation force has a trapezoidal distribution between sampling points.

Example

Consider the step-down force time history, F, defined as

F=[ones(1,100) zeros(1,101)];

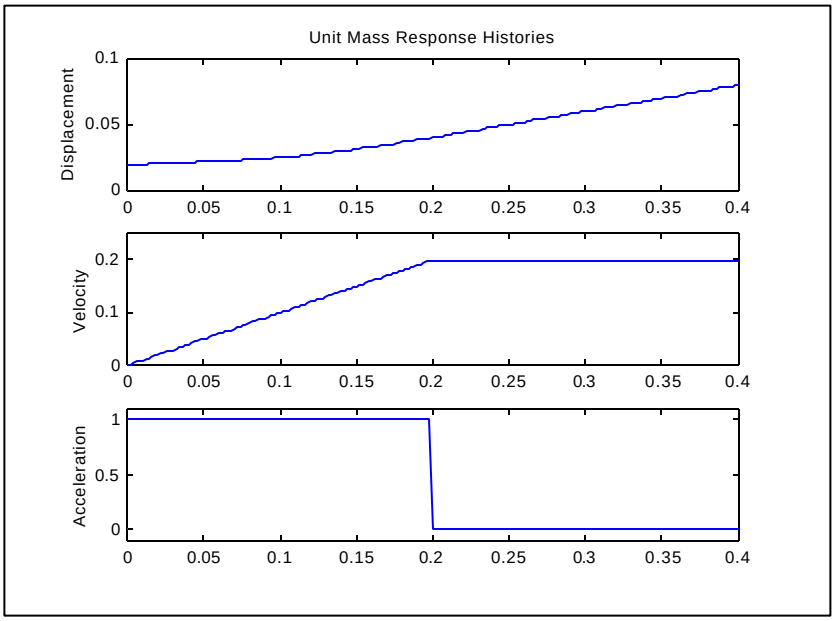
with sampling rate, dt=0.002, as shown below:



The response of a sdof unit mass to this excitation with initial conditions, x(0)=.02 and v(0)=0, is calculated with the command:

[x,v,a]=sdof(F',.02,0,dt);

The displacement response is illustrated below:



Reference

See Subsection 1.2.3

Purpose

Calculate the time history response of a first order, single degree of freedom dynamic system.

Syntax

[x,v]=sdof1(F,l ,dt,n1);

Description

sdof calculates the time history displacement, **x**, and velocity, **v**, responses of a first order, single degree of freedom dynamic system excited by a force history, **F**. The system is described by the differential equation,

$$\dot{x} - \lambda x = F(t)$$

The excitation force, **F**, is a column vector having entries at uniform sampling increments, **dt**. The input parameter, **n1**, is used to specify an initial condition which is either, (b) n1=0 for a static initial condition, x(0) = -F(0)/λ, or (b) n1=1 for zero initial displacement. An exact numerical time history solution is calculated based on the assumption that the excitation force has a trapezoidal distribution between sampling points.

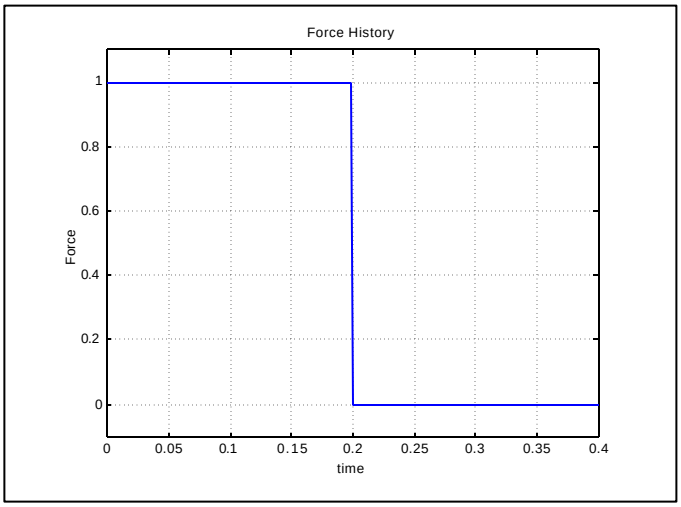
This routine is used to calculate complex modal coordinate solutions for multi-degree of freedom systems with complex modes (see Section 1.4.6)

Example

Consider the step-down force time history, F, defined as

F=[ones(1,100) zeros(1,101)];

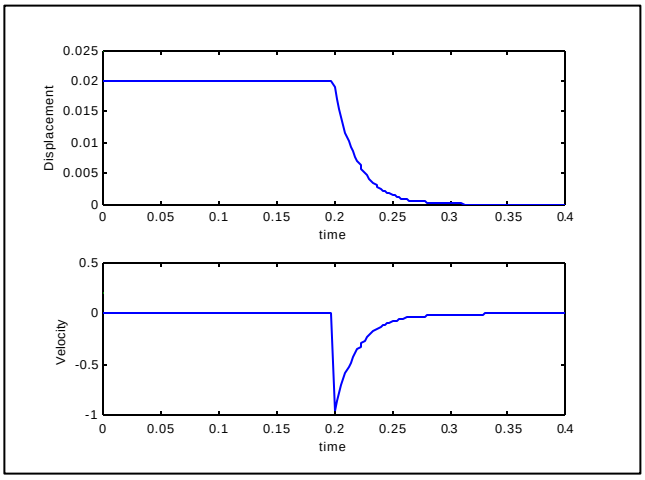
with sampling rate, dt=0.002, as shown below:



The response of a first order dynamic system with $\lambda = -50$ to this excitation with static initial conditions is calculated with the command:

[x,v]=sdof(F',-50,dt,0);

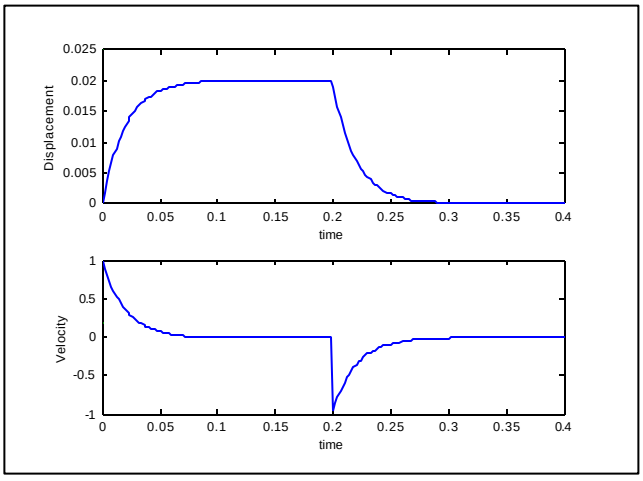
The displacement and velocity responses are illustrated below:



For the case of zero initial displacement , the command

[x,v]=sdof(F',-50,dt,1);

results in the displacement and velocity responses shown below:



Purpose

Compute the normalized response or shock spectrum associated with a force or acceleration history

Syntax

```
[freq,amx,amn,vmx,vmn,xmx,xmn] =
                    shock(F,dt,df,fmax,zeta,n1,ns);
```

Description

shock calculates a normalized response or shock spectrum, which is associated with a specified excitation time history, **F**, which is sampled in uniform time increments, **dt**. Various types of response and shock spectra are calculated at frequencies, **freq**, (increment **df** to maximum frequency, **fmax**) consisting of maximum and minimum absolute acceleration (**amx, amn**), maximum and minimum velocity (**vmx, vmn**), and maximum and minimum displacement (**xmx, xmn**) spectra. A reference SDOF system damping value, **zeta**, is specified for the calculations. The input parameter, **n1**, is used to select an initial condition (0 for static equilibrium, 1 for zero displacement and velocity). The input parameter, **ns**, is used to indicate whether a response (ns=0) or shock (ns=1) is desired.

The response or shock spectrum is calculated as a unit amplitude normalized quantity by setting the peak input value to unity. If non-normalized spectra are desired, multiply the normalized spectra by the peak absolute amplitude of the excitation, F.

Example

Consider the single cycle force history, F(t), defined as

$F = \sin(4\pi t)$ for $t \leq 0.5$ seconds

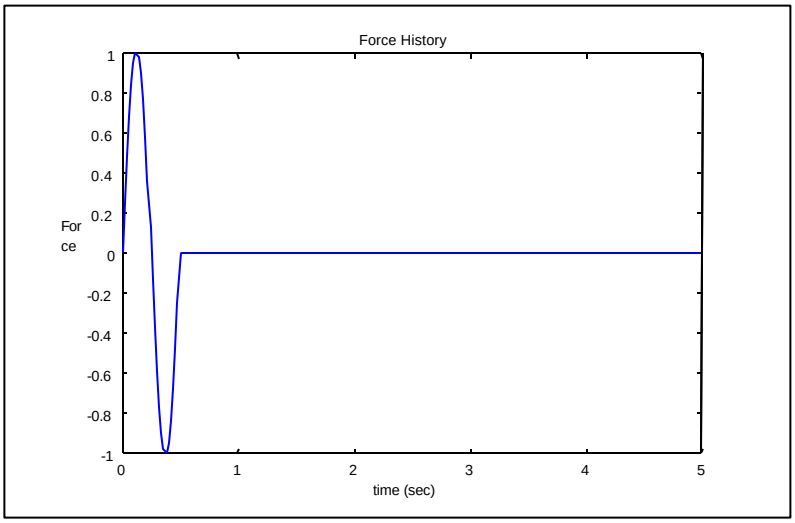
$F = 0$ for $t > 0.5$ seconds,

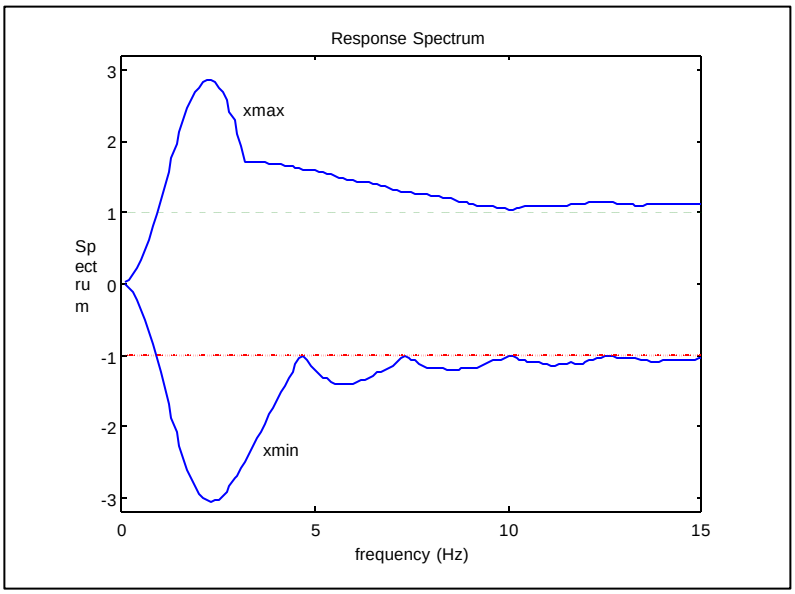
over uniform time increments, dt=0.02 seconds. Invoke the command:

```
[ff,amax,amin,vmax,vmin,xmax,xmin]=
                    shock(F',dt,0.1,15,dt,1,0);
```

Note that F has been defined as a row array. Therefore, since shock requires a column excitation history, F' is specified in the command.

The force history and normalized displacement response spectra are illustrated below:





Reference: See Subsection 1.2.4

Purpose

Generate a TAM substructure modal bar graph display

Syntax

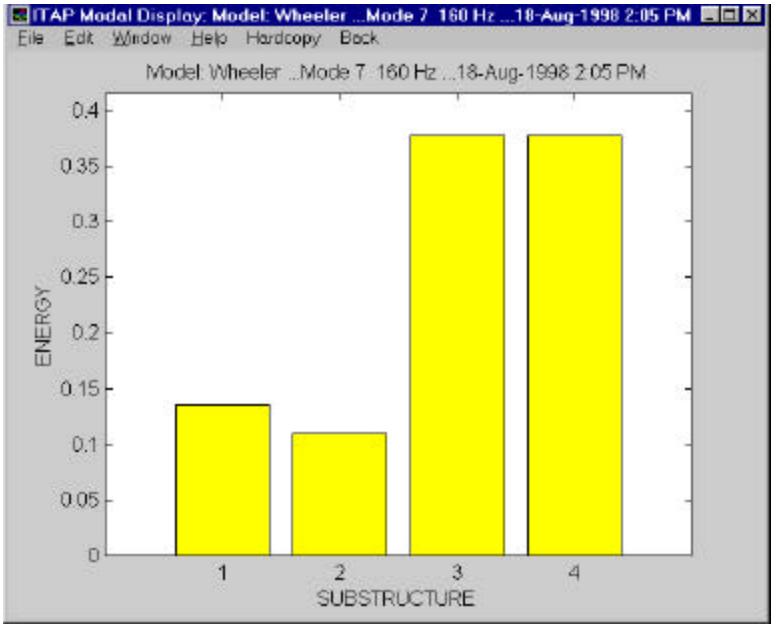
smodbar(ESVEC,titl)

Description

smodbar generates a bar graph display for a specific ITAP-A TAM normalized substructure modal kinetic energy distribution vector, **ESVEC**. A descriptive alphanumeric title, "**titl**", is specified as an input variable. This routine is called within the ITAP-A routine, **modplots**, as one of the menu pick options. While the user may invoke this function module on its own, it is recommended that **modplots** be employed to maximize user plot option versatility.

Example

Shown below is a typical substructure modal bar graph display



Reference

See Subsection 3.2.4

Purpose

Modal test data and TAM modal comparison plot generation menu (substructure allocation)

Syntax

imod=1;smodpl12

Description

smodpl12 generates a test/TAM mode comparison plot and menu picks for selection of additional modal plot displays. This routine is specifically designed for operations on ITAP-A Modal Test Data and TAM files. The user initiates a "**smodpl12**" session by specifying "**imod=1**" to begin at the first test mode. Modal test and TAM mode pairs are sorted according to the cross-orthogonality matrix, which is calculated by the **modtest2** function module.

Example

See Subsection 4.3.5

Purpose

generate a test-analysis model (TAM) substructure allocation map

Syntax

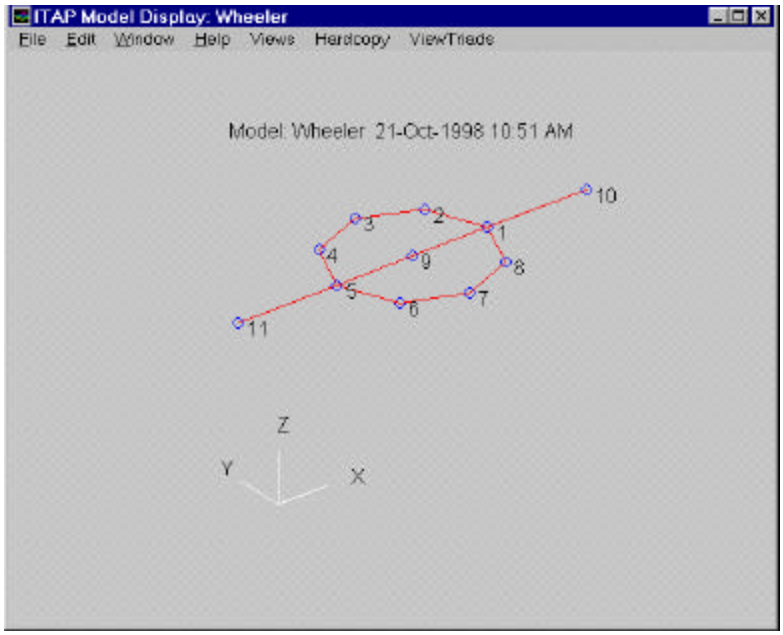
submap

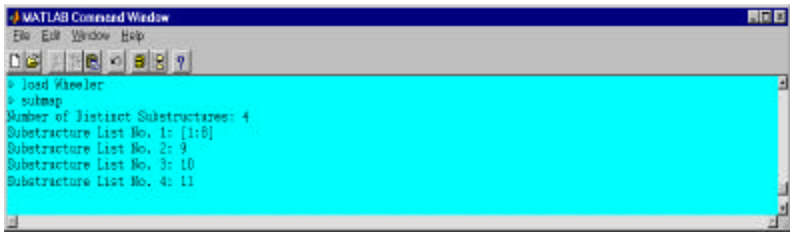
Description

submap generates a TAM model substructure allocation map. This routine is called by function module, **makmodl**. It may optionally be invoked, from the MATLAB command window, to redefine an existing TAM substructure allocation map.

Example

Consider the illustrative example, “Wheeler” described in Chapter 3. By invoking the command **submap**, after loading the model file, the display, shown below, appears and the user responds to the command prompts as indicated.





The generated substructure allocation arrays are as follows:

Sub1 = 1 2 3 4 5 6 7 8

Sub2 = 9

Sub3 = 10

Sub4 = 11

Purpose

Modal test data mode plot generation menu

Syntax

i=1;tmodplot

Description

tmodplot generates an orthographic test modal deformation plot and menu picks for selection of additional modal plot displays. This routine is specifically designed for operations on an ITAP-A modal test data file. The user initiates a "**tmodplot**" session by specifying "**i=1**" to begin at the first test mode. This routine is nearly identical to **modplots**, the primary difference being that it displays modal damping as well as modal frequency.

Example

See Subsection 4.3.4

Purpose

Read modal test data from a SDRC Universal File (or other source). Save results to a standard ITAP-A test data file.

Syntax

unvmodal;

Description

unvmodal translates specific contents of a SDRC Universal File (or other format file), "**namin**", and creates an ITAP-A .mat file, "**namout**", containing modal test data. The file, "**namin**", is located in the folder (or directory) specified by the input name, "**namfold**".

unvmodal is a "place holder" module which is supplied by the ITAP-A user. Since uniformity of SDRC Universal File formats is not guaranteed, the user must supply a preferred translator version which is appropriate for his/her modal test data source.

Example

A simple **unvmodal** routine is supplied with ITAP-A. This routine is for provided for illustrative purposes only. It is assumed that an ASCII modal test data file consists of a single table or matrix with successive columns corresponding to individual modes. The first row of data corresponds to modal frequencies; the second row corresponds to modal critical damping ratios. The remaining rows correspond to the modal matrix. The lising for this example **unvmodal** routine is provided below:

```
function unvmodal;

input('Name of Input Modal Test Data File: ','s');
namin=ans;
input('Name of ITAP-A Modal Test Data File: ','s');
namout=ans;

str=['load ' namin '.dat -ascii;modata=' namin ';'];
eval(str);

[nrows,nmodes]=size(modata);

FREQ=modata(1,:)';
ZETA=modata(2,:)';
PHI=modata([3:nrows],:);

str = ['save ' namout ' FREQ ZETA PHI'];
eval(str);
```

6.0 ITAP-T Theory

6.1 Introductory Remarks

In this chapter, the theoretical foundations of measured time history data analysis, as applied in the ITAP-T Toolbox, are described. Useful single data channel mathematical functions, which describe general quality and content of measured information, are introduced. Effective means of preliminary data quality and content evaluation, utilizing these functions are illustrated. Foundations of spectral and correlation analysis are outlined. System frequency response and coherence function relationships, which involve single or multiple inputs and single or multiple outputs, are reviewed in detail. Recently published applications of spectral and correlation analysis to nonlinear system estimation are also outlined. A highly effective strategy for estimation of modal parameters from estimated frequency response functions, namely the Simultaneous Frequency Domain (SFD) method is described. This method has evolved to permit estimation of real or complex modes for single and multiple input based frequency response functions.

6.2 Preliminary Measured Data Analysis Fundamentals

When engaged in the process of measured data analysis, the essential first task is to review this information for quality and overall content. Once this task is completed, either (a) additional measurements may be deemed necessary or (b) detailed data analyses consistent with project objectives are conducted. The material presented in the following sections provides an overview of preliminary data analysis fundamentals. Attention is focused on classification and evaluation of individual measured data records. Basic statistical quantities and "signature" functions used in preliminary data analysis are described.

6.2.1 Classification of Time History Data Records

In the present discussion, classifications of time history data records are briefly described. An extensive discussion of this subject matter is found in Reference 1. Physical phenomena are either deterministic or random (non-deterministic). A deterministic time history record follows an explicit mathematical relationship (e.g. a sinusoid) while a random time history can only be known in terms of statistical characteristics (e.g. mean value, standard deviation). Deterministic data classifications are summarized in the table below:

Deterministic Time History Records		
Class	Sub-Class	Example
Periodic	Sinusoidal	$A \cdot \sin(\omega t)$
	Complex Periodic	$\sum_{n=1}^{N} A_n \sin(n\omega_0 t + \lambda_n)$
Non-Periodic	Almost Periodic	$\sum_{n=1}^{N} A_n \sin(\omega_n t + \lambda_n)$ where ω_{n+1}/ω_n is not an integer multiple
	Transient	$\sum_{n=1}^{N} A_n e^{-\sigma_n t} \sin(\omega_n t + \lambda_n)$

Several descriptive terms for random data records are now introduced. A collection of time history records from separate "tests" is called an ensemble. A single test record is called stationary if the statistical properties (e.g. mean, standard deviation, etc.) are the same for all sub-records of "reasonable" duration (selected segments from the entire test record). Otherwise, the record is classified as non-stationary. If each data record within an ensemble is stationary and the statistical properties of all

data records are the same, the ensemble is classified as ergodic. Note that all ergodic processes are stationary, but all stationary processes are not ergodic.

Random data classifications are summarized in the table below:

Random Time History Records		
Class	Sub-Class	Example
Stationary	Ergodic	All tests have the same standard deviation
	Non-Ergodic	All tests are stationary, but individual record "intensities" differ
Non-Stationary	See Reference 1, Chapter 12	See Reference 1, Chapter 12

An effective approach for initial classification of time history data records involves review of time history traces of the entire record and selected sub-records. The spectrogram function included in the MATLAB Signal Processing Toolbox (Reference 2) provides a time-frequency content graphic which indicates overall classification of a data record.

In addition to the classifications of measured time history data records defined above, several key functions and parameters are used to describe data quality and content. This subject matter is thoroughly discussed in Reference 1, and key quantities employed in ITAP-T are described in the subsections which follow.

6.2.2 Mean, Variance and Standard Deviation

The (sample) mean value, μ, of a time history data record, [x], with "N" samples, is calculated within MATLAB (the "mean" function) as

$$\mu = (1/N)\sum_{n=1}^{N} x(n)$$

The (sample) variance, σ^2, of a time history data record is calculated as

$$\sigma^2 = (1/N)\sum_{n=1}^{N} [x(n) - \mu]^2$$

The (sample) standard deviation, σ, is simply the square root of the (sample) variance. This quantity is calculated within MATLAB by the “std” function.

6.2.3 Normalized Probability Density and Ideal Gaussian Distribution

The normalized probability density function for a measured time history record is calculated based on the normalized data record, z, defined as

$$z = (x - \mu)/\sigma$$

By dividing the normalized record into “n” equally spaced containers or “bins” between the minimum and maximum values of z, the MATLAB histogram function, h(z), is calculated using the “hist” function. For a data record with a total of N_t samples, the normalized probability density function is defined as

$$p(z) = \left(\frac{n}{z_{max} - z_{min}} \right) \cdot \left(\frac{h(z)}{N_t} \right)$$

The ideal normalized Gaussian probability density function, $p_G(z)$, is defined as

$$p_G(z) = \left(1/\sqrt{2\pi}\right) \cdot e^{-(z^2/2)}$$

Both the actual normalized probability density and normalized Gaussian probability density functions have the integral property,

$$\int_{-\infty}^{\infty} p(z)dz = \int_{-\infty}^{\infty} p_G(z)dz = 1$$

6.2.4 Total Probability Function

The total probability that the normalized time history will fall within a specified range is defined as

$$P(z_1 \le z \le z_2) = \int_{z_1}^{z_2} p(z)dz$$

and the probability that the value of z will not exceed an upper bound, z_2, is defined as

$$P(-\infty \le z \le z_2) = \int_{-\infty}^{z_2} p(z)dz$$

6.2.5 Autospectrum or Power Spectral Density Function

The autospectrum or power spectral density function is defined in the literature by a variety of relationships. For a thorough exposition of this function, see Reference 1. A physically descriptive definition of the autospectrum is based on the way this quantity was estimated prior to the popular use of digital signal processing techniques. The analog process for estimating autospectrum involved four steps, namely:

1. Filter a time history, x(t), with a narrow bandpass filter of bandwidth, Δf, and center frequency, f (Hz), resulting in x(f, Δf, t).

2. Form the square of the filtered signal, $x^2(f, \Delta f, t)$.

3. Calculate the average value of the squared, filtered signal,

$$\overline{x}^2(f,\Delta f,t) = \frac{1}{T}\int_0^T x^2(f,\Delta f,t)dt$$

4. Divide the result by the filter bandwidth, Δf, resulting in the autospectum,

$$G_{xx}(f) = \frac{\overline{x}^2(f,\Delta f,t)}{\Delta f} = \frac{1}{(\Delta f \cdot T)}\int_0^T x^2(f,\Delta f,t)dt$$

The (real valued) autospectrum or power spectral density function provides information on the frequency distribution of the standard deviation of a time history record. It therefore has the following property:

$$\sigma^2 = \int_0^\infty G_{xx}(f)df$$

A modern, state-of-the-art method (implemented in the MATLAB Signal Processing Toolbox, Reference 2) for calculating the autospectrum employs the finite Fourier transform of the time history record. By subdividing the record into "n_d" distinct sub-records (of duration, ΔT) the autospectrum is defined as

$$G_{xx}(f) = \frac{2}{(n_d \Delta T)} \cdot \sum_{i=1}^{n_d} X_i(f) * X_i^*(f)$$

where X(f) is the finite Fourier transform (efficiently computed as a Fast Fourier Transform or FFT).

This function is calculated at discrete frequencies,

$$f_k = k \cdot \Delta f \quad (0 \le k \le n_w/2)$$

where $\Delta f = 1/\Delta T$, $n_w = \Delta T/\Delta t$, and Δt is the sampling time.

6.2.6 Response and Shock Spectrum Functions

In Section 1.2.4, response and shock spectrum functions were defined as "signature" functions, which map the response of sdof oscillators to a subject time history record. The ITAP-A function **shock** is included in ITAP-T for analysis of measured time history records (See Section 1.2.4 and Chapter 10 for additional details). A specific, limited application of response spectrum calculation and display has been implemented for preliminary data analysis in the function routine, **srspec**, which is described in Chapter 10.

6.2.7 Random Decrement Signature Function

Random Decrement theory was developed by H.A. Cole (Reference 3) in 1968. In 1977, S.R. Ibrahim introduced the concept of auto- and cross-Random Decrement functions and applied this more generalized methodology to modal parameter identification of structures (Reference 4). Recently J.C. Asmussen completed a thorough study of Random Decrement theory and applications (Reference 5), which serves as an excellent exposition of this class of functions.

The Random Decrement or Randomdec function for a time history record (assumed to be the stationary random response of a linear dynamic system) is calculated by simple averaging of specific sub-records of the complete record. Each sub-record of duration, tr, begins at a "trigger" time defined by a selected initial displacement or velocity condition. In ITAP-T, estimation of a Randomdec signature is implemented using a "positive zero crossing" trigger. The average of all sub-records, which is the Randomdec signature, can be proven to approximate the free decay response, h(tr), of the subject system.

The ITAP-T function **randec** incorporates several innovations, which enhance convergence of a computed Randomdec signature. Moreover, **randec** permits computation of auto- and cross-Randomdec functions for a multi-channel time history record (see Chapter 10).

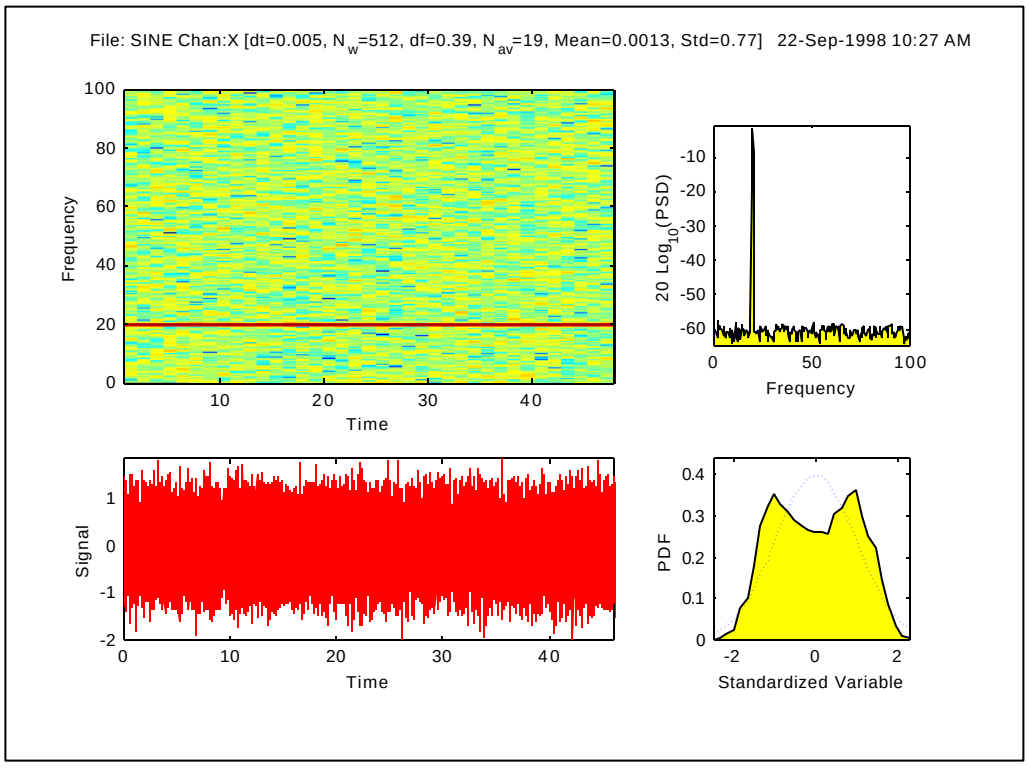
Noting that the Randomdec signature corresponds (approximately) to the free decay response of the subject dynamic system, its Fourier transform resembles the drive point frequency response, H(f), of that system. The ITAP-T function **rdec** computes and displays the Randomdec signature and its Fourier transform for a selected time history record.

6.3 Preliminary Measured Data Analysis Examples

A series of illustrative examples are provided to demonstrate application of preliminary data analysis fundamentals in various situations. Overall classification of individual time history records is effected through calculation and display of the time history, spectrogram, autospectrum and probability density function. Data quality and content is evaluated by closer examination of the autospectrum, probability density and total probability, response spectrum and Randomdec signature as judged necessary by the ITAP-T user.

6.3.1 Sinusoidal Time History Record with Background Random Noise

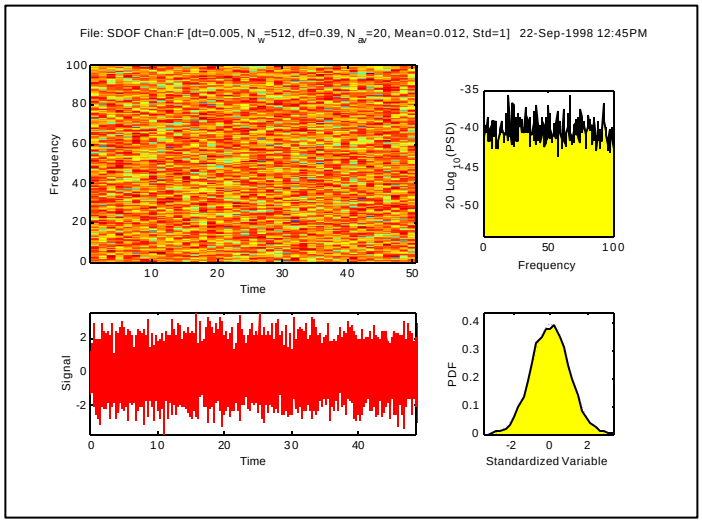
Consider a measured time history record consisting of a 20 Hz sinusoidal signal, contaminated by broad band random noise, and sampled at dt=.005 second over a duration of 50 seconds. The (ITAP-T standard) preliminary data analysis display shown below includes a color spectrogram (upper left), time history trace (lower left), autospectrum (upper right) and probability density (lower right). The dashed curve in the probability density plot indicates the ideal Gaussian probability density function. In addition, the title indicates the window length (N_W), frequency bandwidth (Δf) and number of distinct averages (N_{av}) of autospectrum sub-records, as well as the mean and standard deviation (Std) values for the data record.

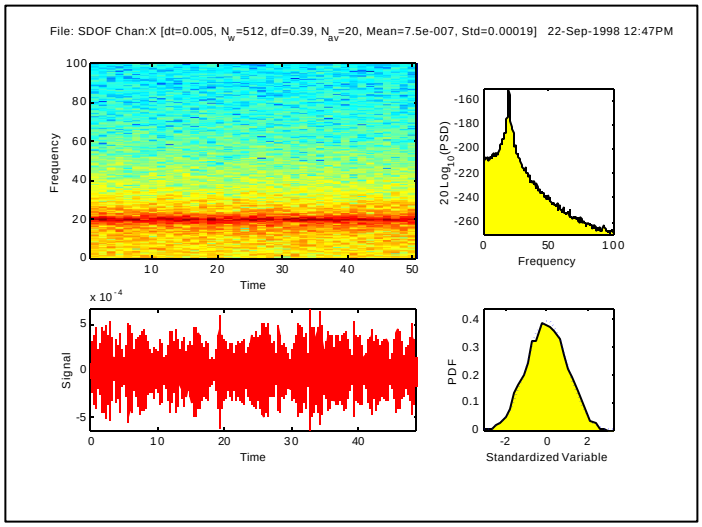


Based on general uniformity of the spectrogram and time history plots with respect to time, the data record is judged stationary. Both the spectrogram and autospectrum indicate that the record is dominated by a 20 Hz sinusoidal signal. The record, in spite of the presence of broad band noise, may be classified as predominantly deterministic due to general consistency of the sinusoidal component. The probability density function differs substantially from the ideal Gaussian distribution in a manner suggesting a sinusoid (see Reference 1, Chapter 3).

6.3.2 SDOF Linear System Response to Broad Band Random Excitation

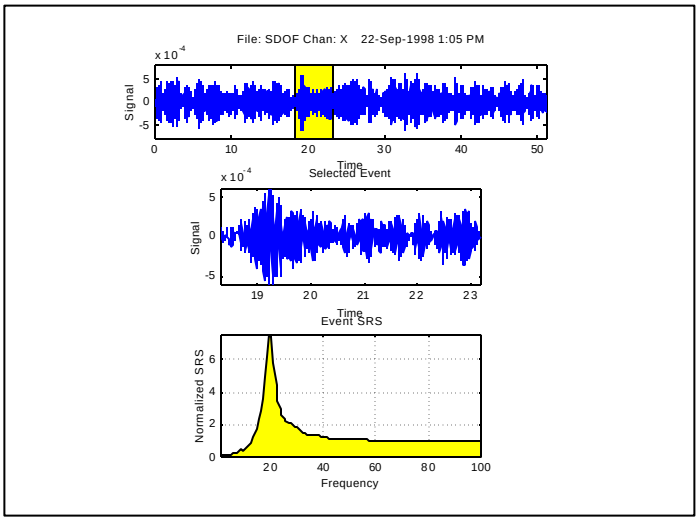
A SDOF linear system is subjected to broad band random excitation, F(t), and has displacement response, x(t). Both excitation and response are sampled at dt=.005 second over a duration of 50 seconds. The (ITAP-T standard) preliminary data analysis displays for excitation, F(t) and response, x(t), are shown below.





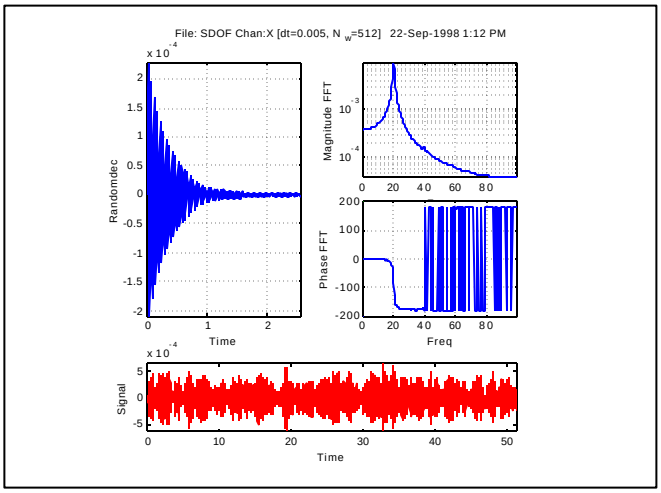
General uniformity of the spectrogram and time history plots with respect to time for both excitation and response records indicate that the process is stationary. Both the spectrogram and autospectrum for the excitation indicate broad band frequency content. The Gaussian probability density distribution indicates that the excitation is random. The output, x(t), spectrogram and autospectrum reveal narrow band random response character. Gaussian probability density distribution for the response history suggests that the system is linear (See Reference 6, Chapter 2).

General frequency content of system response is indicated by the response spectrum plot, which is shown below:



The upper plot in the above figure is the complete response record with a user selected time segment highlighted. The central plot presents the selected time history segment, and the lower plot is the normalized response spectrum associated with the selected time history segment.

Finally, results of Randomdec analysis of the response time history are shown below:



The above figure includes a plot of the Randomdec signature (upper left), magnitude and phase of the Randomdec's Fourier transform (upper right), and the response time history (lower). Note that the Randomdec's Fourier transform resembles a drive point frequency response function.

6.3.3 Nonlinear SDOF System Response to Random Excitation

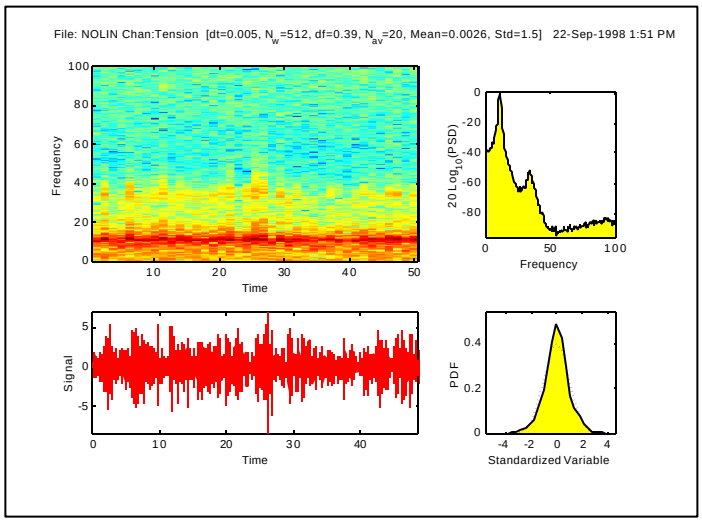
Consider a nonlinear SDOF system, of a Duffing type (Reference 5), described by the following differential equation and internal loading (tension) relationship:

$$M\ddot{x} + B\dot{x} + Kx + K_3 x^3 = F(t)$$

$$\text{Tension} = T = Kx + B\dot{x} + K_3 x^3$$

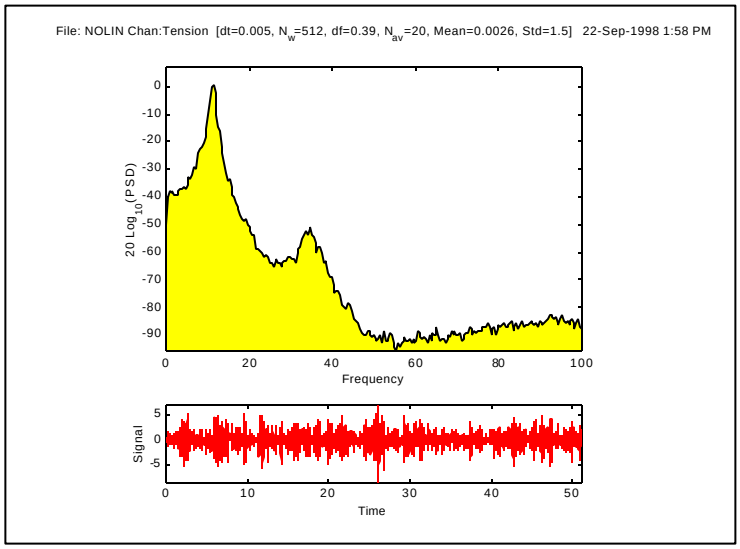
where M=1.0, B=6.2832, K=3.948 X 10^3, K_3=3.948 X 10^9. This system is subjected to a broad band random excitation, F(t). Time history records for F(t), x(t) and T(t) are measured.

Preliminary analysis displays (not shown) for F(t) and x(t) are similar to those from the previous linear SDOF example problem. The results for tension loading, T(t), however provide revealing information about this system as shown below:

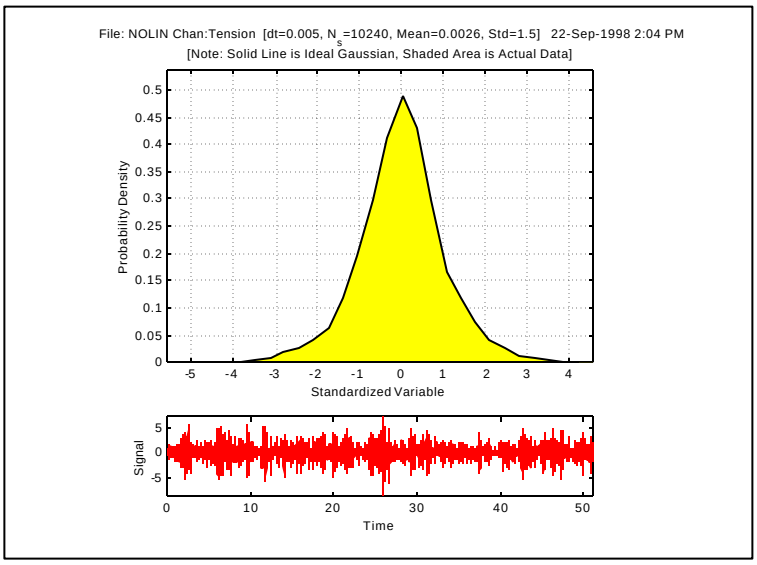


As for the previous example problem, the histogram and time history plots suggest a stationary process. The autospectrum and probability density merit closer review.

A detailed display of the autospectrum for tension, T(t), shown below indicates pronounced activity at ~11 and ~33 Hz (three times the fundamental harmonic).



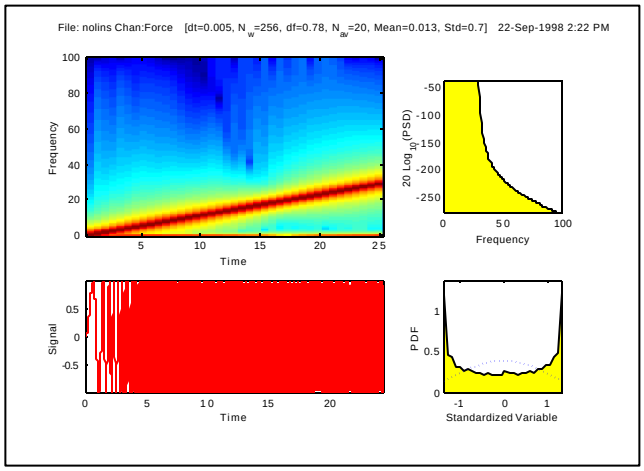
The probability density associated with tension, T(t), shown below reveals deviation from the ideal Gaussian distribution indicative of a “hardening” spring (see Reference 6, Chapter 2).

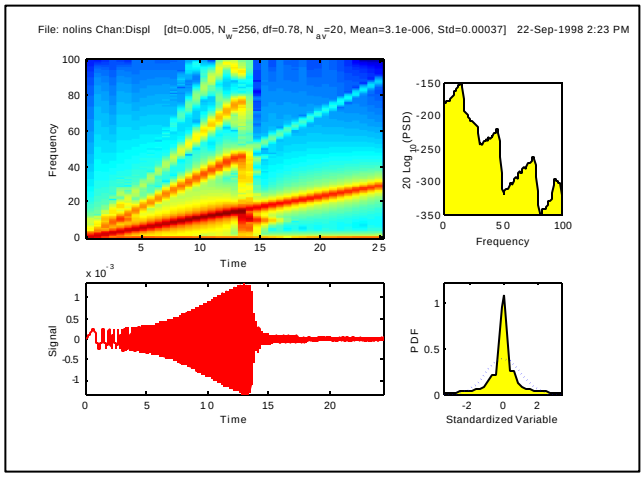


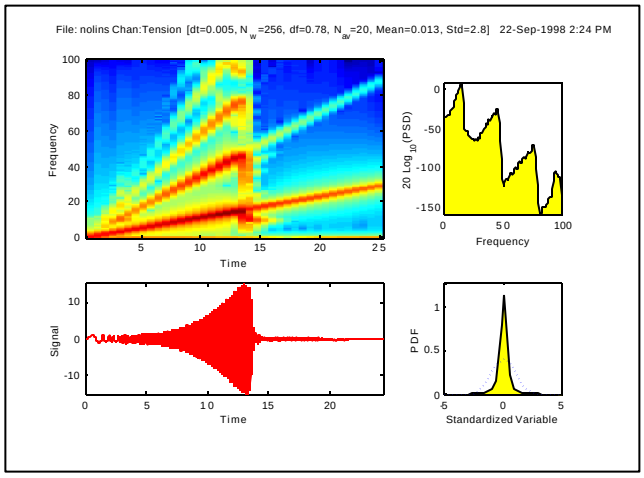
Based on the autospectrum and probability density functions, additional data analyses focusing on estimation of nonlinear system characteristics are warranted.

6.3.4 Nonlinear SDOF System Response to Swept Sine Excitation

The nonlinear system studied in the previous example problem is now subjected to a swept sine excitation force, F(t), and time history data is recorded. Results of preliminary data analysis of excitation force, F(t), displacement response, x(t), and tension loading, T(t), respectively, are shown below:



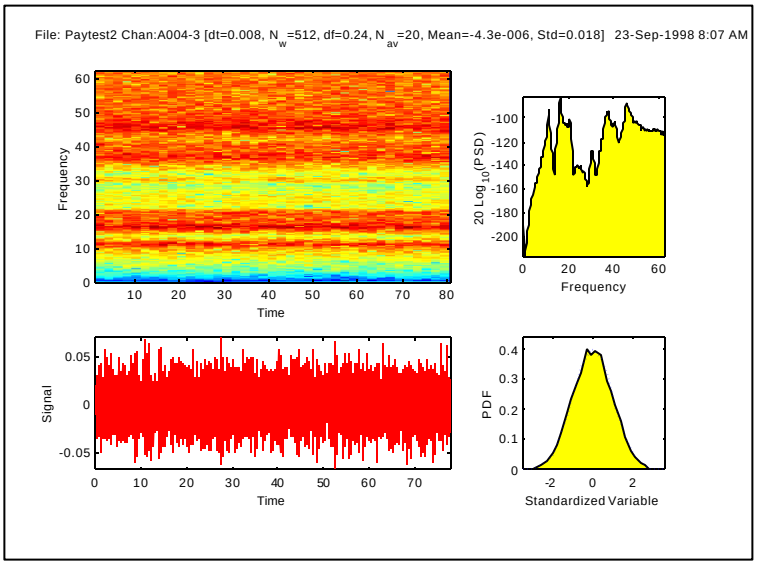




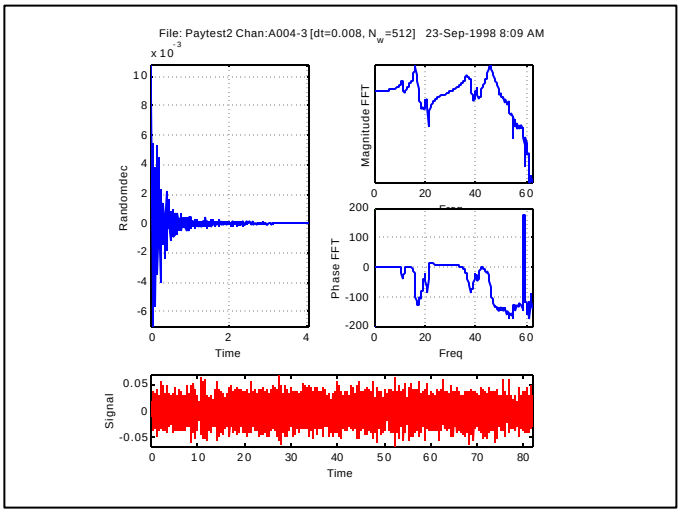
The spectrograms and time histories for all three individual data records clearly indicate that the process is non-stationary. In addition, the presence of multiple harmonics in x(t) and T(t) in response to excitation, F(t), which consists of a single swept harmonic suggests nonlinear system behavior.

6.3.5 Linear MDOF System Response to Broad Band Random Excitation

Shown below are preliminary data analysis results for an individual response channel of a linear MDOF system. The spectrogram and time history plots indicate that the process is stationary. The autospectrum plot illustrates the presence of a series of vibration modes. The probability density plot is approximately Gaussian suggesting the system is linear.



It is also of interest to note the Randomdec signature for this response channel as shown below:



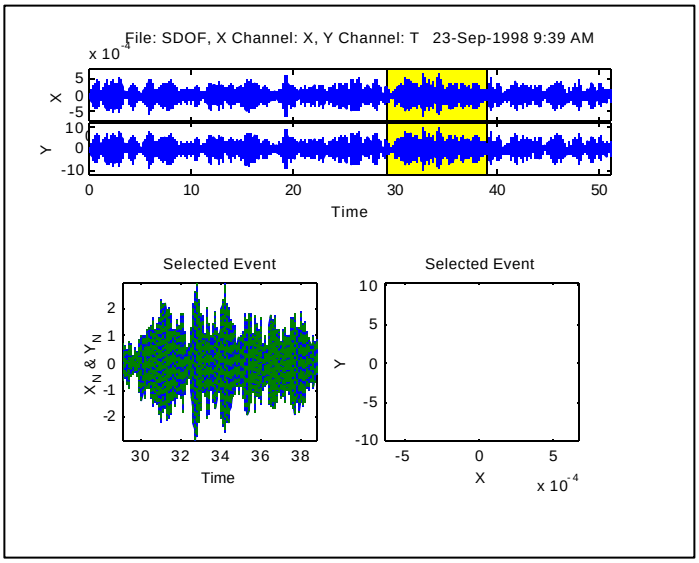
6.3.6 Preliminary Channel Pair Analysis

In certain situations, pairs of measured data records may be reviewed graphically for the preliminary evaluation of suspected nonlinear behavior. The relevant display is a plot of one time history record (X) with respect to another (Y). When internal (member) loading and (member) displacement data is available, a clear picture of linear or nonlinear behavior is obtained. The ITAP-T function **pairevu** permits the user to select a channel (X,Y) pair and time segment of interest and display X versus Y. Several examples are provided herein to demonstrate utility of this type of preliminary data analysis.

Consider the linear SDOF system, discussed in Subsection 6.3.2 and described by the following differential equation and internal loading:

$$M\ddot{x} + B\dot{x} + Kx = F(t)\ ,\ \text{Tension} = T = Kx + B\dot{x}$$

This system is subjected to a broad band random excitation, F(t). Time history records for F(t), x(t) and T(t) are measured. The channel pair display for x versus T is shown below:

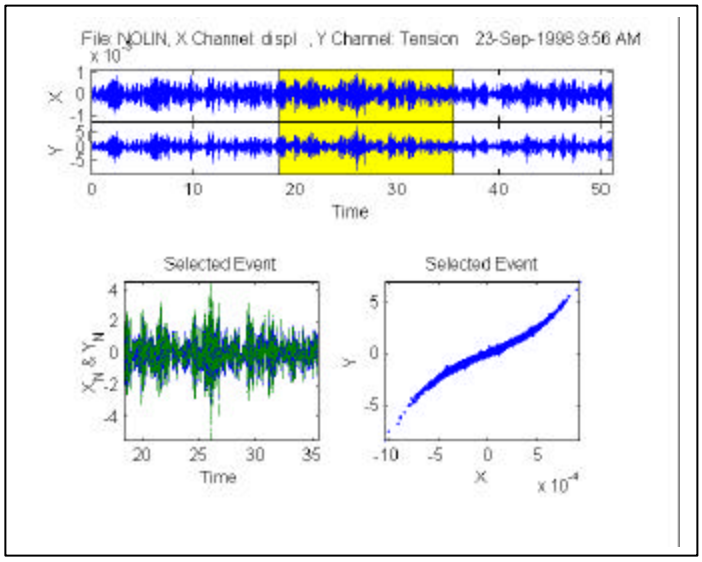


The above X-Y plot clearly indicates a linear tension (T) versus displacement (x) relationship. The width of the tension versus displacement “cloud” is narrow yet “elliptic” in form indicative of viscous damping.

Consider the nonlinear SDOF system (which was discussed in Subsection 6.3.3), described by the following differential equation and internal loading (tension) relationship:

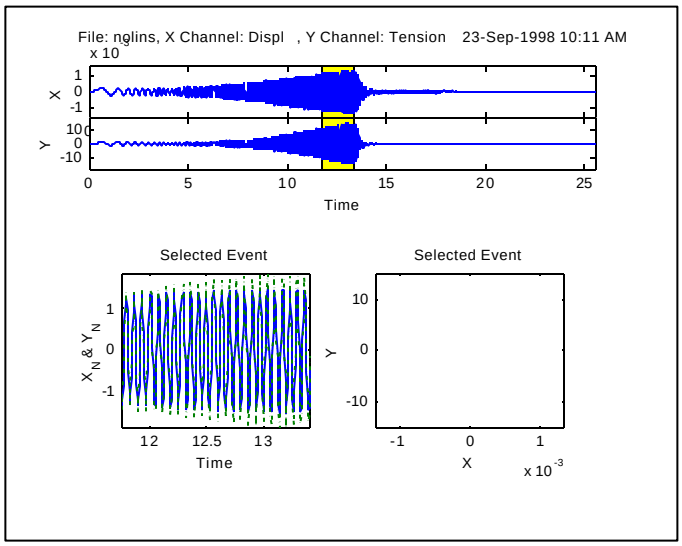
$$M\ddot{x} + B\dot{x} + Kx + K_3 x^3 = F(t)\,, \quad \text{Tension} = T = Kx + B\dot{x} + K_3 x^3$$

This system is subjected to a broad band random excitation, F(t). Time history records for F(t), x(t) and T(t) are measured. The channel pair display for x versus T is shown below:



The above X-Y plot clearly indicates a nonlinear tension (T) versus displacement (x) relationship.

Preliminary evaluation of the above nonlinear system may be continued with swept sine excitation testing (discussed previously in Subsection 6.3.4). By selecting a near peak amplitude time segment for that data, a very clear picture of nonlinear tension versus displacement is obtained as shown below:



6.4 SI/SO and SI/MO Spectral and Correlation Analysis

After completion of preliminary data evaluations, detailed analyses are often performed to estimate input-output characteristics of a subject dynamic system. The following sections provide an overview of single input/single output (SI/SO) and single input/multiple output (SI/MO) data analysis procedures as they are implemented in ITAP-T. Spectral and correlation analysis of SI/SO and SI/MO data records is the primary tool used for estimation of system frequency response function (FRF) characteristics.

6.4.1 The Cross Spectral Density Function

The cross (power) spectral density function between two records, x(t) and y(t), is defined in a manner similar to the autospectrum or power spectral density (Reference 1). A modern, state-of-the-art method for calculating cross-spectral density (implemented in the MATLAB Signal Processing Toolbox, Reference 2) employs the finite Fourier transform of time history records. By subdividing the records into "n_d" distinct sub-records (of duration, ΔT) the (complex) cross-spectral density function is defined as

$$G_{yx}(f) = \frac{2}{(n_d \Delta T)} \cdot \sum_{i=1}^{n_d} Y_i(f) * X_i^*(f)$$

where Y(f) is the finite Fourier transform of y(t) and X*(f) is the complex conjugate finite Fourier transform of x(t). The subscript notation differs from that adopted in References 1 and 6 for reasons which will become clear in the discussion of MI/SO and MI/MO analysis.

6.4.2 Optimum SI/SO Frequency Response Function Estimation

A SI/SO dynamic system is defined by the (convolution integral) relationship

$$y(t) = \int_0^t h(t-\tau)x(\tau)d\tau + n(t)$$

where x(t) is the input or excitation, y(t) is the output or response and n(t) is "noise" which is assumed to be "uncorrelated" with respect to x(t). The Fourier transform of the above equation is

$$Y(f) = H(f) \cdot X(f) + N(f)$$

Consider the situation in which time history records x(t) and y(t) are measured over a duration, $0 < t < T$.

If the total records are subdivided into "n_d" distinct sub-records (of duration, ΔT) the finite Fourier transforms of sub-records are

$$Y_i(f) = H(f) \cdot X_i(f) + N_i(f)$$

After multiplying the above equation by "$2X_i^*(f)/(n_d \Delta T)$, and averaging all sub-records, we obtain

$$\frac{2}{(n_d \Delta T)} \cdot \sum_{i=1}^{n_d} Y_i(f) * X_i^*(f) = H(f) \cdot \frac{2}{(n_d \Delta T)} \cdot \sum_{i=1}^{n_d} X_i(f) * X_i^*(f) + \frac{2}{(n_d \Delta T)} \cdot \sum_{i=1}^{n_d} N_i(f) * X_i^*(f)$$

Recalling the definitions of autospectrum and cross-spectral density (Subsections 6.2.5, 6.4.1), the above equation is written as

$$G_{yx}(f) = H(f) \cdot G_{xx}(f) + G_{nx}(f)$$

Since the input, x(t), and noise, n(t), are uncorrelated, $G_{nx}(f)=0$, and

$$H(f) = G_{yx}(f) / G_{xx}(f)$$

The frequency response function, calculated by the above relationships, has been proven in Reference 1, Chapter 6 to be an optimum estimate.

6.4.3 The Ordinary Coherence Function

Recalling the sub-record finite Fourier transform relationship

$$Y_i(f) = H(f) \cdot X_i(f) + N_i(f),$$

form the product of $Y_i(f) \cdot Y_i^*(f)$, and average all sub-records, to obtain

$$G_{yy}(f) = |H(f)|^2 \cdot G_{xx}(f) + G_{nn}(f) .$$

where coherent output autospectrum is

$$G_{vv}(f) = G_{yy}(f) - G_{nn}(f) = |H(f)|^2 G_{xx}(f)$$

The ordinary coherence function is defined as the ratio of autospectra

$$\gamma_{yx}^2(f) = G_{vv}(f) / G_{yy}(f) = 1 - G_{nn}(f) / G_{yy}(f) = |G_{yx}(f)|^2 / (G_{xx}(f) \cdot G_{yy}(f))$$

which is positive valued with an upper bound of 1.0 (for the case of no noise, G_{nn}=0).

6.4.4 Windowing and Overlap Processing

The MATLAB Signal Processing Toolbox offers the user flexibility in the calculation of SI/SO data processing. Windowing (see Reference 2, Chapter 4) helps to reduce a phenomenon called “leakage” as will be demonstrated in illustrative examples. The default window function in the MATLAB Signal Processing Toolbox function **spectrum** is the Hanning window. In addition, segmentation and averaging of data records into windows of length “n_w” may be performed on overlapping windows. For example if a data record has N_t=10240 time samples and a window length, n_w = 1024 is selected, **spectrum** performs (default) averaging of disjoint records for 10 windows [1-1024], [1025-2048],etc. If the “overlap” option is selected, averaging (50 percent is typically recommended (see Reference 1, Chapter 11) is performed on records [1-1024], [513-1536], [1025-2048],etc.

6.4.5 SI/MO Analysis

Single Input/Multiple Output or SI/MO analysis represents a simple extension to SI/SO analysis. If a set of time histories associated with a system test article are measured where x(t) is the input or excitation source and $y_1(t)$, $y_2(t)$, $y_3(t)$,…,$y_N(t)$ are the outputs or responses, then SI/MO analysis consists of a series of SI/SO analyses on the input-output pairs [x(t), $y_1(t)$], [x(t), $y_2(t)$], [x(t), $y_3(t)$],…, [x(t), $y_N(t)$]. The ITAP-T Toolbox function **mimo** performs SI/MO calculations on the time history data arrays,

[X] = (N_t X 1) input matrix (single input),

[Y] = (N_t X N_{out}) matrix (N_{out} = number of outputs)

with user specified parameters,

NFFT = window length,

NOVL= overlap index (0 for no ovelap, NFFT/2 for 50% ovelap).

The output frequency array is formed as,

[frq] = [0,1,2,NFFT/2] · df = (1 X N_f) matrix

df = 1/(NFFT·dt) = band width resolution

and the following results matrices are calculated:

[H] = (N_f X N_{out}) frequency response function matrix

[COH] = (N_f X N_{out}) ordinary coherence function matrix.

The default window for the ITAP-T function routine **mimo** is “Hanning”.

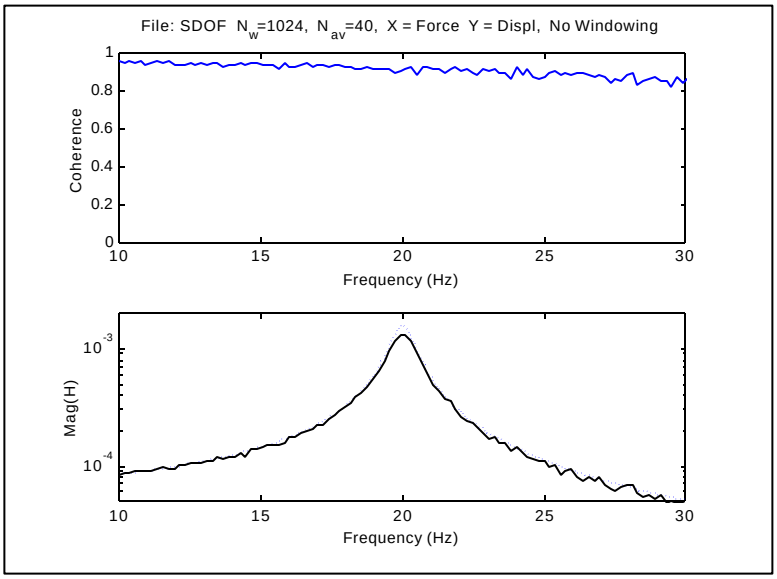
6.5 SI/SO and SI/MO Analysis Examples

Several illustrative examples are provided to demonstrate various aspects of SI/SO and SI/MO analysis. The linear SDOF system discussed in Subsections 6.3.2 and 6.3.6 is used to illustrate aspects of averaging, windowing, overlap processing. The nonlinear SDOF system discussed in Subsections 6.3.3 and 6.3.6 is used to demonstrate the effects of nonlinearity on ordinary coherence. Finally, excitation and response records for a linear 3 DOF system are used to demonstrate SI/MO analysis.

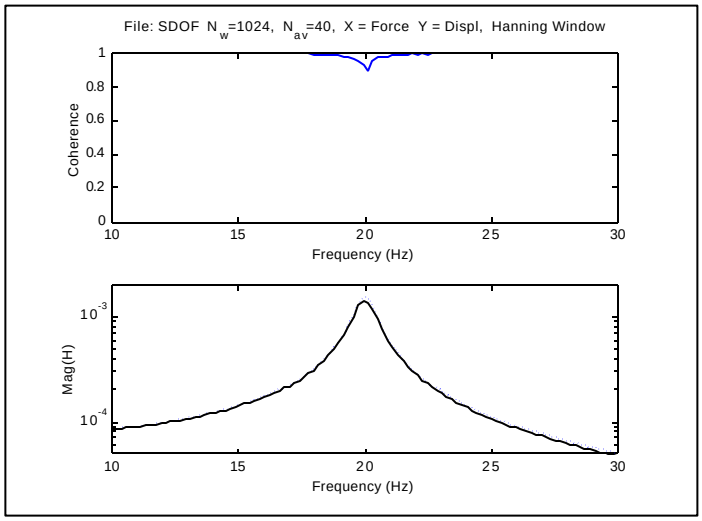
6.5.1 Linear SDOF System SI/SO Analysis

The example linear SDOF system has an undamped natural frequency, f_n=20 Hz, and critical damping ratio, ζ_n=0.02. Simulated test data records for random excitation and displacement response were calculated for a record length, N_t=40960 at a sampling rate, dt=0.005 Sec.

SI/SO analysis was first performed for disjoint sub-record windows of length, n_w=1024 (40 averages). When no windowing is employed, the ordinary coherence and frequency response function estimate are as illustrated below:



The dashed curve in the frequency response plot corresponds to the exact frequency response function. When a Hanning window is employed, the coherence function increases to nearly unit level (with a narrow band dip near the resonant frequency) and the frequency response estimate has decreased random error, as shown below:



Employment of a Hanning window generally improves SI/SO estimates when the data record is of a continuous random nature. If the data record consists of a series of transient decays (as occurs in impulsive testing), it can be shown that no windowing may be more appropriate.

Employment of overlap processing generally improves the quality of frequency response function estimates. However, negligible improvement is realized in the present case.

The user always makes a trade-off decision on window length, n_w, versus the number of averages, $n_d = N_t/n_w$, and frequency resolution bandwidth is $df = 1/(n_w \cdot dt)$. As a general guideline, the bandwidth resolution required to adequately resolve a resonant response peak is

$$(df)_{min} \approx \zeta_n f_n / 2$$

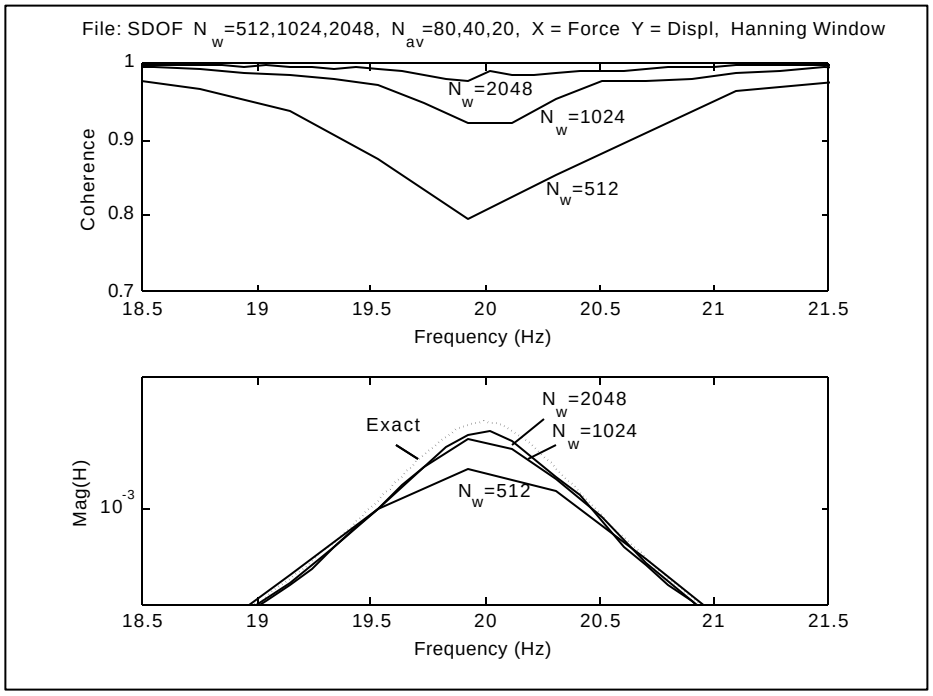
For the present example system the minimum requirement is

$$(df)_{min} \approx \zeta_n f_n / 2 = .02 \cdot 20 / 2 = 0.2$$

and the required minimum window length is

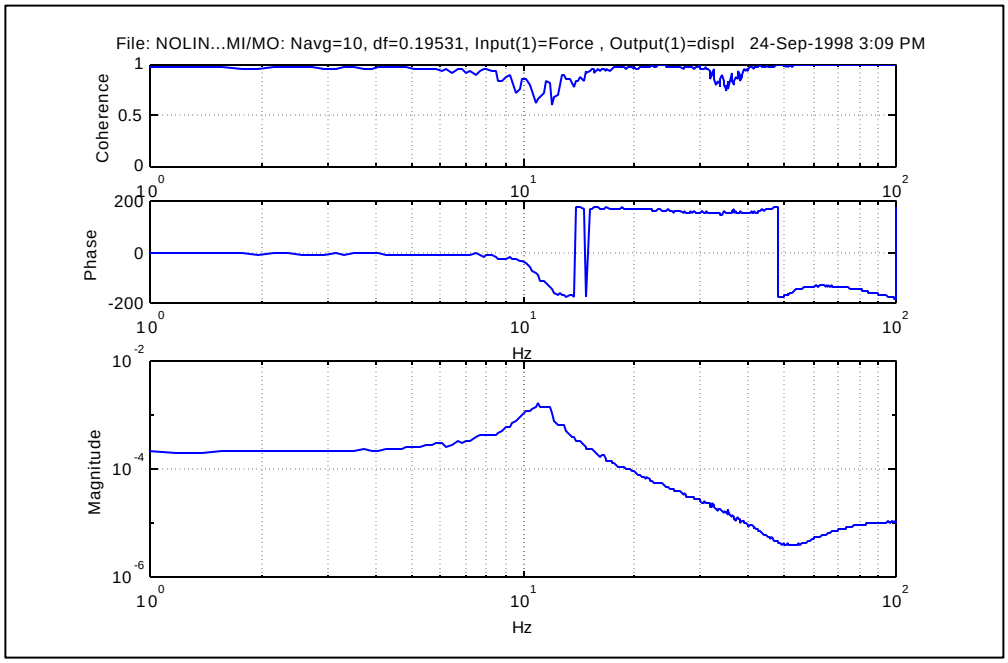
$$n_w \approx 1/(df \cdot dt) = 1/(0.2 \cdot 0.005) = 1000$$

SI/SO results in the frequency band 18.5<f<21.5 are illustrated below for various selections of window length (bandwidth resolution):



6.5.2 Nonlinear SDOF System SI/SO Analysis

The example nonlinear SDOF system introduced in Section 6.3.3 has a record length, N_t=10240 and sampling rate, dt=0.005 Sec. SI/SO analysis of this system (input=force, output=displacement) for a window length, n_w=1024, is shown below:



Reduced coherence in the 8-16 Hz and 30-40 Hz frequency bands is associated with the presence of nonlinearity, the presence of which is strongly suspected based on the preliminary evaluation discussed in Subsection 6.3.6.

6.5.3 Linear MDOF System SI/MO Analysis

Consider a 3 DOF linear system described by the following matrix equation set:

$$[M]\{\ddot{U}\}+[B]\{\dot{U}\}+[K]\{U\}=[\Gamma]\cdot F(t)$$

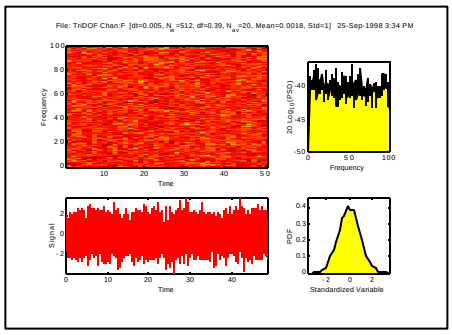
where

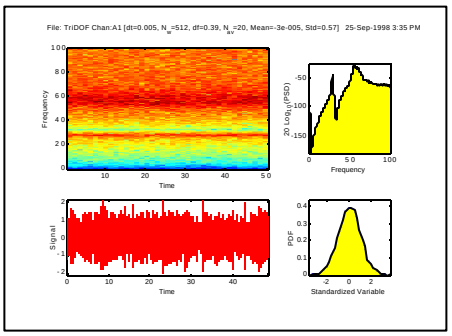
$$[M]=\begin{bmatrix}5.0 & 0 & 0\\ 0 & 1.0 & 0\\ 0 & 0 & 1.4\end{bmatrix},\quad [K]=\begin{bmatrix}60 & -5 & -5\\ -5 & 10 & -5\\ -5 & -5 & 10\end{bmatrix}\cdot 10^4,$$

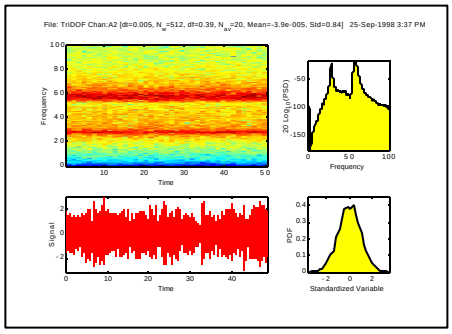
$$[B]=\begin{bmatrix}60 & -5 & -5\\ -5 & 10 & -5\\ -5 & -5 & 10\end{bmatrix},\quad [\Gamma]=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$$

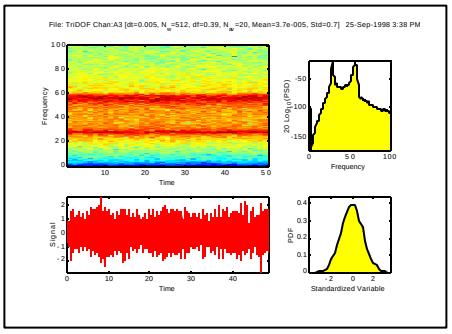
This system is excited by broad band random excitation, F(t), of 51.2 Sec duration and sampled at dt=.005 Sec. Response accelerations for the three DOFs was calculated by numerical simulation using the ITAP-A function **mtresp**.

Results of preliminary data analysis are shown below:

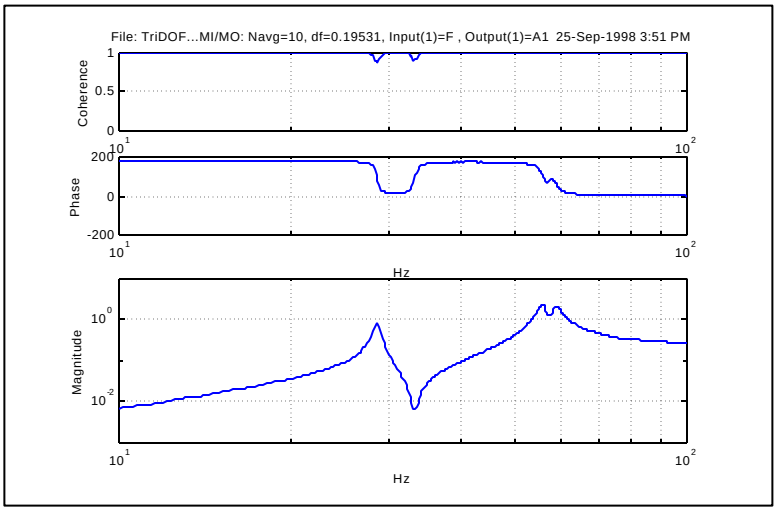


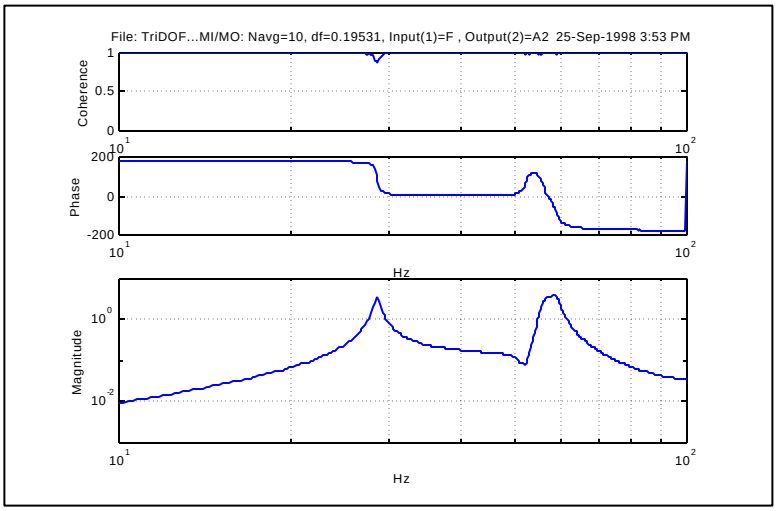


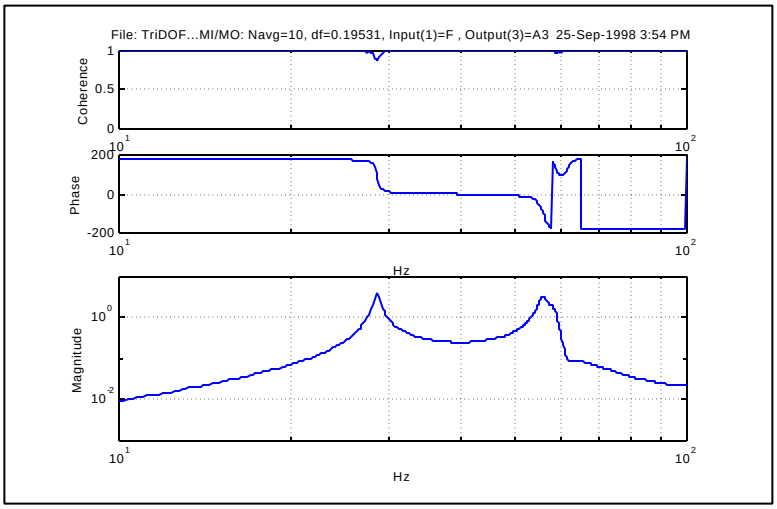




SI/MO analysis was performed on the time history data records using the ITAP-T function **mimo** to obtain frequency response functions associated with the input and three respective acceleration responses, shown below:







Inspection of the three frequency response functions suggests that the system has three responding modes. Additional understanding of system characteristics is realized by the use of various graphical displays; this subject is discussed in Subsection 6.8.1.

6.6 MI/SO and MI/MO Spectral and Correlation Analysis

The following sections provide an overview of multiple input/single output (MI/SO) and multiple input/multiple output (MI/MO) data analysis procedures as they are implemented in ITAP-T. Spectral and correlation analysis procedures employed in MI/SO and MI/MO analyses build upon the formulations described in the sections on SI/SO and SI/MO data analysis.

6.6.1 Linear System Response due to Multiple Excitations

Consider a linear system, which responds to a series of applied excitations. The response of a single output variable, y(t) to the excitations, $x_1(t)$, $x_2(t)$, …, $x_N(t)$ is of the Duhamel integral form

$$y_j(t) = \sum_{k=1}^{N}\left[\int_0^t h_{jk}(t-\tau)x_k(\tau)d\tau\right] + n_j(t)$$

The Fourier transform of the above equation is

$$Y_j(f) = \sum_{k=1}^{N}[H_{jk}(f)\cdot X_k(f)] + N_j(f)$$

Describing the above equation n matrix form (dropping the “j” subscript for simplicity) the multiple input-single output (MI/SO) relationship is

$$Y(f) = \left[H_{YX}(f)\right]\cdot\{X(f)\} + N(f)$$

where $[H_{YX}(f)]$ is the “row” frequency response function matrix with column dimension corresponding to the number of excitations.

Following the manipulations employed in the development of SI/SO spectral equations, the total record array is subdivided into “n_d” distinct sub-records (of duration, ΔT) yielding the finite Fourier transform relationship for sub-records

$$Y(f)\big|_i = \left[H_{YX}(f)\right]\cdot\{X(f)\}_i + N(f)\big|_i$$

After multiplying the above equation by “$2[X^*]_i/(n_d\Delta T)$, and averaging all sub-records, we obtain

$$\frac{2}{(n_d\Delta T)}\sum_{i=1}^{n_d} Y_i * \left[X^*\right]_i = \left[H_{YX}\right]\frac{2}{(n_d\Delta T)}\sum_{i=1}^{n_d} X_i * \left[X^*\right]_i + \frac{2}{(n_d\Delta T)}\sum_{i=1}^{n_d} N_i * \left[X^*\right]_i$$

where the frequency dependence symbol is dropped for simplicity. The above equation is written in “cross-spectral” matrix form as

$$[G_{yx}(f)] = [H_{YX}(f)] \cdot [G_{xx}(f)] + [G_{nx}(f)]$$

Since the inputs, $x_k(t)$, and noise, n(t), are uncorrelated, $[G_{nx}(f)]=[0]$, and

$$[G_{YX}(f)] = [H_{YX}(f)] \cdot [G_{xx}(f)]$$

We are tempted to solve for the frequency response matrix by direct inversion of the input cross-spectral matrix, $[G_{XX}(f)]$. However there are several shortcomings to such a strategy, namely,

(1) If any excitation pairs are fully (or nearly) correlated, $[G_{XX}(f)]$ will be singular (or nearly singular).

(2) Multiple input coherence will not be in a useful, diagnostic form.

6.6.2 MI/SO Analysis using Triangular Decomposition

The concept of MI/SO analysis using triangular decomposition, along with an explanation of key results, is thoroughly developed by Bendat and Piersol in Reference 1, Chapters 6 and 7. That approach has been applied in the ITAP-T Toolbox using MATLAB matrix operations as described in the present section.

Cholesky factorization of the input cross-spectral matrix, $[G_{XX}(f)]$ yields

$$[G_{xx}(f)] = [\Gamma_{XZ}(f)] \cdot [\Gamma_{ZX}(f)]$$

where $[\Gamma_{XZ}(f)]$ is a complex, lower triangular matrix and $[\Gamma_{ZX}(f)]$ is its upper triangular, complex conjugate transpose (note that $[G_{XX}(f)]$ is a Hermitian matrix). A triple product decomposition version of Cholesky factors is formed by unit diagonal normalization of $[\Gamma_{XZ}(f)]$ and $[\Gamma_{ZX}(f)]$ resulting in

$$[G_{xx}(f)] = [L_{XZ}(f)] \cdot [G_{ZZ}(f)] \cdot [L_{ZX}(f)]$$

where $[L_{XZ}(f)]$ and $[L_{ZX}(f)]$ are lower and upper triangular, unit diagonal matrices, respectively, and $[G_{ZZ}(f)]$ is a positive-definite, diagonal matrix.

Physical significance of the normalized triangular factors is recognized by noting the input variable trasformation

$$\{X(f)\} = [L_{XZ}(f)] \cdot \{Z(f)\}$$

The successive terms in {Z(f)} are (see Reference 1)

$Z_1(f) = X_1(f)$

$Z_2(f) = X_2(f)$ "swept" of contributions from $X_1(f)$

$Z_3(f) = X_3(f)$ "swept" of contributions from $X_1(f)$ and $X_2(f)$

etc.

Therefore, $[G_{ZZ}(f)]$ is the diagonal autospectrum matrix of uncorrelated, generalized inputs, {Z(f)}.

Substitution of the triple product $[G_{XX}(f)]$ decomposition relationship into the MI/SO spectral equation set results in the frequency response functions associated with uncorrelated generalized inputs, {Z(f)},

$$[H_{YZ}] = [G_{YZ}] \cdot [G_{ZZ}]^{-1}$$

where

$$[H_{YZ}] = [H_{YX}] \cdot [L_{XZ}] \text{ , } [G_{YZ}] = [G_{YX}] \cdot [L_{ZX}]^{-1}$$

The frequency responses associated with physical inputs, {X(f)}, are

$$[H_{YX}] = [H_{YZ}] \cdot [L_{XZ}]^{-1}$$

6.6.3 The Cumulative Coherence Function Family

The generalized MI/SO equation in terms of generalized inputs is

$$Y(f) = \left[H_{YZ}(f)\right] \cdot \left\{Z(f)\right\} + N(f)$$

Since the inputs, $Z_i(f)$, are uncorrelated, the output autospectrum is

$$G_{YY} = \left|H_{YZ_1}\right|^2 \cdot G_{Z_1Z_1} + \left|H_{YZ_2}\right|^2 \cdot G_{Z_2Z_2} + ... + \left|H_{YZ_N}\right|^2 \cdot G_{Z_KZ_K} + G_{NN}$$

Coherent output autospectra and cumulative coherence function pairs are therefore defined as follows:

For input, $x_1(t)$ -

$$(G_{YY})_1 = \left|H_{YZ_1}\right|^2 \cdot G_{Z_1Z_1}$$

$$\gamma^2_{Y1} = \left(\left|H_{YZ_1}\right|^2 \cdot G_{Z_1Z_1}\right) / G_{YY}$$

For inputs, $x_1(t)$ and $x_2(t)$

$$\left(G_{YY}\right)_2 = \left|H_{YZ_1}\right|^2 \cdot G_{Z_1Z_1} + \left|H_{YZ_2}\right|^2 \cdot G_{Z_2Z_2}$$

$$\gamma^2_{Y2} = \left(\left|H_{YZ_1}\right|^2 \cdot G_{Z_1Z_1} + \left|H_{YZ_2}\right|^2 \cdot G_{Z_2Z_2}\right) / G_{YY}$$

For inputs, $x_1(t)$, $x_2(t)$, … , and $x_K(t)$ -

$$\left(G_{YY}\right)_K = \left|H_{YZ_1}\right|^2 \cdot G_{Z_1Z_1} + \left|H_{YZ_2}\right|^2 \cdot G_{Z_2Z_2} + ... + \left|H_{YZ_N}\right|^2 \cdot G_{Z_KZ_K}$$

$$\gamma^2_{YK} = \left(\left|H_{YZ_1}\right|^2 \cdot G_{Z_1Z_1} + \left|H_{YZ_2}\right|^2 \cdot G_{Z_2Z_2} + ... + \left|H_{YZ_N}\right|^2 \cdot G_{Z_KZ_K}\right) / G_{YY}$$

The cumulative coherence function family has the property

$$0 \le \gamma^2_{Y1} \le \gamma^2_{Y2} \le ... \le \gamma^2_{YK} \le 1.0$$

6.7 MI/SO and MI/MO Analysis Examples

Several illustrative examples are provided to demonstrate various aspects of MI/SO and MI/MO analysis.

6.7.1 Linear MDOF System MI/MO Analysis

Consider the 3 DOF linear system (introduced in Section 6.6.3) described by the following matrix equation set:

$$[M]\{\ddot{U}\}+[B]\{\dot{U}\}+[K]\{U\}=[\Gamma]\cdot\{F(t)\}$$

where

$$[M]=\begin{bmatrix}5.0 & 0 & 0\\ 0 & 1.0 & 0\\ 0 & 0 & 1.4\end{bmatrix},\quad [K]=\begin{bmatrix}60 & -5 & -5\\ -5 & 10 & -5\\ -5 & -5 & 10\end{bmatrix}\cdot 10^4\ ,$$

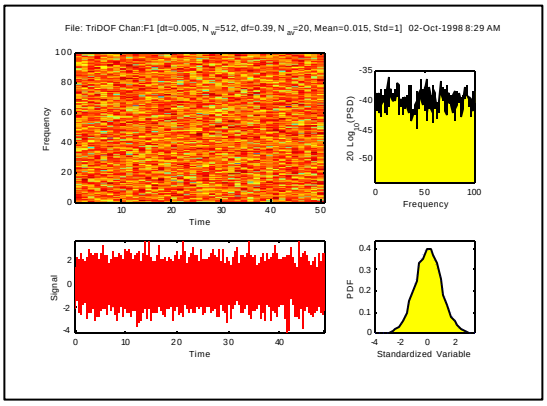
$$[B]=\begin{bmatrix}60 & -5 & -5\\ -5 & 10 & -5\\ -5 & -5 & 10\end{bmatrix},\quad [\Gamma]=\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}$$

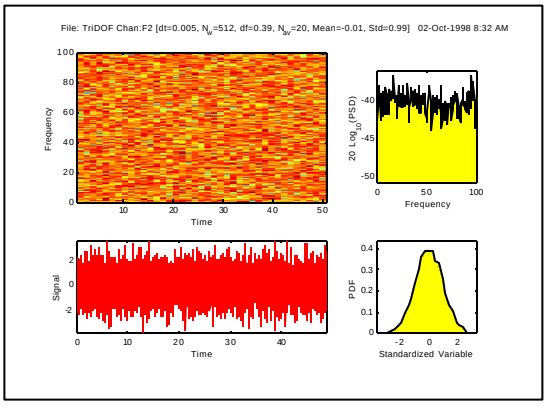
This system is excited by two separate broad band random excitation forces,

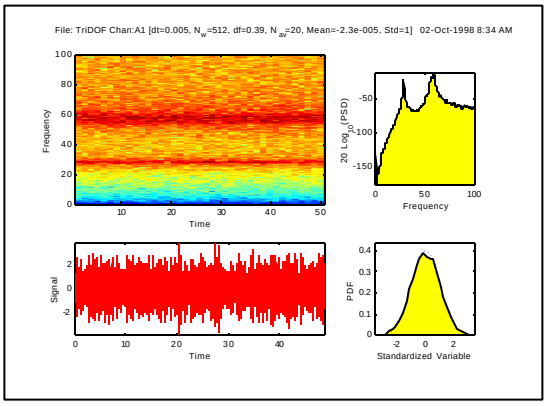
$$\{F(t)\}=\begin{Bmatrix}F_1(t)\\ F_2(t)\end{Bmatrix}$$

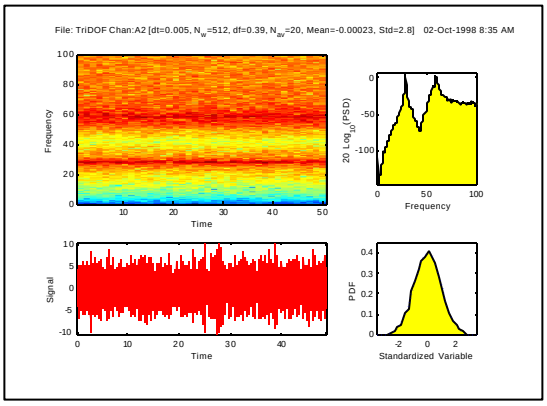
of 51.2 Sec duration and sampled at dt=.005 Sec. Response accelerations for the three DOFs was calculated by numerical simulation using the ITAP-A function **mtresp**.

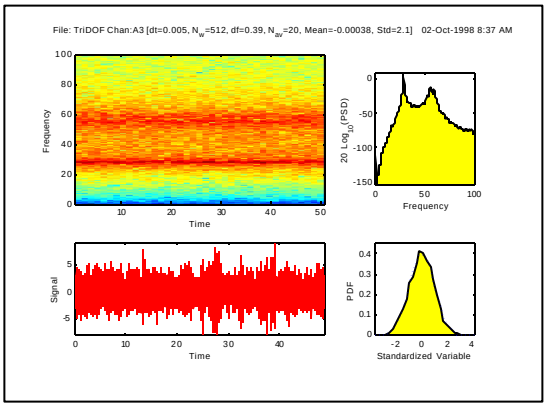
Results of preliminary data analysis are shown below:



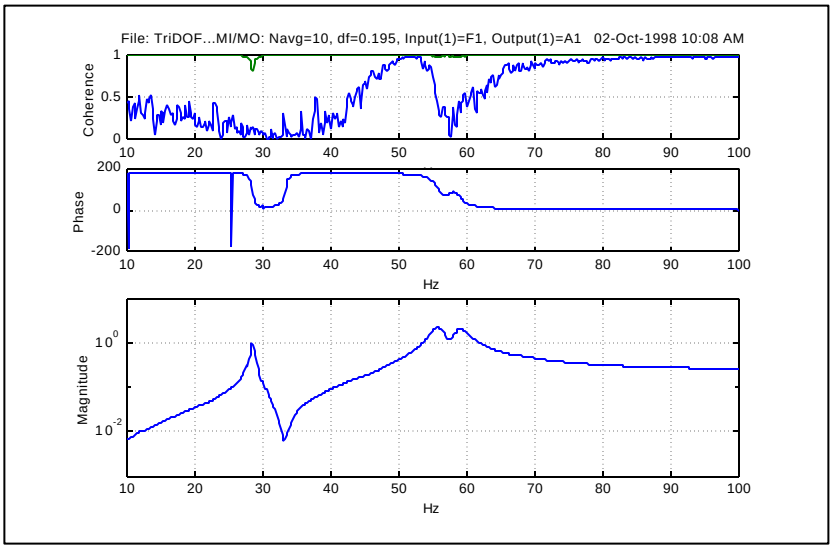


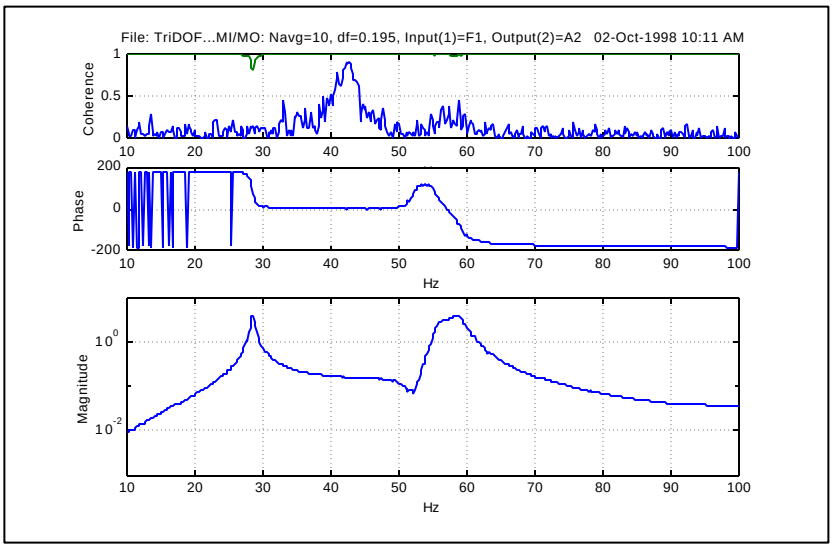


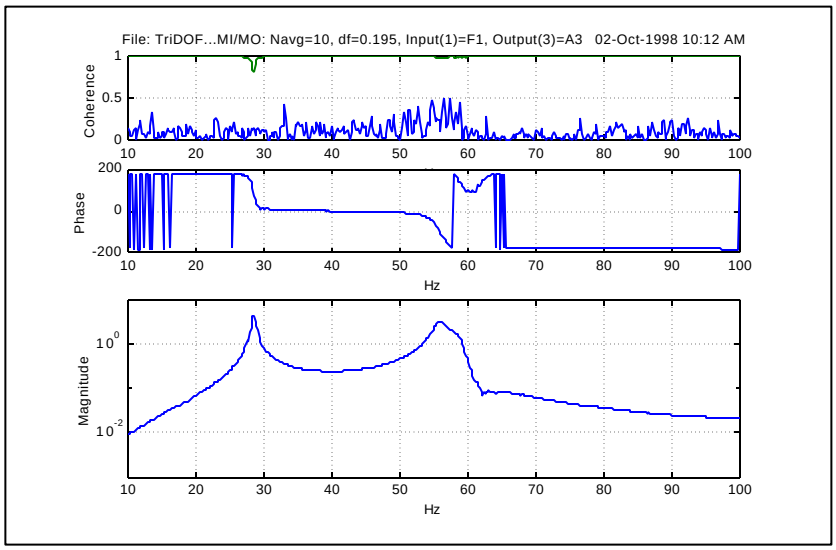


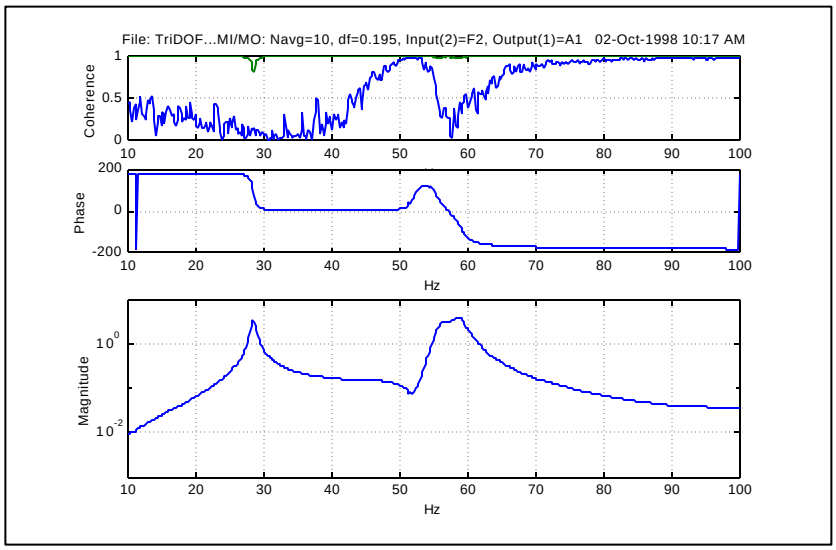


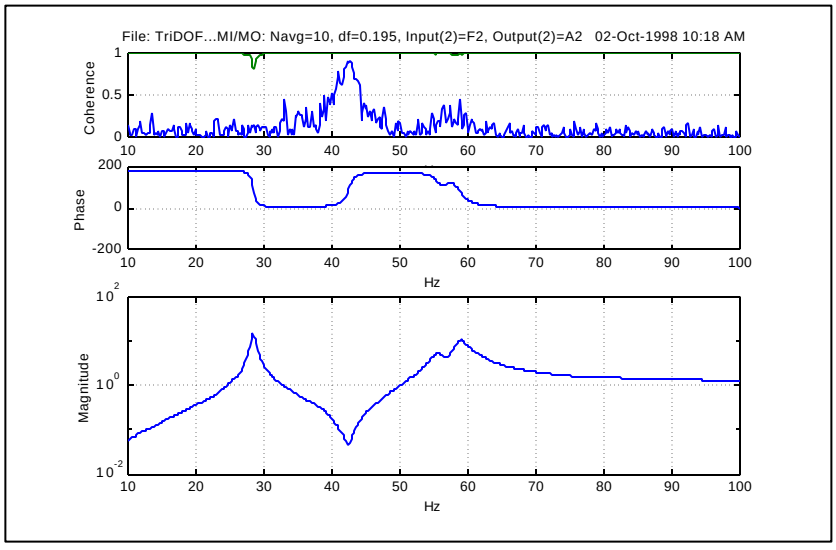
MI/MO analysis was performed on the time history data records using the ITAP-T function **mimo** to obtain frequency response functions associated with the two excitation force inputs and three respective acceleration responses. Shown below are **mimo** frequency response functions and cumulative coherence associated with the two forces and three acceleration responses:

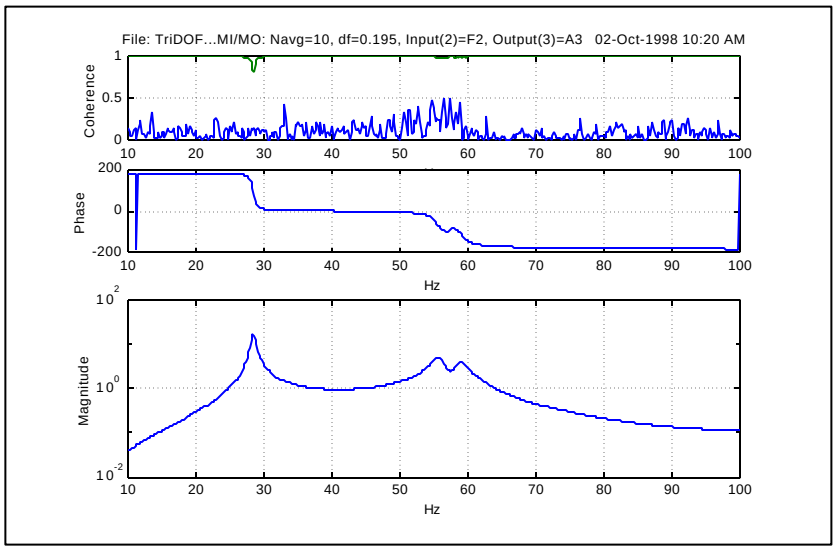





File: TriDOF...MI/MO: Navg=10, df=0.195, Input(1)=F1, Output(3)=A3 02-Oct-1998 10:12 AM
Coherence
Phase
Magnitude
Hz


File: TriDOF...MI/MO: Navg=10, df=0.195, Input(2)=F2, Output(1)=A1 02-Oct-1998 10:17 AM
Coherence
Phase
Magnitude
Hz


File: TriDOF...MI/MO: Navg=10, df=0.195, Input(2)=F2, Output(2)=A2 02-Oct-1998 10:18 AM
Coherence
Phase
Magnitude
Hz



Inspection of the above frequency response functions suggests that the system has three responding modes. The cumulative coherence function family, shown with each of the (magnitude and phase) frequency response results, indicates the influence of each excitation on the response autospectrum.

Additional understanding of system characteristics is realized by the use of various graphical displays; this subject is discussed in Subsections 6.8.1 and 6.8.2.

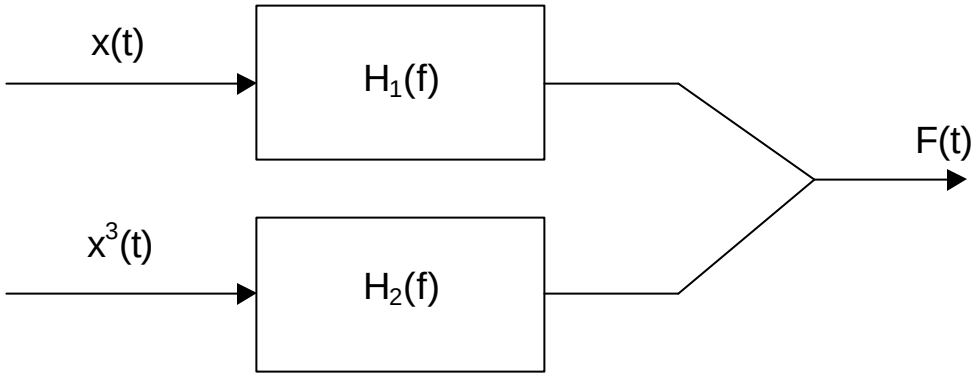
6.7.2 Nonlinear SDOF System MI/SO Analysis (Reverse Dynamic System)

The SDOF nonlinear system, which was discussed in Subsections 6.3.3, 6.3.6, and 6.5.2, is now examined using MI/SO spectral and correlation analysis. Preliminary data analyses (see Subsections 6.3.3 and 6.3.6) suggest the presence of "hardening" spring nonlinearity. Several MI/SO analysis models are hypothesized below to estimate characteristics of this system.

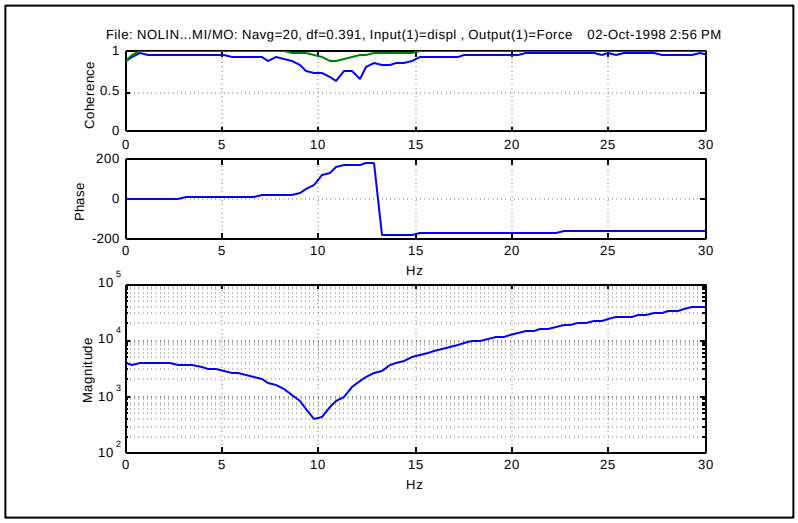
Consider a reverse dynamic system, which describes the nonlinear ordinary differential equation

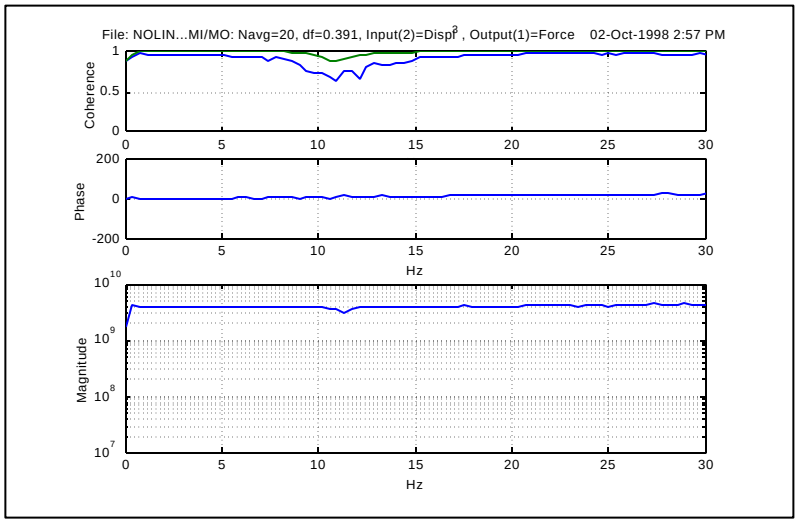
$$M\ddot{x} + B\dot{x} + Kx + K_3 x^3 = F(t)$$

where x(t) and $x^3(t)$ are inputs and F(t) is the output. The input-output schematic describing the reverse dynamic system is



MI/SO analysis of this system was performed by first forming the $x^3(t)$ time history signal from the measured time history record for x(t). The ITAP-T function **mimo** yields the following results for $H_1(f)$ and $H_2(f)$:





The results of MI/SO analysis closely agree with the exact reverse MI/SO dynamic system frequency response functions which are

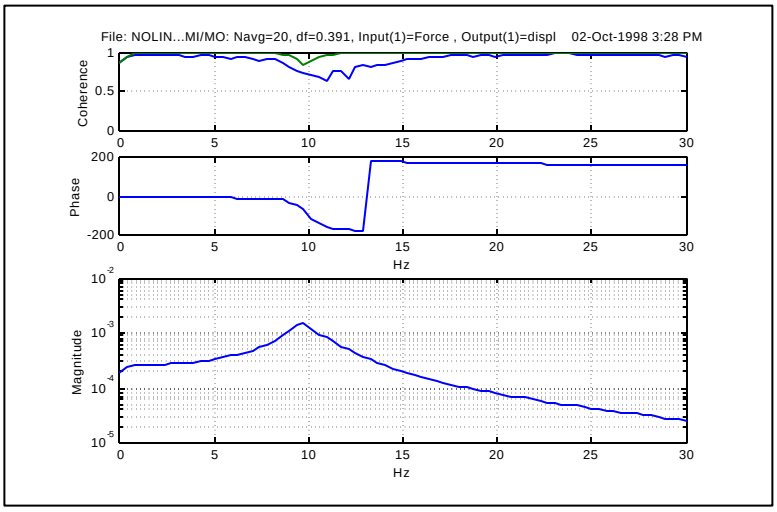
$$H_1(f) = (K - (2\pi f)^2 \cdot M) + i \cdot (2\pi f) \cdot B \quad , \quad H_2(f) = K_3$$

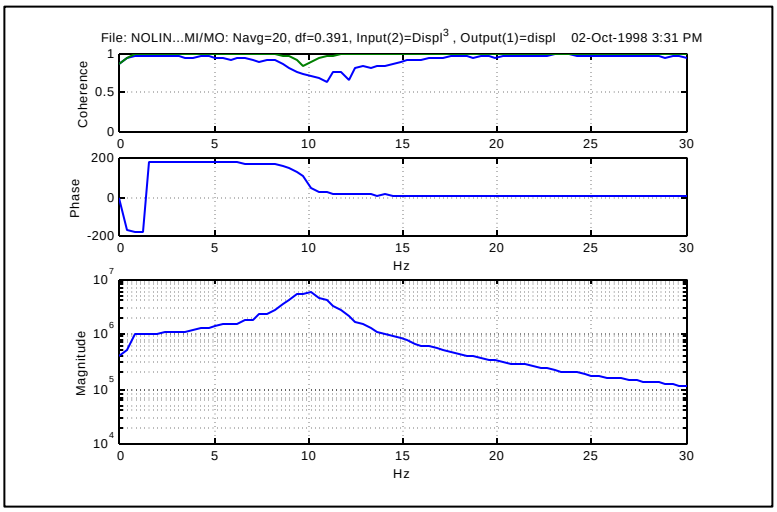
6.7.3 MI/SO Analysis (Dynamic System with Nonlinear "Feedback")

The SDOF nonlinear system, treated in the previous subsection, is now viewed as a nonlinear "feedback" described by the nonlinear ordinary differential equation

$$M\ddot{x} + B\dot{x} + Kx = F(t) - K_3 x^3$$

where F(t) and $x^3(t)$ are inputs and x(t) is the output. MI/SO analysis of the measured time history records produces the following results:





The estimated H1(f) and H2(f) functions are proportional to one another in accordance with the exact theoretical results,

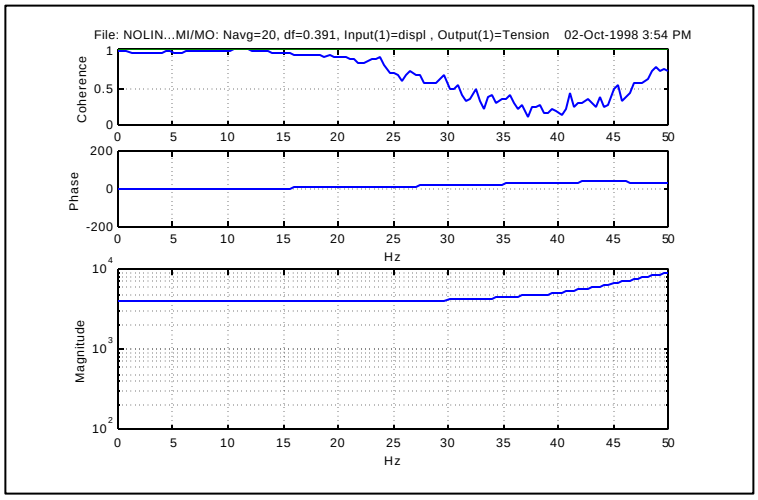
$$H_1(f) = (K - (2\pi f)^2 \cdot M) + i \cdot (2\pi f) \cdot B \quad , \quad H_2(f) = -K_3 \cdot H_1(f)$$

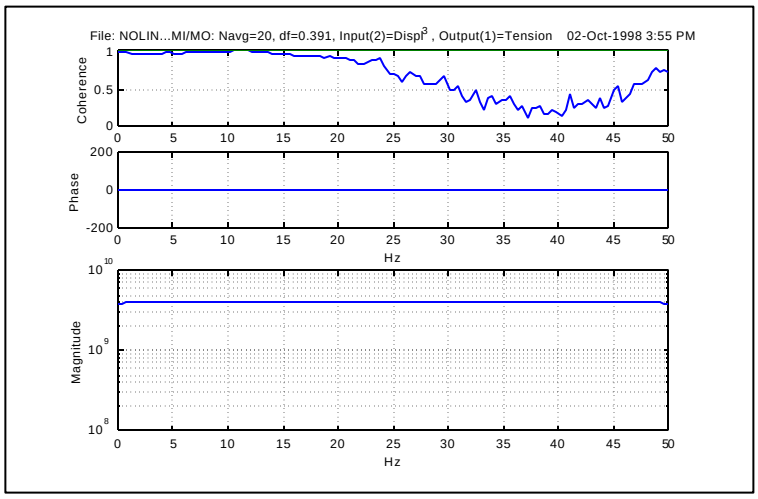
6.7.4 MI/SO Analysis (Nonlinear Tension-Displacement Relationship)

If the internal "tension" load, T(t), is measured, it is possible to directly estimate the nonlinear tension-displacement relationship

$$T = Kx + B\dot{x} + K_3 x^3$$

MI/SO analysis of this system where x(t) and x^3(t) are inputs and T(t) is the output produces the following results:





The above results are consistent with the exact theoretical results, which are

$$H_1(f) = K + i \cdot (2\pi f) \cdot B \quad , \quad H_2(f) = K_3$$

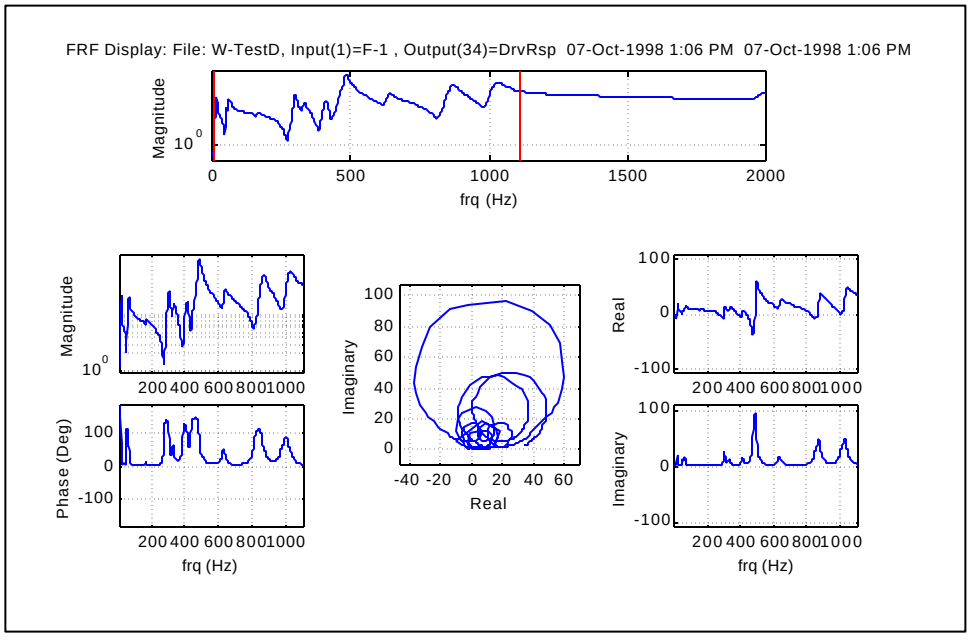
6.8 Modal Parameter Estimation

In Chapter 1 of this manual, theoretical formulations of linear structural and mechanical dynamic systems were discussed. Mathematical properties of real and complex modes (described in terms of modal parameters) were shown to offer opportunities for understanding behavior of linear dynamic systems and for efficient computation of system response to excitation environments. In the present chapter of this manual, procedures for analysis of measured data have been described and demonstrated. By employing SI/MO and MI/MO spectral and correlation methods, frequency response functions of linear (as well as nonlinear) systems, from measured time history records, are obtained. In the present section, methodology for estimation of modal parameters from frequency response functions is introduced.

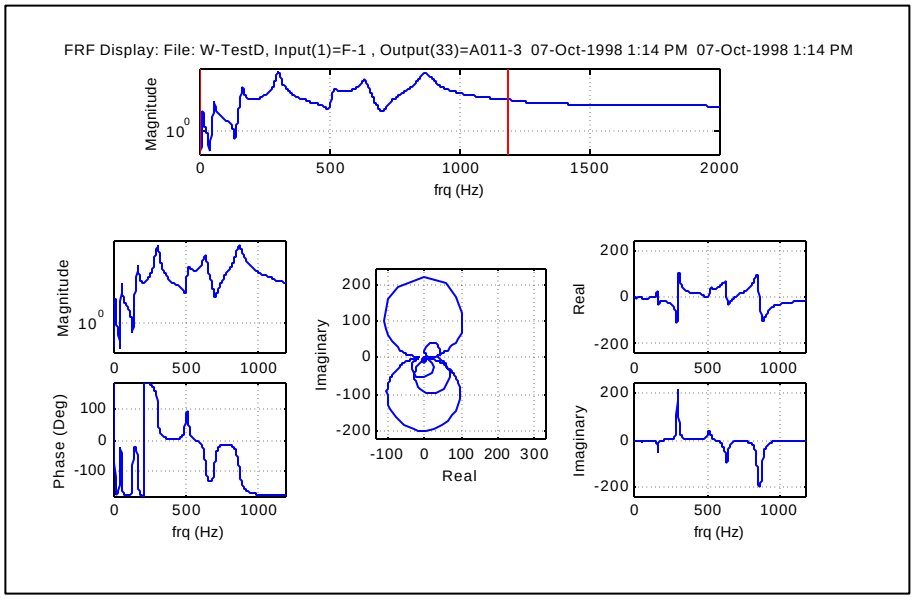
6.8.1 Preliminary Single Channel Modal Analysis

Experimental modal analysis is an engineering discipline, which had its beginnings in the aircraft industry in the 1940s (see Reference 7, Chapter 21). Before the advent of modern digital computers, experimental modal analysis placed heavy emphasis on graphical analysis tools. Today, such tools serve as means for preliminary experimental modal analysis. In the present discussion, a variety of graphical displays, which are included ITAP-T are discussed.

Shown below is a set of plots for a drive point frequency response function (output acceleration / input force).

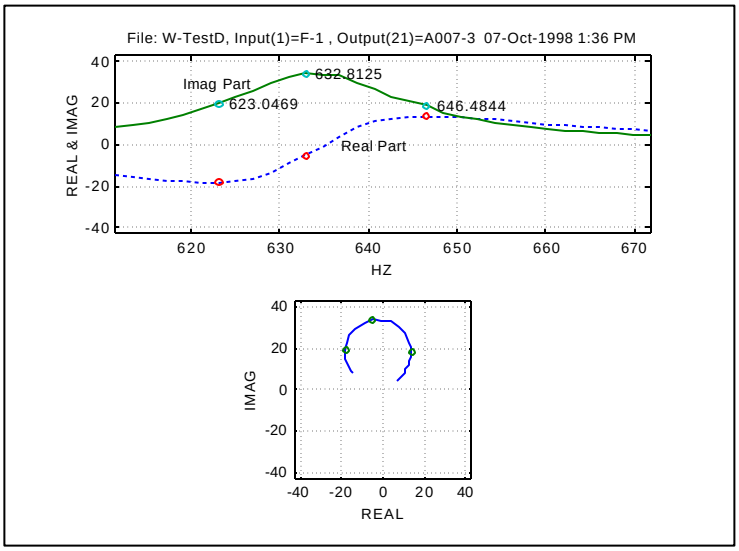


In addition, plot displays associated with a non-drive point frequency response function (FRF) are shown below.



Both frequency response functions indicate peak magnitude values at modal frequencies. In addition, the displays of real versus imaginary components trace “circles” (or “circle arcs”) in the vicinities of modal frequencies. A distinction between drive point and all other FRFs is that the phase of drive point acceleration FRF is always greater than zero; this is not necessarily the case for the other responses.

When modes are well-spaced, natural frequency and damping may be estimated from real and imaginary parts of the frequency response function, as shown below for a narrow frequency band (non-drive point FRF):



In the above plot, local extrema in both real and imaginary FRF components have been marked. The frequency values are as follows:

	Frequency (Hz)	Real or Imaginary
f_1	623.05	Real
f_2	632.81	Imaginary
f_3	648.48	Real

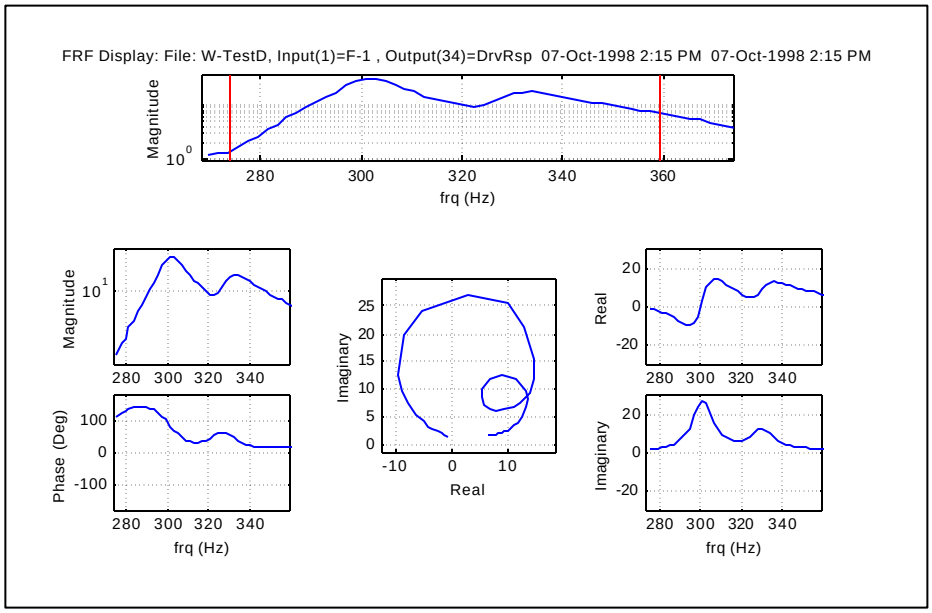
Using the following well-known formulae, modal frequency and critical damping ratio is estimated:

$$f_n \approx (f_1 + f_3)/2 = 635.8\,, \qquad \zeta_n \approx (f_3 - f_1)/(f_3 + f_1) = 0.02$$

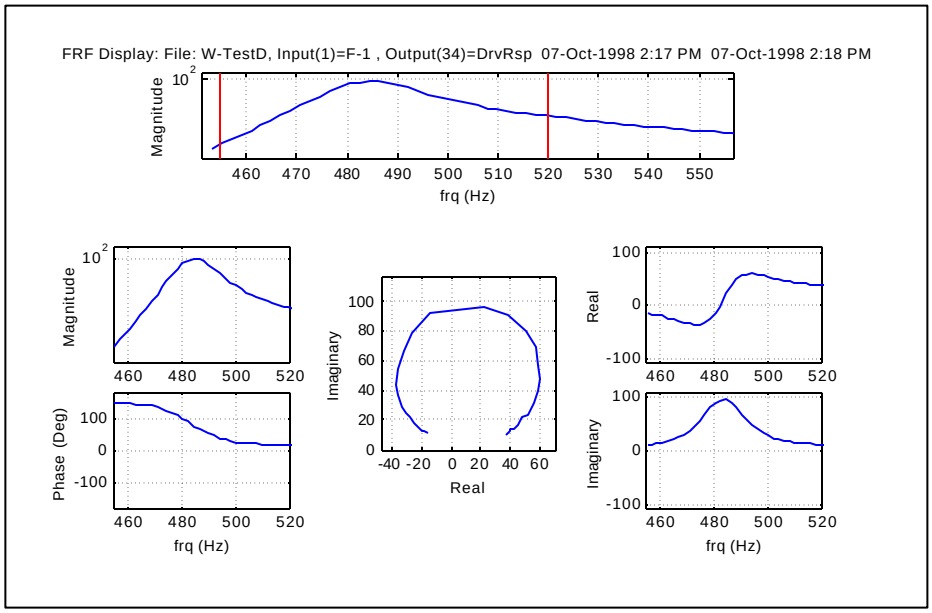
[Note: The exact modal frequency and critical damping ratio are, 633.78 and 0.02, respectively].

These simple formulae become difficult to apply when modes are closely spaced as illustrated in the following two frequency bands (drive point FRF).

Moderately Close Mode Spacing



Very Close Modal Spacing



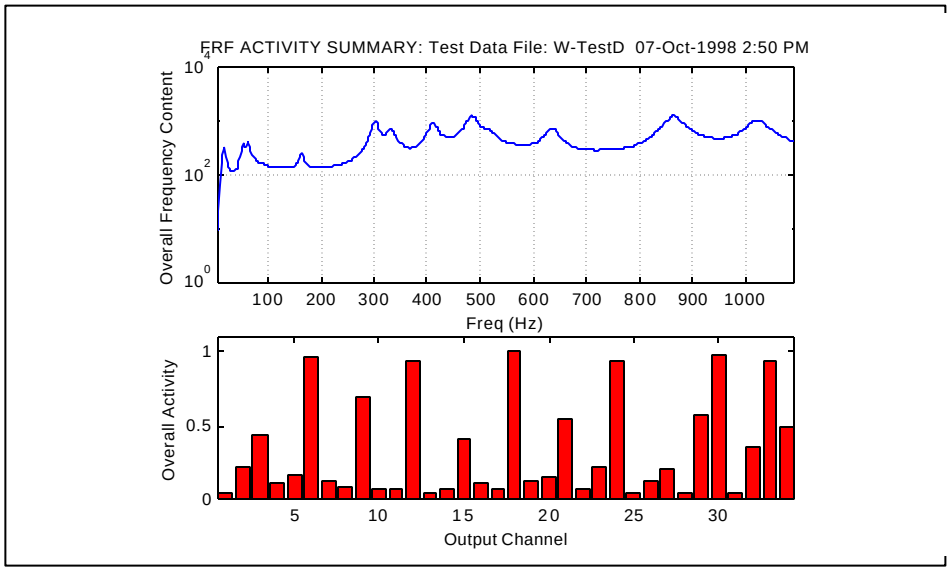
When modes are very closely spaced as in the above frequency band, the presence of multiple modes may be noticed as a distortion (from a circular arc) in the real versus imaginary plot.

Difficulties in the task of estimating closely-spaces modal frequency and damping parameters led to the establishment of more sophisticated experimental modal analysis algorithms.

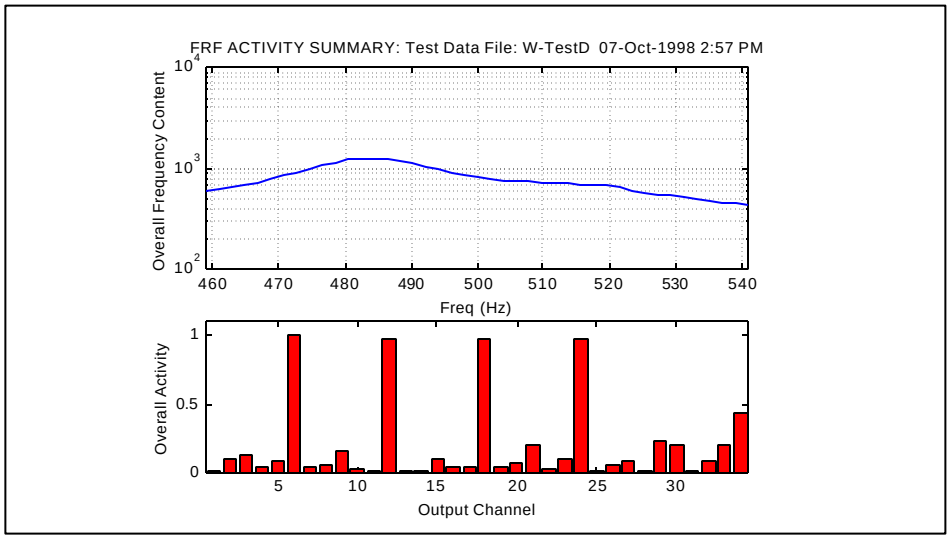
6.8.2 Preliminary Multi-Channel Modal Analysis

A variety of multi-channel (and multi-exciter) strategies have been developed to address difficulties associated with closely spaced modes (see Reference 7, Chapter 21). Several elementary techniques have been incorporated in ITAP-T which permit the user to (at least) note the presence of closely spaced modes.

The first elementary technique, contained in the ITAP-T function **frfsum**, calculates (1) the absolute sum of all frequency response functions (single- and multi-exciter situations) and (2) the absolute sum over all frequencies of each response channel FRF (set). This technique may be applied to selected frequency bands. Illustrated below are results of a wide frequency band **frfsum** analysis.



A 460-540 Hz frequency band **frfsum** analysis is shown below.



This localized frequency band display indicates the presence of two modes by distinct “bumps” in the overall frequency content plot. Moreover, the overall activity bar plot indicates the output (response) channels which dominate global response in this frequency band.

6.8.3 The Simultaneous Frequency Domain (SFD) Method Overview

A variety of systematic procedures for estimation of modal parameters from multi-channel measured FRF have been developed since the early 1970's (Reference 7). The ITAP-T toolbox features the Simultaneous Frequency Domain (SFD) method (References 8 and 9), which addresses a wide variety of situations. The SFD method as formulated for ITAP-T will estimate modal parameters for (a) single or multiple excitation FRF data sets, and (b) FRF data associated with applied force or base acceleration excitations. In addition, the user may select real or complex modal vector estimation.

The SFD method is composed of four successive mathematical operations, namely:

(0) Select FRF data set frequency bands which indicate possible modal activity (by analysis of a modal indicator function).

(1) Within each selected frequency band, determine dominant vectors and associated generalized frequency response functions (GFRFs) which describe FRF data set acivity in the frequency band (using a spectral decomposition procedure).

(2) From the GFRFs construct an optimal dynamic system which describes system dynamics in the frequency band. Calculate complex eigenvalues of the effective dynamic system, which are modal frequency and damping estimates in the selected frequency band.

(3) Collect modal frequency and damping estimates from all selected frequency bands.Calculate global (real or complex) modal vectors using a linear least squares fit procedure to optimally reconstruct the FRF data set.

Each of the following four sections describe successive SFD mathematical operations.

6.8.4 SFD Step 0: The Global Skyline Modal Indicator Function

A function of the FRF array, which has been found to indicate the presence of modal activity, is the global skyline function (Reference 9). The global skyline (modal indicator function) is defined as

$$S(f) = \left(\sum_{k=1}^{N} Real(H_k(f))^2 \right) + i \cdot \left(\sum_{k=1}^{N} Imag(H_k(f))^2 \right)$$

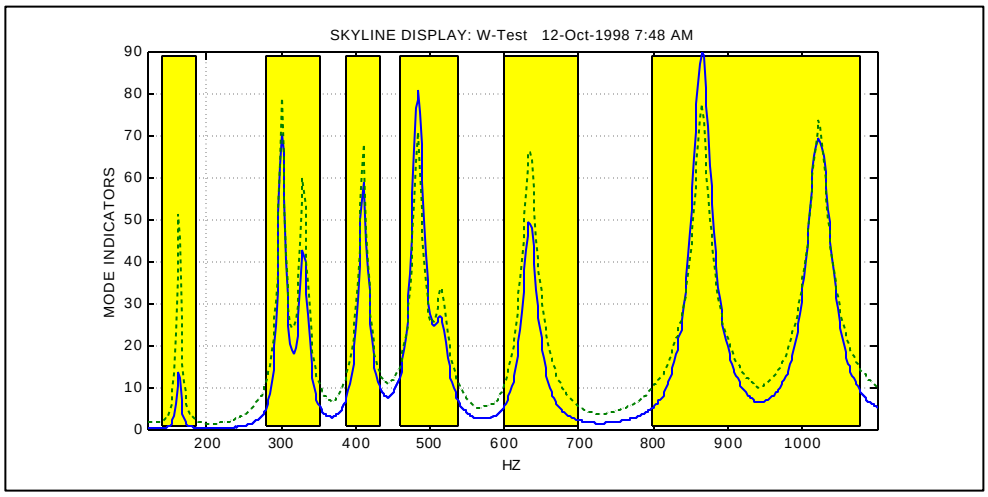
where "N" is the total number of individual FRFs equal to the product of input chaneels and output channels.

This function has the following properties:

(1) The global skyline function magnitude, |S(f)|, is equal to the sum of the squares of all FRF magnitudes.

(2) The global skyline phase angle can take on values from 0 to 90 degrees. The phase angle tends to peak at responding modal frequencies (to 90 degress if a single mode is "purely" excited).

The ITAP-T function **skyline** calculates and displays properties of the global skyline function. Moreover, it provides the user with graphical means to highlight (select) frequency bands where modal response is indicated. An example **skyline** display is shown below:



The solid curve represents the normalized (to 90 maximum value) imaginary part of S(f), and the dashed curve is the phase angle of S(f).

6.8.5 SFD Step 1: Dominant Vectors and Generalized FRFs

The viewpoint adopted by SFD is that structural acceleration responses described by the frequency response function set, [H(f)], are expressible as linear combinations of generalized acceleration FRFs, $\left[\ddot{Q}(f)\right]$ as follows:

$$[H(f)] = [V] \cdot \left[\ddot{Q}(f)\right] + [R(f)]$$

where [V] is a set of dominant vectors and [R(f)] is a residual error matrix. In order to solve for [V] and $\left[\ddot{Q}(f)\right]$, the above complex matrix equation is expressed as the real algebraic equation set

$$[X] = [V] \cdot [Z] + [R]$$

where

$$[X] = \left[[H(f)]_{REAL} \quad [H(f)]_{IMAG}\right], \quad [Z] = \left[\left[\ddot{Q}(f)\right]_{REAL} \quad \left[\ddot{Q}(f)\right]_{IMAG}\right]$$

Both the dominant vectors and generalized FRFs are identified from solutions of an algebraic eigenvalue problem (singular value decomposition). Details of the strategy for singular value decomposition and its use in this particular application (the ITAP-T function **ranc**) are found in Chapter 10 of this manual. The dominant vectors, [V], are an orthonormal set, $V^T V = [I]$, and the overall activity (or "power" level) associated with each successive generalized FRF is directly related to a particular eigenvalue, that is,

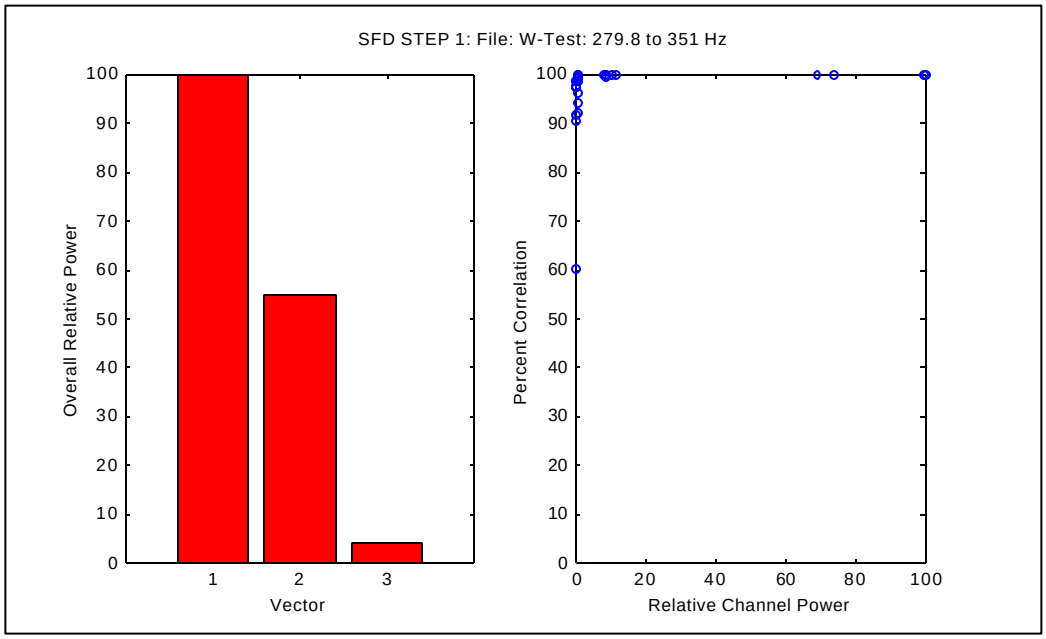
$$\lambda_K = \sum_J \left|\ddot{Q}_K(f_J)\right|^2 \quad \text{where } \lambda_1 \geq \lambda_2 \geq \lambda_3, \text{ etc.}$$

The number of dominant vectors and associated generalized FRFs retained for further SFD analysis is set by a tolerance parameter (tol=0.01, default based on extensive experience with actual test data); all components for which $\lambda_K / \lambda_1 \geq \text{tol}$ are retained.

The quality level to which each individual FRF is reconstructed by the product of dominant vectors and associated generalized FRFs is calculated as a correlation coefficient ($0 \leq C_J \leq 1$). The correlation coefficients are defined as

$$C_J = \left(\sum_K Y_{JK}^2\right) / \left(\sum_K X_{JK}^2\right) \quad \text{where} \quad [Y] = [V] \cdot [Z]$$

Continuing with the example FRF data used to demonstrate the skyline calculation, results of SFD Step 1 analysis (in the second selected frequency band)are displayed below:



The plot on the left indicates the overall relative power level of three dominant vectors and generalized FRFs identified in the 280-350 Hz frequency band. The plot on the right indicates the ability of the three vector, three generalized FRF description to reconstruct the FRF array in this frequency band (individual channel correlation coefficient versus overall channel “power” in the frequency band).

6.8.6 SFD Step 2: Effective Dynamic System and Associated Eigenvalues

The generalized FRFs, $\left[\ddot{Q}(f)\right]$, are associated with an effective dynamic system in the selected frequency band, which is described in terms of the matrix equation set

$$\left\{\ddot{Q}(f)\right\}+[B]\cdot\left\{\dot{Q}(f)\right\}+[K]\cdot\{Q(f)\}=[\Gamma]\cdot\{F(f)\}$$

Since the generalized FRFs are in the frequency domain, the following relationships are used to defined excitation and response quantities:

$$\left\{\dot{Q}(f)\right\}=\left\{\ddot{Q}(f)\right\}/(2\pi i\cdot f)\ ,\ \{Q(f)\}=-\left\{\ddot{Q}(f)\right\}/(2\pi f)^2\ ,\ F(f)=1$$

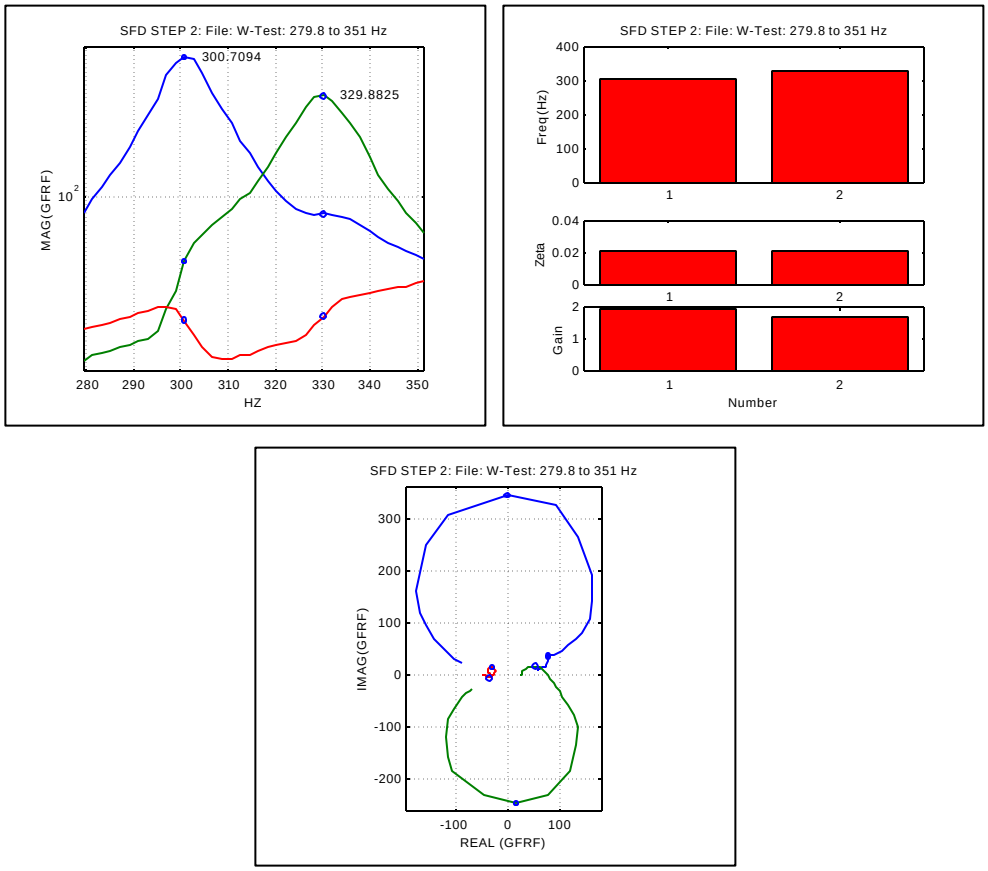
The unknown matrices, [B], [K], [Γ], are estimated directly by linear least squares analysis.

Effective dynamic system complex eigenvalues are determined by solution of the algebraic eigenvalue problem

$$[A]\{\phi\}=\{\phi\}\lambda\ \text{ where }\ [A]=\begin{bmatrix}-B & -K\\ I & 0\end{bmatrix}$$

Complex eigenvalues with imaginary frequency component within the frequency band of the effective dynamic system are candidates for actual system modal frequencies. As part of an ITAP-T modal identification session, the user selects “acceptable” candidate eigenvalues from the computed set. The acceptable candidates are accumulated from all selected frequency band based effective dynamic systems.

Results of SFD Step 2 analysis (for the illustrative example FRF data in the 280-351 Hz selected frequency band)are displayed below:



The plot on the upper left side illustrates generalized FRF magnitudes versus frequency with the identified candidate natural frequencies labeled. The plot on the upper right side summarizes the candidate natural frequencies, critical damping ratios and modal gains in bar chart form. Finally, the lower real versus imaginary generalized FRF plot indicates the presence of modal activity in the most dominant first two generalized FRFs.

6.8.7 SFD Step 3: Calculation of Modal Vectors

The theoretical relationship between FRFs and modal parameters (assuming that modal vectors are real) is

$$[H(f)] = [\Phi] \cdot [\ddot{q}(f)]$$

where $[\Phi]$ is the unknown real modal matrix and $[\ddot{q}(f)]$ is the modal acceleration matrix. The terms of $[\ddot{q}(f)]$ are defined as

$$[\ddot{q}(f)]_{JK} = \frac{-\omega_K^2}{\left(\omega_J^2 + 2i\zeta_J\omega_J\omega_K - \omega_K^2\right)}$$

where the forcing frequency is $\omega_K = 2\pi f_K$, the modal frequencies are $\omega_J = 2\pi f_J$, and the modal critical damping ratios are ζ_J. Since the modal acceleration matrix is completely known, the real modal matrix is obtained by linear least squares analysis. At the user's initiation, low and/or high frequency residual modal frequencies may be added to the set of identified eigenvalues to enhance accuracy of modal vector estimates.

The theoretical relationship between FRFs and modal parameters (assuming that modal vectors are complex) is

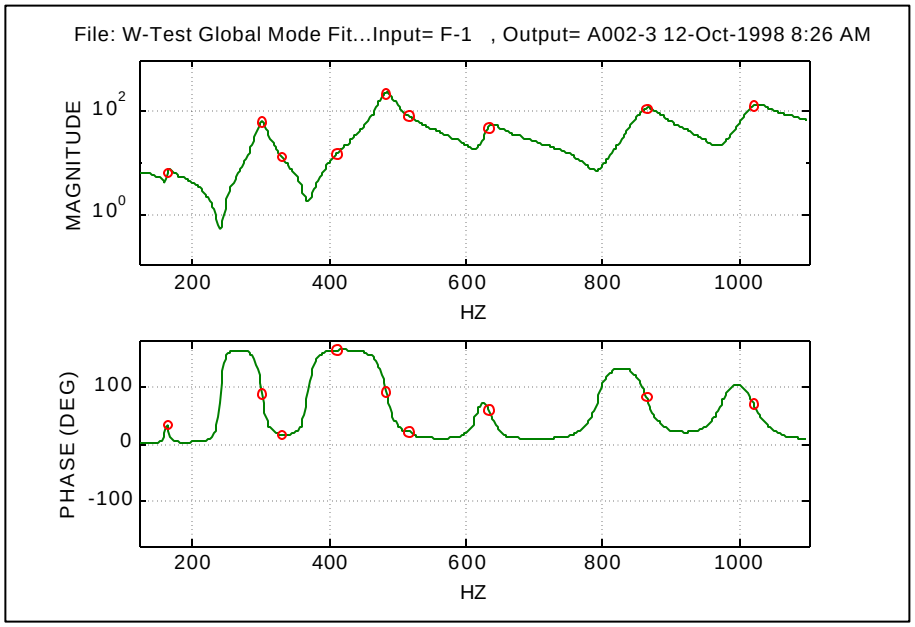
$$[H(f)] = -\omega^2 \cdot \sum_J \left(\frac{\Phi_J}{\lambda_J - i\omega} + \frac{\Phi_J^*}{\lambda_J^* - i\omega} \right)$$

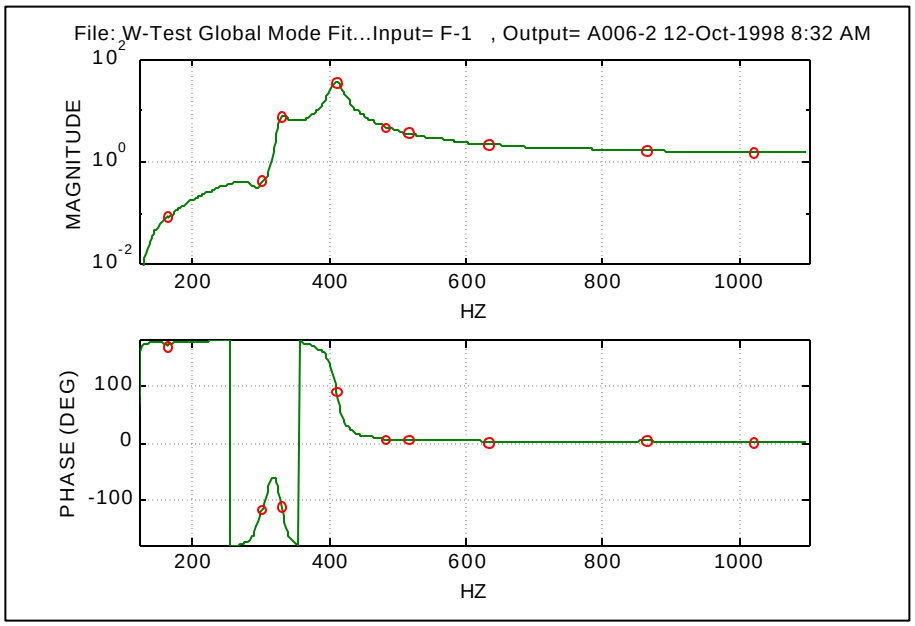
where Φ_J is the unknown J^{th} complex modal residue vector and λ_J is the J^{th} complex eigenvalue. The complex modal residue vectors are proportional to the complex system modes. The forcing frequency, is $\omega = 2\pi f$, and the complex eigenvalues (when critical damping ratio, ζ_J, is less than 1.0) are

$$\lambda_J = -\zeta_J\omega_J + i\omega_J$$

modal frequencies are $\omega_J = 2\pi f_J$, and the modal critical damping ratios are ζ_J. Since the complex eigenvalues are completely known, the complex modal residue vectors (proportional to the complex modal vectors) are obtained by linear least squares analysis. As for the case of real modes, low and/or high frequency residual modal frequencies may be added to the set of identified eigenvalues to enhance accuracy of modal vector estimates.

Regardless of whether the user chooses to estimate real or complex modal vectors, reconstructed FRFs calculated from identified modal parameters serve as a means for a quality check on the overall SFD process. Typical comparisons of original and reconstructed FRFs are illustrated below:





In the above comparison plots, the “dotted” curves represent the original FRF and the solid curves are the reconstructed FRFs. The small circles indicate locations of identified modal frequencies.

6.8.8 Multiple Excitations and Base Motion Excitation

When FRF data is associated with multiple excitation sources, the SFD method, as implemented in ITAP-T, views all of the FRF information as a single matrix array. Therefore, SFD steps 0-2, discussed in the previous sections, perform calculations in the same way as for FRF data associated with a single excitation source. For the most part, SFD step 3 calculations are carried out as for the single excitation case. There is, however, a key difference in the final calculation of modal vectors.

The theoretical relationships between FRFs associated with more than one excitation and global modal parameters (assuming that modal vectors are real) are

$$[H_n(f)] = [\Phi_n] \cdot [\ddot{q}(f)]$$

where $[H_n(f)]$ is the FRF matrix associated with the n^{th} excitation and $[\Phi_n]$ is the real modal matrix estimate (scaled to best fit the FRF data). $[\ddot{q}(f)]$ is the modal acceleration matrix described in Section 6.8.7.

The modal matrix, [Φ], is then calculated as the average of estimates from all "N" excitation sources, i.e.,

$$[\Phi] = \frac{1}{N} \cdot \left(\sum_{n=1}^{N} [\Phi_n] \right).$$

Since the individual modal matrix estimates, $[\Phi_n]$, are scaled to suit the individual n^{th} excitation based FRFs, the averaging process is weighted in favor of strongly excited responses. [Note: The same type of averaging is used for the case of complex modes].

When response is due to base motion excitation, all SFD analysis steps are identical to those for the situation of applied force excitation. To appropriately account for the contribution associated with applied motion excitations, it is suggested that the identified eigenvalue set be augmented with a low frequency residual.

In Chapter 9 a series of illustrative examples are provided to demonstrate usage of SFD for the above cited situations.

6.9 References

1. Bendat and Piersol, Random Data Analysis and Measurement Procedures, 2nd Edition, John Wiley and Sons, 1986

2. MATLAB, Signal Processing Toolbox User's Guide (Version 4), The Math Works Inc., 1996

3. Cole, "On-The-Line Analysis of Random Vibrations, AIAA Paper No 68-288, 1968

4. Ibrahim, "Random Decrement Technique for Modal Identification of Structures", Journal of Spacecraft and Rockets", Vol 14, No 11, Nov 1977, pp 696-700

5. Asmussen, "Modal Analysis Based on the Random Decrement Technique", Ph.D. Thesis, University of Aalborg, Denmark, 1997

6. Bendat, Nonlinear System Techniques and Applications, John Wiley and Sons, 1998

7. Harris, Shock and Vibration Handbook, 3rd Edition, McGraw-Hill Book Company, 1988

8. Coppolino, "A Simultaneous Frequency Domain Technique for Estimation of Modal Parameters from Measured Data", SAE Aerospace Conference and Exposition, Paper 811046, Oct 1981

9. Coppolino and Stroud, "A Global Technique for Estimation of Modal Parameters from Experimental Data", Proceedings of the 4th International Modal Analysis conference, Feb 1986, pp 674-681

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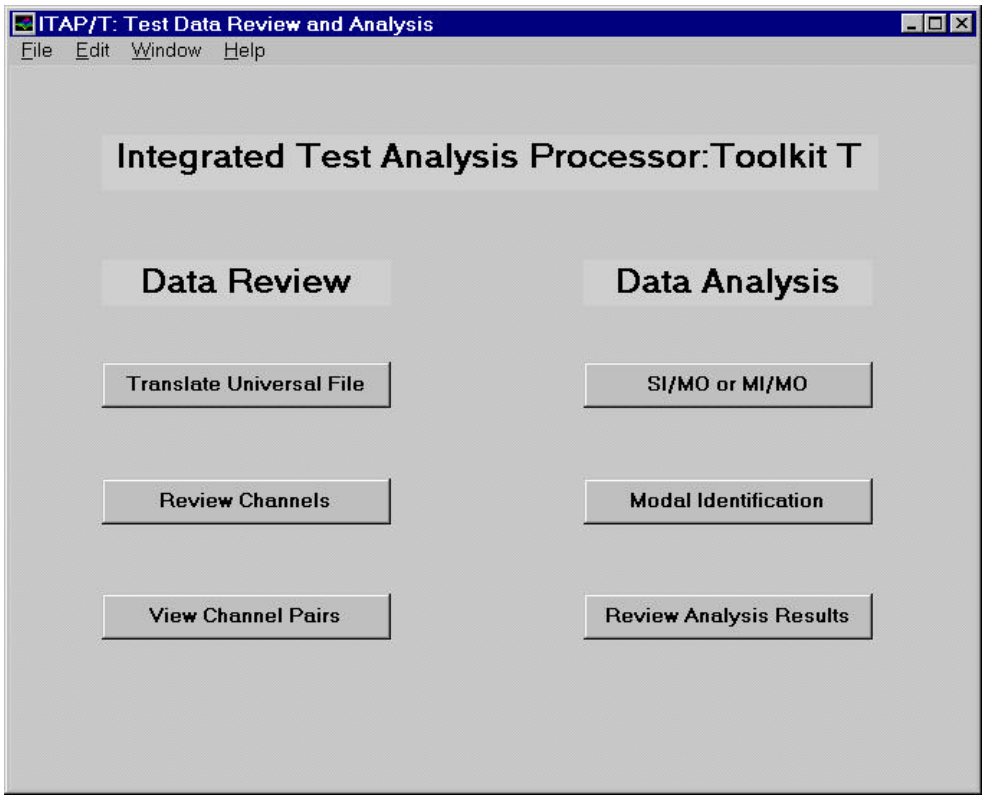
7.0 Preliminary Data Analysis

7.1 Introductory Remarks

This chapter describes features of the ITAP-T toolbox, designed to address preliminary measured data analysis. These features have been incorporated within a menu structure accessed by the user command, **itapt**, which causes the following display to appear:



The three push-down buttons under the heading entitled **Data Review** are the primary functions to be discussed in this chapter. **Translate Universal File** initiates translation of time history data from SDRC Universal File format to a standard ITAP-T time history file structure. **Review Channels** causes a preliminary channel-by-channel data analysis session to begin. **View Channel Pairs** initiates preliminary channel pair graphical analysis, which is often useful for preliminary evaluation of nonlinear systems.

Application of theoretical concepts described in Sections 6.2 and 6.3, within the ITAP-T environment is the primary emphasis of this chapter.

7.2 Measured Data File Structure

In this section, the standard ITAP-T data file structure is introduced. For the purposes of preliminary data analysis, in particular, a standard ITAP-T time history file format and contents is defined. Measured time history data, which comes from an external data acquisition system, must be translated into the ITAP-T standard format prior to initiation of all data analysis tasks. Additional data file contents are generated (and appended) to the basic ITAP-T time history file as a result of detailed data analyses (which are discussed in Chapters 8 and 9). It is convenient to introduce these data structures in the present chapter for completeness. In addition, for the users of ITAP-A, model analysis file variable names (**namch**) may be loaded into the ITAP-T time history file for consistency of analysis and test data files.

7.2.1 The Basic Time History (.mat) Data File

The most fundamental ITAP-T data file is a .mat file which contains single or multiple variable time history data, in the form of a matrix named **tdata**, and a uniform sampling time, **dt**. The matrix named **tdata** consists of time correlated signals (as successive columns), which we will refer to as data channels. The number of rows in **tdata** corresponds to the number of recorded time samples. Data channels may be either excitations or responses, which are distinct sub-groups of channels. It is essential for the purposes of a complete, coordinated ITAP-A and ITAP-T activity, that the sequencing of response variables be the same as for a corresponding mathematical model (i.e., TAM). A third data array called **namch** consists of variable names for successive data channels; this array is not necessary for initiation of channel-by-channel preliminary data analysis (initiated by pressing the **Review Channels** button on the **itapt** main screen). If there is no **namch** data in the basic time history data file, the user is prompted to input variable names during the initial session. If a modal test planning analysis has been completed, including accelerometer and excitation resource allocation (see Chapter 3), an array called **namch** exists in the TAM data file; this array may be copied by the user into the basic time history data file.

In summary, the following arrays comprise the basic time history data file:

[tdata]: number of time samples X number of channels

dt: uniform sampling time

[namch]: number of channels X (up to 8 columns)

7.2.2 Conversion of SDRC Universal File (Time History) Data

Measured time history data may be brought into the ITAP-T environment as an SDRC Universal File (ASCII .unv file format). The ITAP-T routine **unvread** translates basic time history data from “Data Set Type 58” to a .mat file. The three basic arrays, namely, **dt, tdata** and **namch** are written onto a .mat file, which has the same name as the .unv file. This routine may be invoked in the MATLAB command window with the command “**unvread**” or in the ITAP-T main screen by pressing the “**Translate Universal File**” button.

7.2.3 Correspondence with an ITAP-A Test-Analysis Model (TAM) File

If the ITAP user completes a modal test plan (see Chapter 3), the ITAP-A TAM file includes a list of channel names (**namch**), which may be incorporated in the basic time history data file. The **namch** array is formed as part of the ITAP-A **mprepfrf** analysis session during which planned excitation resources are allocated. Sequencing of **namch** contents in this situation is (a) names of excitation forces, (b) names of response degrees of freedom (node – dof triad designations).

The illustrative example structure TAM data file, **Wheeler** (discussed in Chapter 3,) produces the following **namch** array (comments are added here to describe particulars of the array contents):

F-1	(name of single excitation force channel)
A001-1	(accelerometer node 1, dof 1)
A001-2	(accelerometer node 1, dof 2)
A001-3	(accelerometer node 1, dof 3)
…	
A011-1	(accelerometer node 11, dof 1)
A011-2	(accelerometer node 11, dof 2)
A011-3	(accelerometer node 11, dof 3)

7.2.4 Frequency Response Function Data

If a spectral and correlation analysis is performed (initiated by pressing the **SI/MO or MI/MO** button on the **itapt** main screen), results data are appended to the basic time history data file. The contents of the appended file will include the following variables:

Variable	Description	Dimensions
tdata	Time History Array	Time samples X Channels
dt	Sampling Time	1X1
namch	Channel Names	Channels X (up to 8)
nin	Number of Inputs	1X1
Nin	Indices of Input Channels	1Xnin
namin	Input Channel Names	nin X (up to 8)
nout	Number of Outputs	1X1
Nout	Indices of Output Channels	1Xnout
namout	Output Channel Names	nout X (up to 8)
navg	Number of Spectral Averages	1X1
df	Bandwidth Resolution	1X1
frq	Frequency Array	1X (frequency bins)
H	FRF Array	(ninXnout) X (frequency bins)
COH	Cumulative Coherence Array	(ninXnout) X (frequency bins)

The arrangement of information in the FRF matrix (H) is “row” sequenced as

“output 1 due to input 1”
“output 1 due to input 2”
…
”output N due to input 1”
“output N due to input 2”
…

Similarly, the cumulative coherence matrix (COH) is “row” sequenced as

“output 1 due to input 1”
“output 1 due to inputs 1+2”
…
”output N due to input 1”
“output N due to inputs 1+ 2”
…

The appended time history data file will be referred to herein as the “test data file”.

7.2.5 Experimental Modal Analysis Data

If experimental modal parameters are identified (initiated by pressing the **Modal Identification** button on the **itapt** main screen), results data are appended to the test data file. The contents of the appended file will include the following variables:

Variable	Description	Dimensions
tdata	Time History Array	Time samples X Channels
dt	Sampling Time	1X1
namch	Channel Names	Channels X (up to 8)
nin	Number of Inputs	1X1
Nin	Indices of Input Channels	1Xnin
namin	Input Channel Names	nin X (up to 8)
nout	Number of Outputs	1X1
Nout	Indices of Output Channels	1Xnout
namout	Output Channel Names	nout X (up to 8)
navg	Number of Spectral Averages	1X1
df	Bandwidth Resolution	1X1
frq	Frequency Array	1X (frequency bins)
H	FRF Array	(ninXnout) X (frequency bins)
COH	Cumulative Coherence Array	(ninXnout) X (frequency bins)
FREQ	Modal Frequencies	(number of modes) X 1
ZETA	Modal Critical Damping Ratios	(number of modes) X 1
PHI	Modal Matrix	nout X (number of modes)
FRFIT	Fitted FRF array from modes	(ninXnout) X (modal freq. bins)
frqx	Fitted FRF frequency array	1X (modal frequency bins)
PHIRAW	Raw Modal Matrix Data	(ninXnout) X (number of modes)

The arrangement of information in the Raw Modal Matrix (**PHIRAW**) is "row" sequenced as

"output 1 due to input 1"
"output 1 due to input 2"
…
"output N due to input 1"
"output N due to input 2"
…

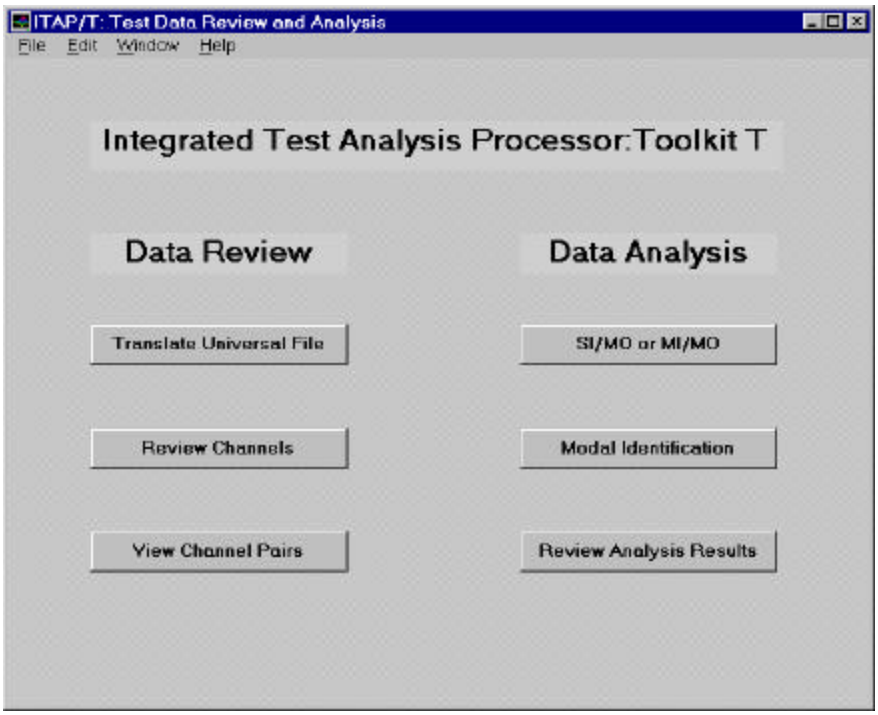
PHIRAW is saved as a test data file matrix for advanced user analyses and evaluations when modal data is estimated from multiple excitation based data.

7.3 Quality and Content of Individual Time Histories

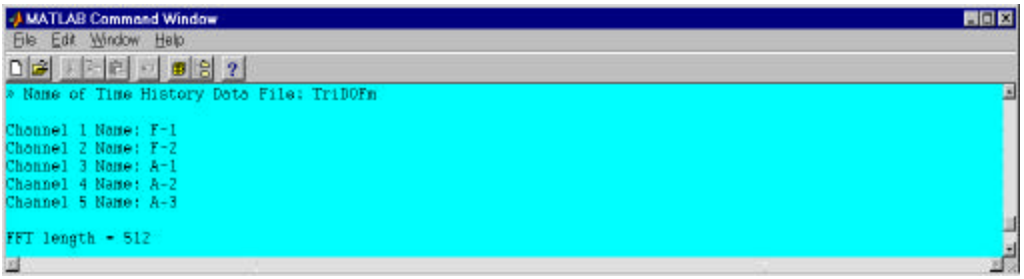
When engaged in the process of measured data analysis, the essential first task is to review this information for quality and overall content. Once this task is completed, either (a) additional measurements may be deemed necessary or (b) detailed data analyses consistent with project objectives are conducted. Fundamentals of (single channel) preliminary data analysis have been discussed in Sections 6.2 and 6.3 of this manual. The material presented in the following sections describes implementation and usage of such methodology within the ITAP-T environment. The ITAP-T function module **prelim** coordinates all individual channel preliminary review and evaluation operations.

7.3.1 Illustrative Example "Prelim" Session

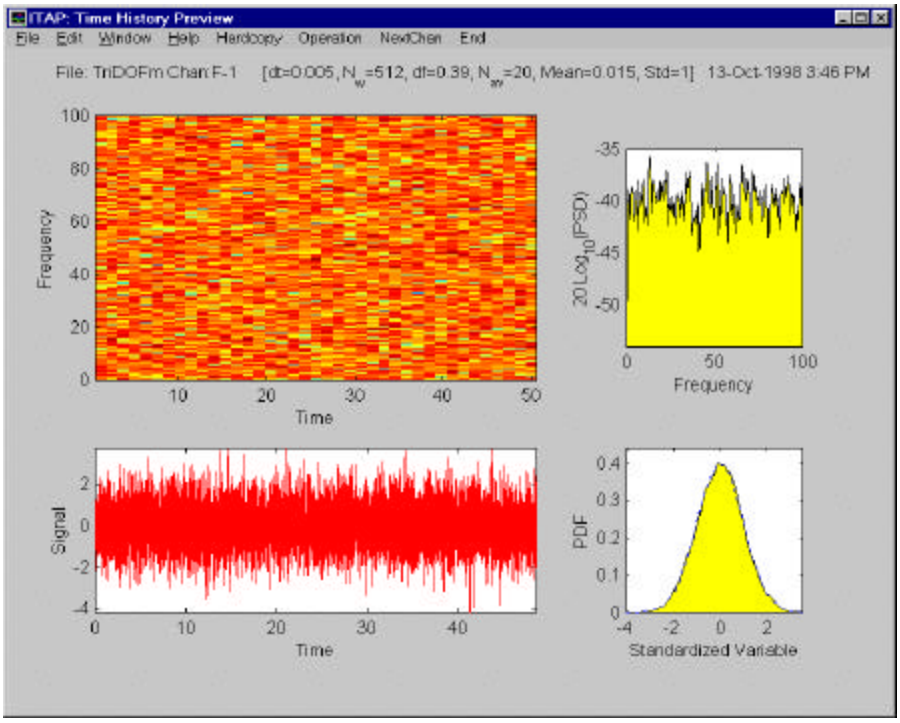
From the MATLAB command window, the user types "**itapt**" to initiate the **prelim** session. A screen with push-button choices appears as shown below:



Upon pressing the "**Review Channels**" button, successive requests for information appear on the MATLAB command window as shown below:

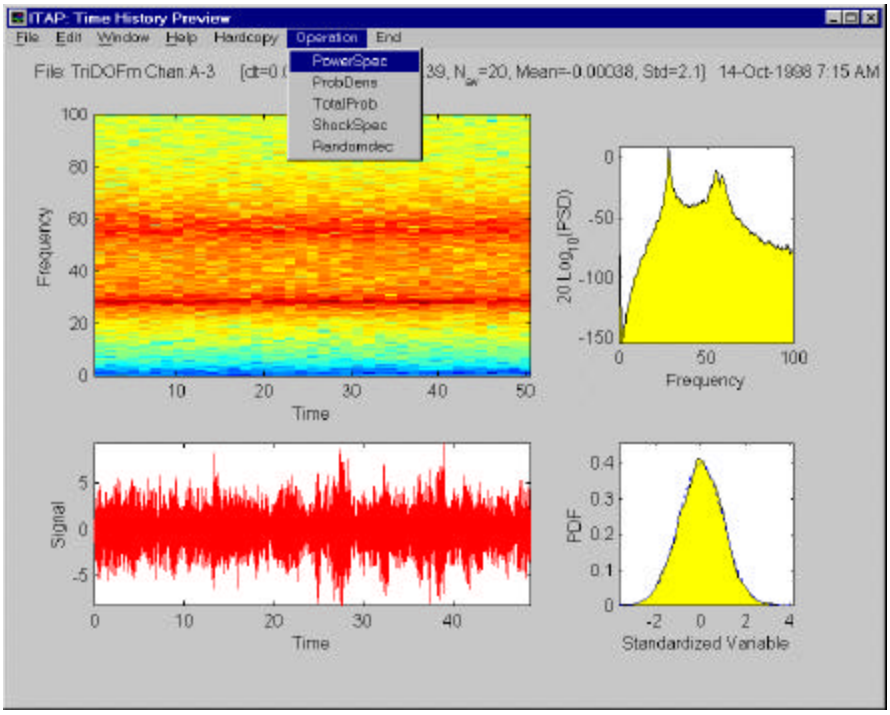


The requests for channel names are made only if the **namch** array does not exist in the data file. The requested "FFT length" parameter is required for calculation of the spectrogram display function and autospectrum.

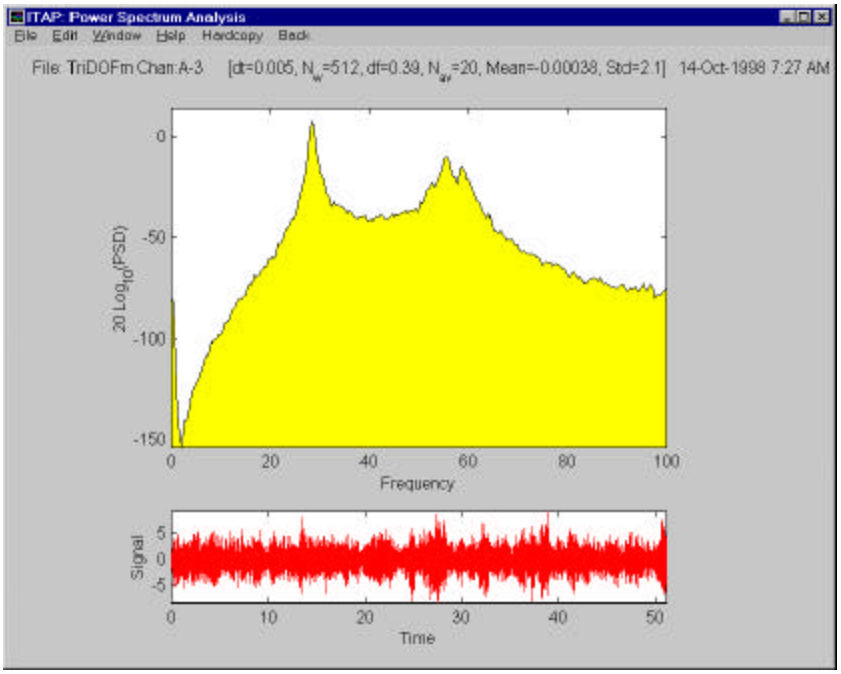


The above "ITAP Time History Preview" display contains spectrogram, time history trace, autospectrum (or PSD) and probability density function (PDF) plots associated with the subject channel. In addition, the display title includes key numerical data (data file name, window length ($\mathbf{N_w}$), bandwidth resolution (**df**), mean value, and standard deviation).

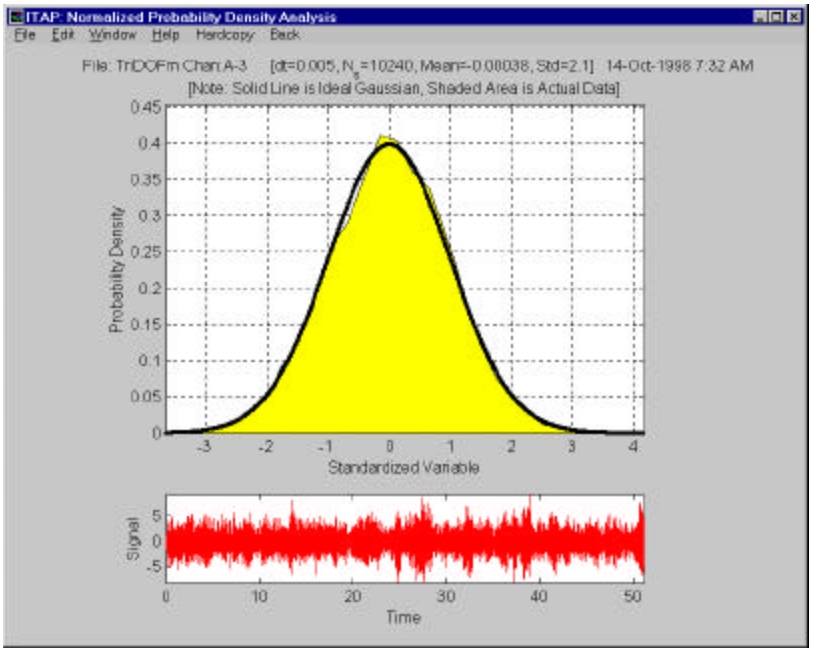
In order to scroll through successive data channels, the user presses the "**NextChan**" menu pick on the ITAP Time History Preview display. Shown below is the display for the fifth data channel (A-3):



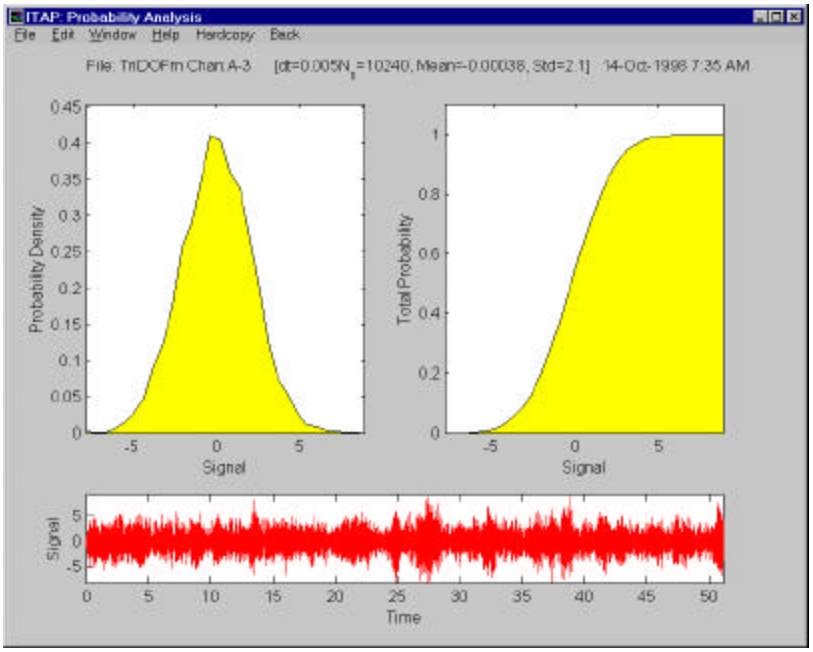
Note that the pull-down menu called "**Operation**" has been accessed in the Time History Preview display. By selecting the "**PowerSpec**" option, the following autospectrum display is generated:



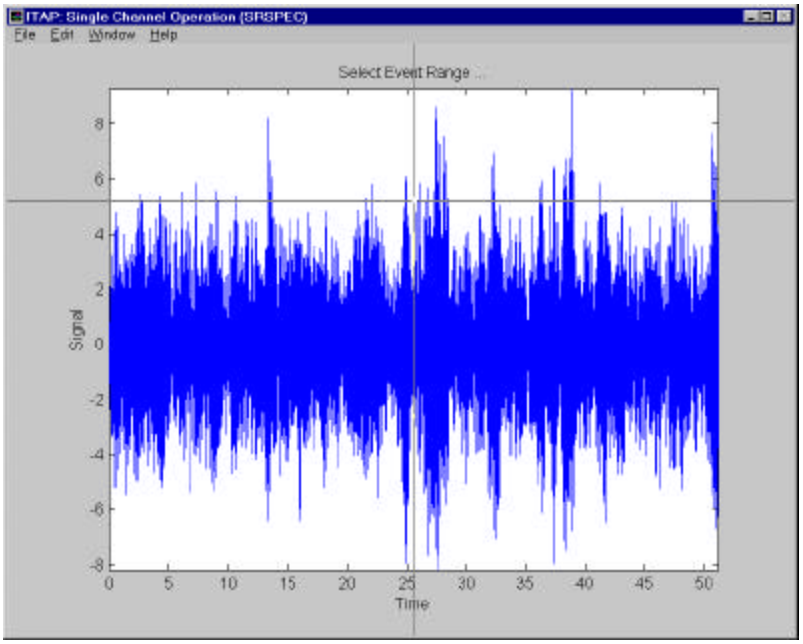
By pressing the “**Back**” button on the display, the ITAP Time History display is restored to the screen. Selection of the “**Operation/ProbDens**” option results in the following probability density display:



By pressing the “**Back**” button on the display, the ITAP Time History display is restored to the screen. Selection of the “**Operation/TotalProb**” option results in the following display of probability data:

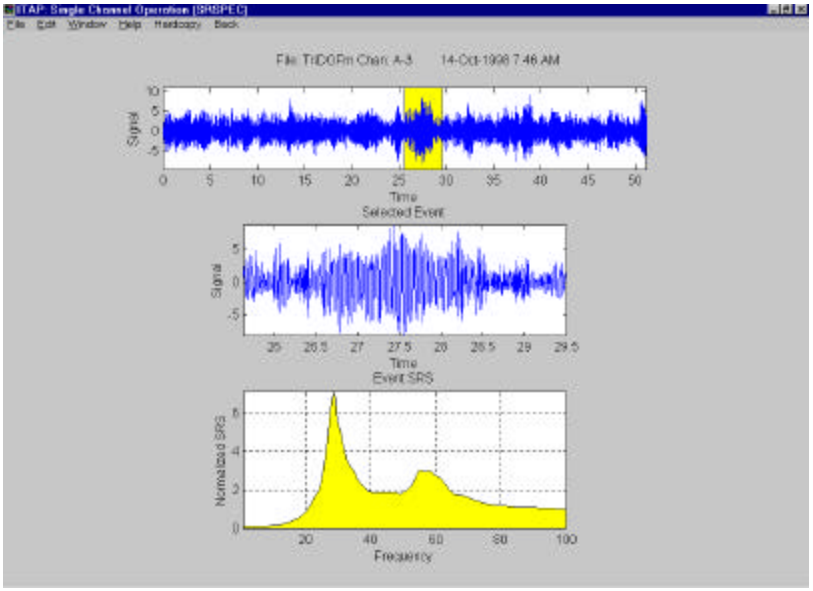


By pressing the “**Back**” button on the display, the ITAP Time History display is restored to the screen. Selection of the “**Operation/ShockSpec**” option results in the following time history display (from which a time segment of interest (event range) is selected for shock spectrum analysis):

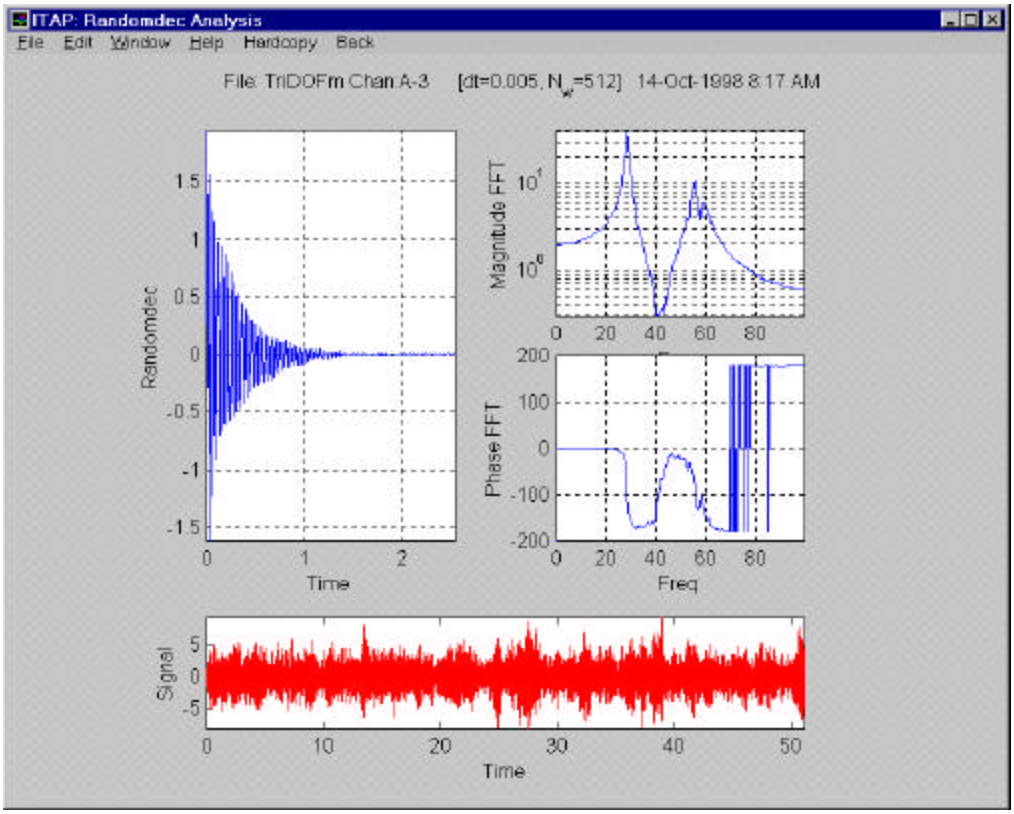


:

Upon marking the event range limits, the normalized shock spectrum (see Section 1.2.4 for theoretical background) is calculated and displayed as shown below:



By pressing the “**Back**” button on the display, the ITAP Time History display is restored to the screen. Selecting the “**Operation/Randomdec**” option results in the following display:



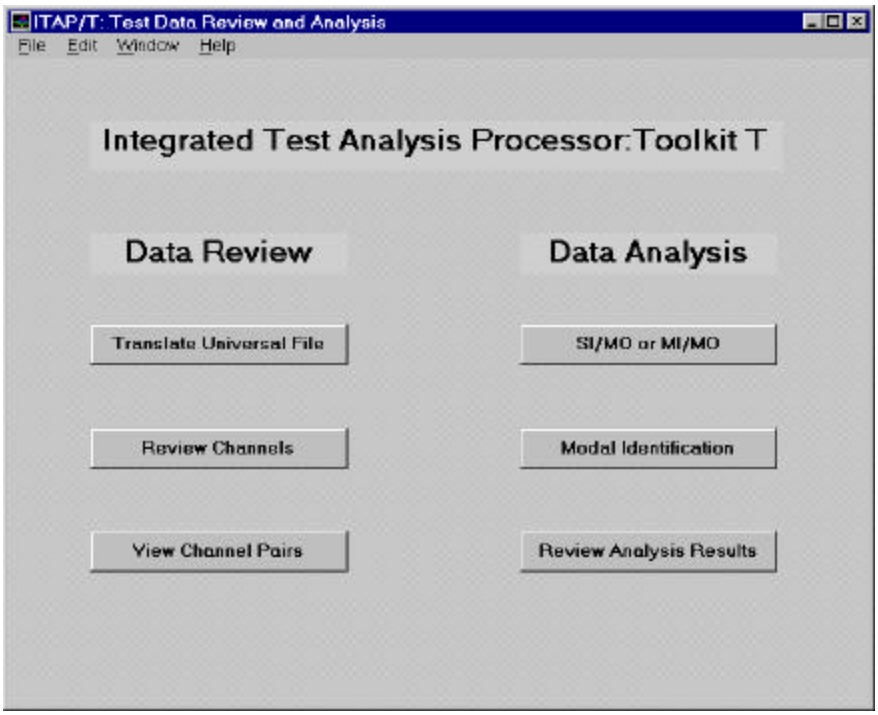
The upper left plot is the randomdec signature associated with the time history signal (the bottom plot). The two plots on the right are the magnitude and phase of the randomdec signature, which resemble a drive point frequency response function.

7.4 Preliminary Channel Pair Analysis

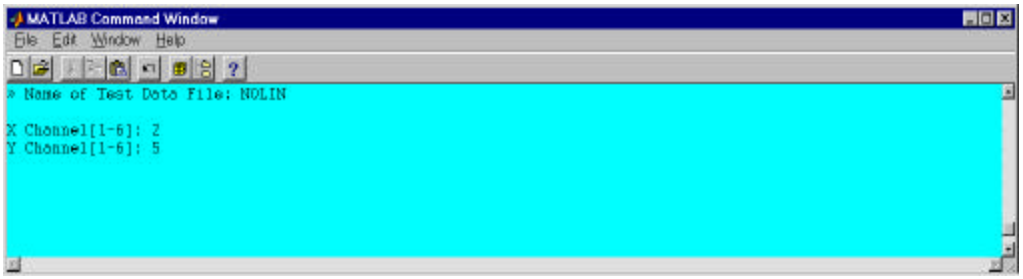
Pairs of measured data records may be reviewed graphically for the preliminary evaluation of suspected nonlinear behavior. The relevant display is a plot of one time history record (X) with respect to another (Y). When internal (structural member) loading and (structural member) displacement data is available, a clear picture of linear or nonlinear behavior is obtained. The ITAP-T function **pairevu** permits the user to select a channel (X,Y) pair and time segment of interest and display X versus Y.

7.4.1 Illustrative Example "pairevu" Session

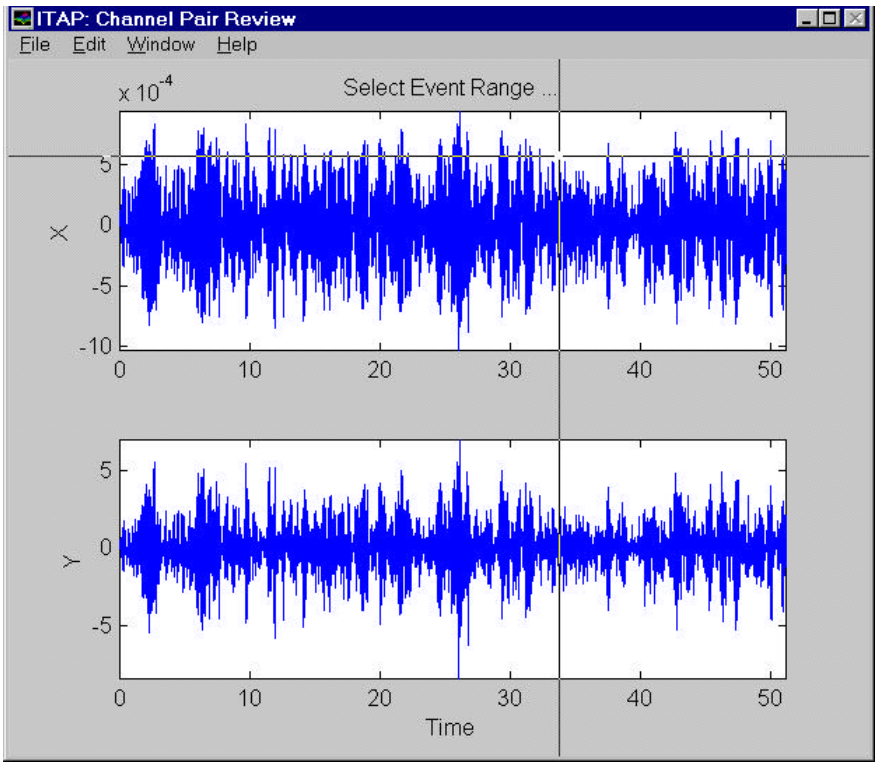
From the MATLAB command window, the user types "**ITAP-T**" to initiate the **pairevu** session. A screen with push-button choices appears as shown below:



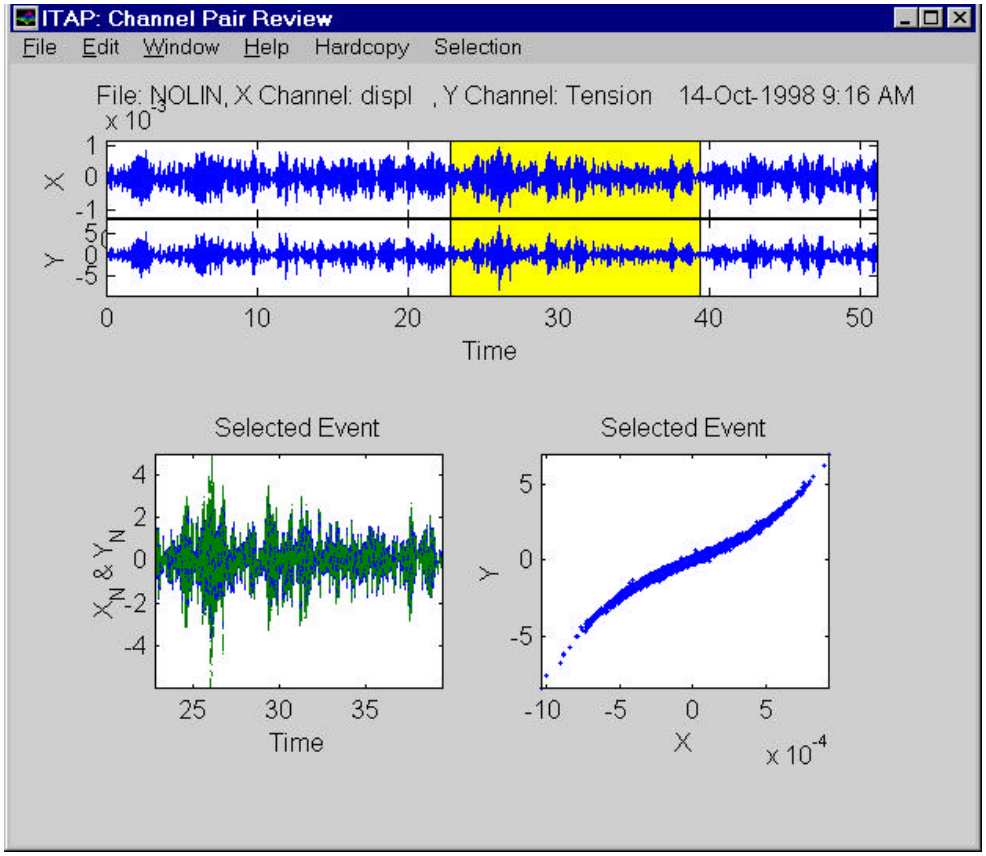
Upon pressing the "**View Channel Pairs**" button, successive requests for information appear on the MATLAB command window as shown below:



The following display of "X" and "Y" channel histories (from which the user selects an event range for further analysis):



Upon marking the event range limits, the channel pair display, shown below is generated:



Upon pressing the "**Selection**" menu pick, the user is prompted either to select another channel pair or end the session.

8.0 Spectral and Correlation Analysis

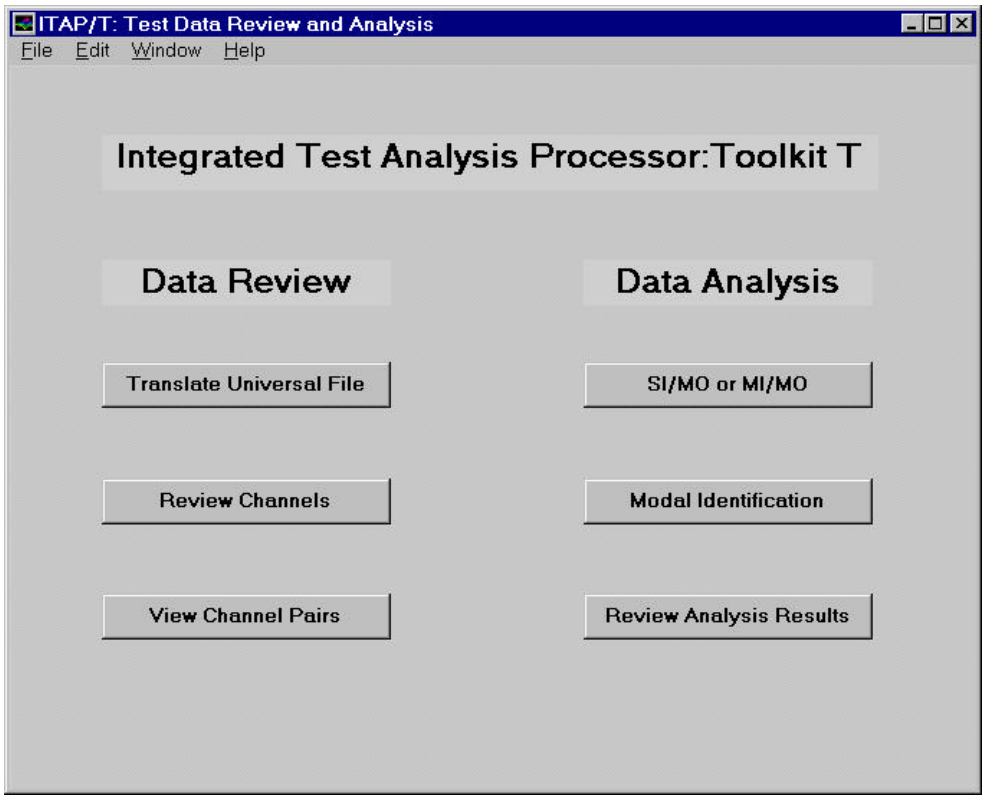
8.1 Introductory Remarks

8.2 Illustrative Example SI/MO and MI/MO Sessions

8.3 Review of SI/MO and MI/MO Analysis Results

8.1 Introductory Remarks

This chapter describes features of the ITAP-T toolbox, designed to address spectral and correlation analysis of measured data. These features have been incorporated within a menu structure accessed by the user command, **itapt**, which causes the following display to appear:



The three push-down buttons under the heading entitled **Data Analysis** are the primary functions to be discussed in this chapter. **SI/MO or MI/MO** initiates spectral and correlation analysis of a Basic Time History Data file.

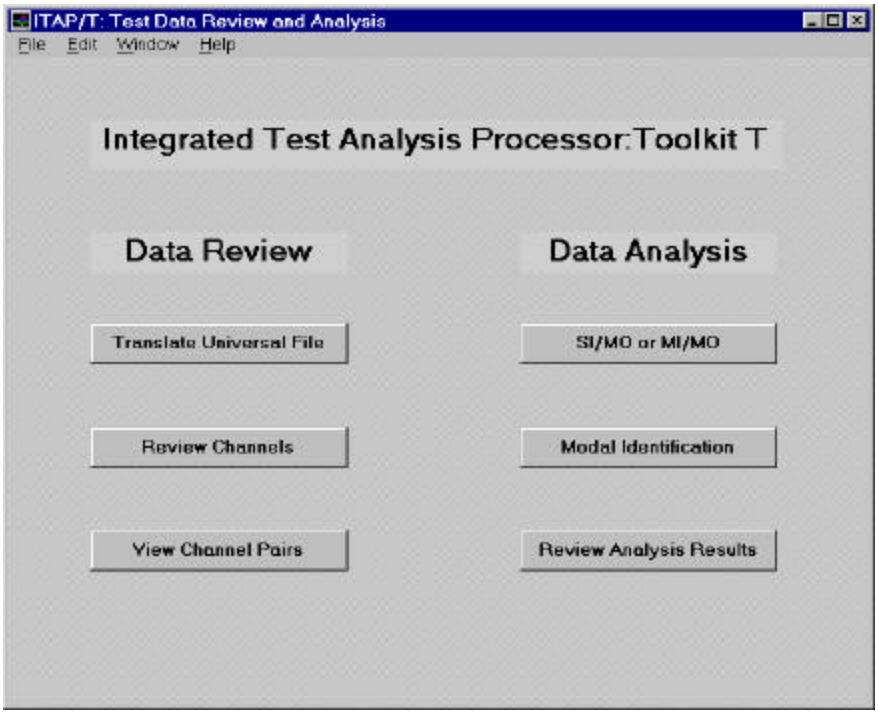
Application of theoretical concepts described in Sections 6.4 through 6.7, within the ITAP-T environment is the primary emphasis of this chapter.

8.2 Illustrative Example SI/MO and MI/MO Sessions

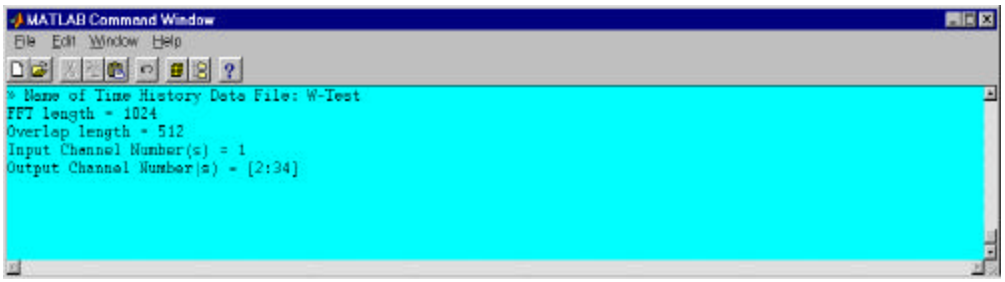
Before proceeding with spectral and correlation analysis of a Basic Time History Data file, raw time histories are reviewed and evaluated according to the procedures demonstrated in the previous chapter. Once the overall quality and content of time history data has been confirmed, key decisions are made regarding spectral and correlation analysis. In particular, the user must specify the FFT window length for spectral averaging and an overlap processing index (usually half the FFT window length is most appropriate). Fundamentals of spectral and correlation analysis have been discussed in Sections 6.4 through 6.7 of this manual. The material presented in the following sections describes implementation and usage of such methodology within the ITAP-T environment. The ITAP-T function module **mimo** coordinates all SI/SO, SI/MO, MI/SO and MI/MO computations, and output display and data storage operations.

8.2.1 Illustrative Example "SI/MO" Session

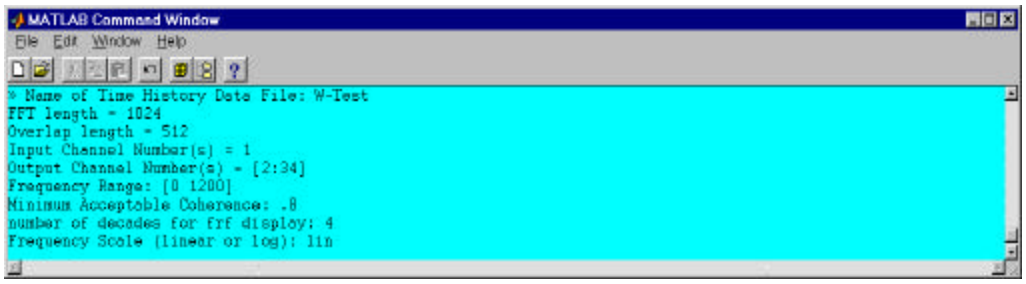
From the MATLAB command window, the user types "**itapt**" to initiate the **SI/MO** session. A screen with push-button choices appears as shown below:



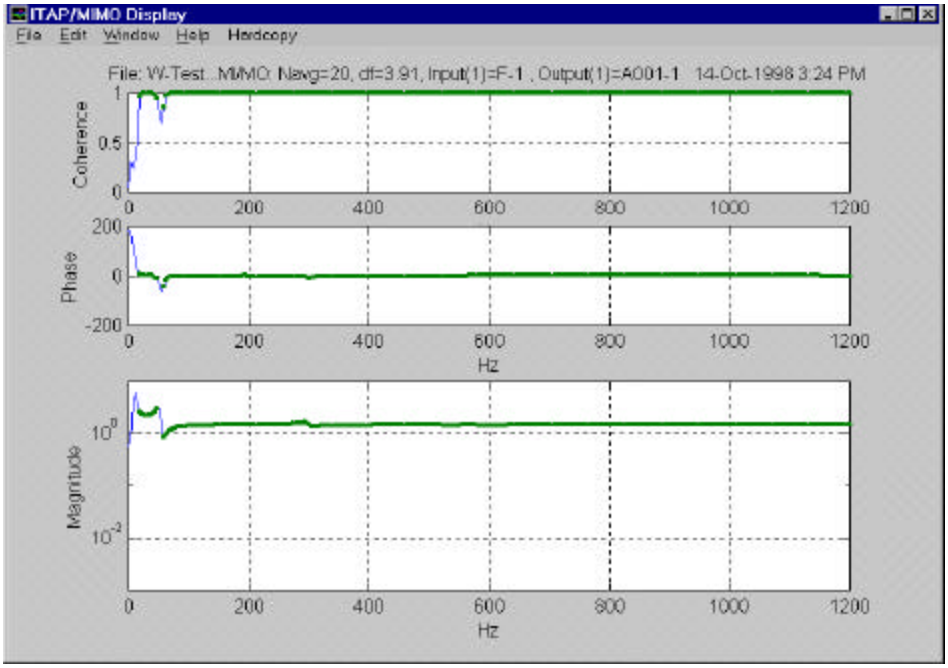
Upon pressing the "**SI/MO and MI/MO**" button, successive requests for information appear on the MATLAB command window as shown below:



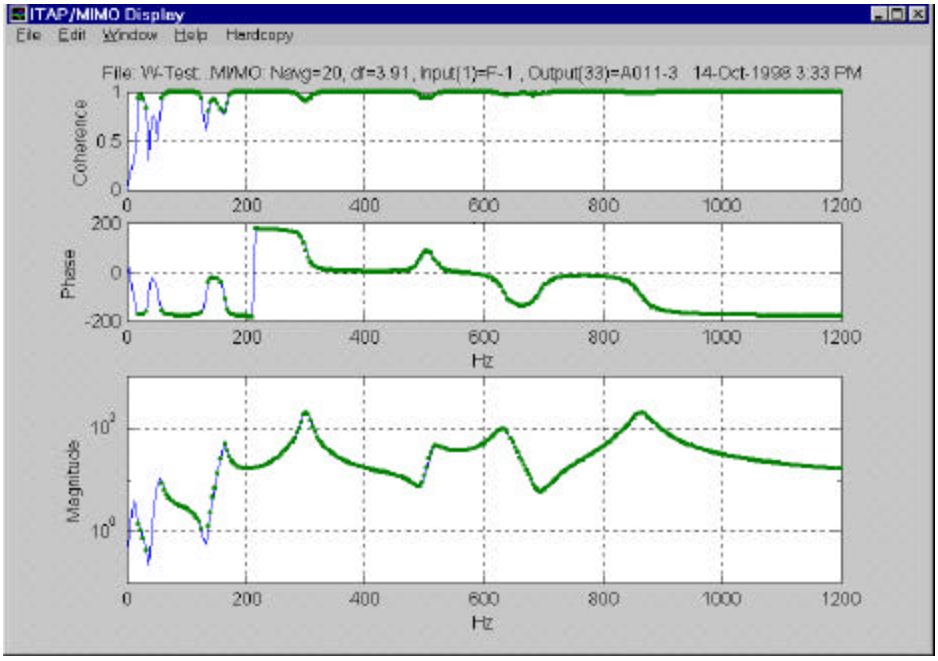
Requests for the time history data file name, FFT length and overlap length, input and output channel numbers channel names appear in the command window. SI/MO calculations are launched after all of the data requests are answered. When these calculations are completed, additional parameters related to output displays are requested in the command window as shown below:



The first ITAP/MIMO FRF and coherence display (associated with the first output channel) then appears on the screen, as illustrated below:



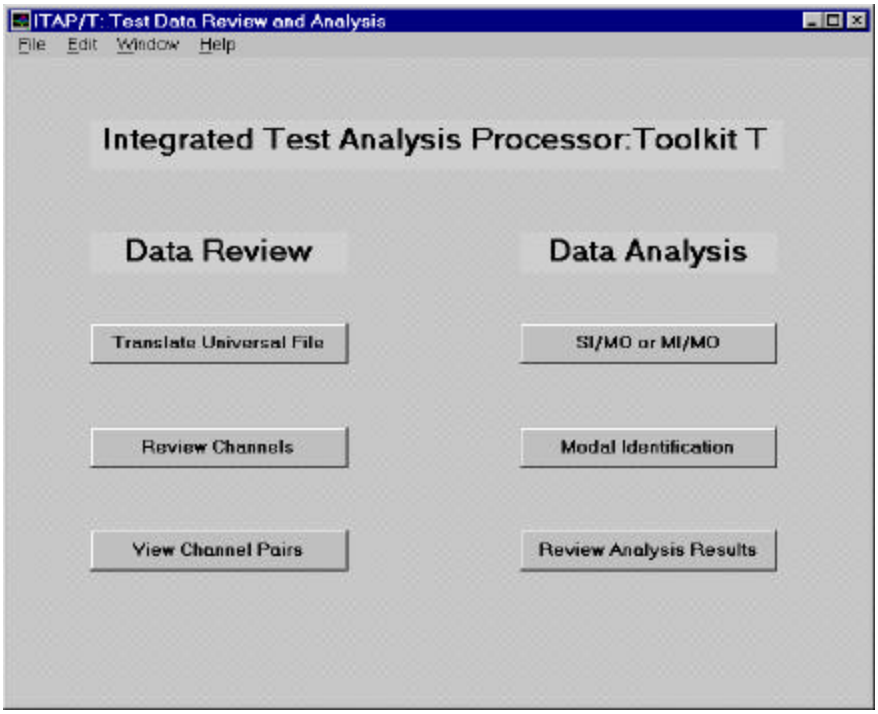
By pressing the “enter” or “return” key, successive output channel FRFs and associated coherences are displayed. Shown below is the final FRF and coherence display:



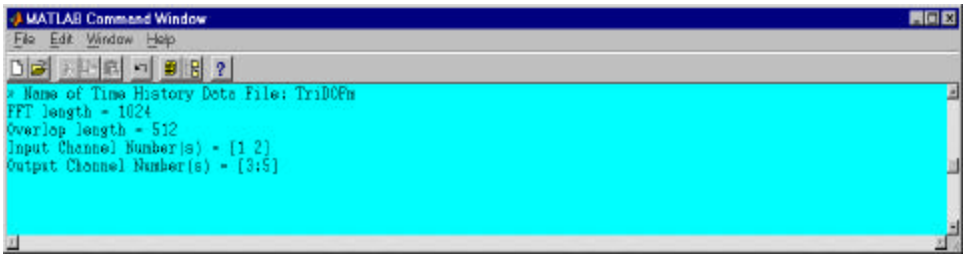
Note in the above plot that FRF and coherence data with coherence above the specified “minimum acceptable level=0.8” is highlighted with dots.

8.2.2 Illustrative Example "MI/MO" Session

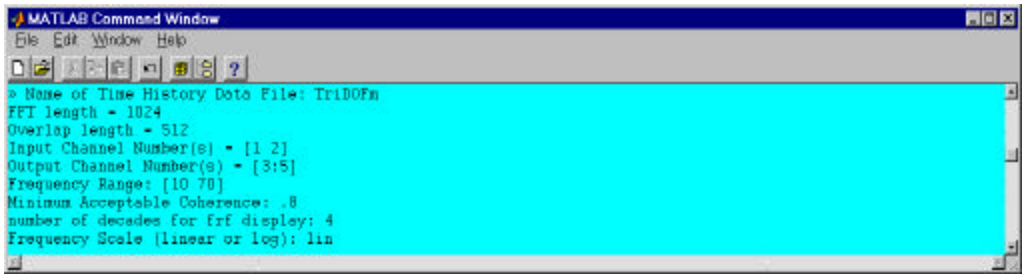
From the MATLAB command window, the user types "**itapt**" to initiate the **MI/MO** session. A screen with push-button choices appears as shown below:



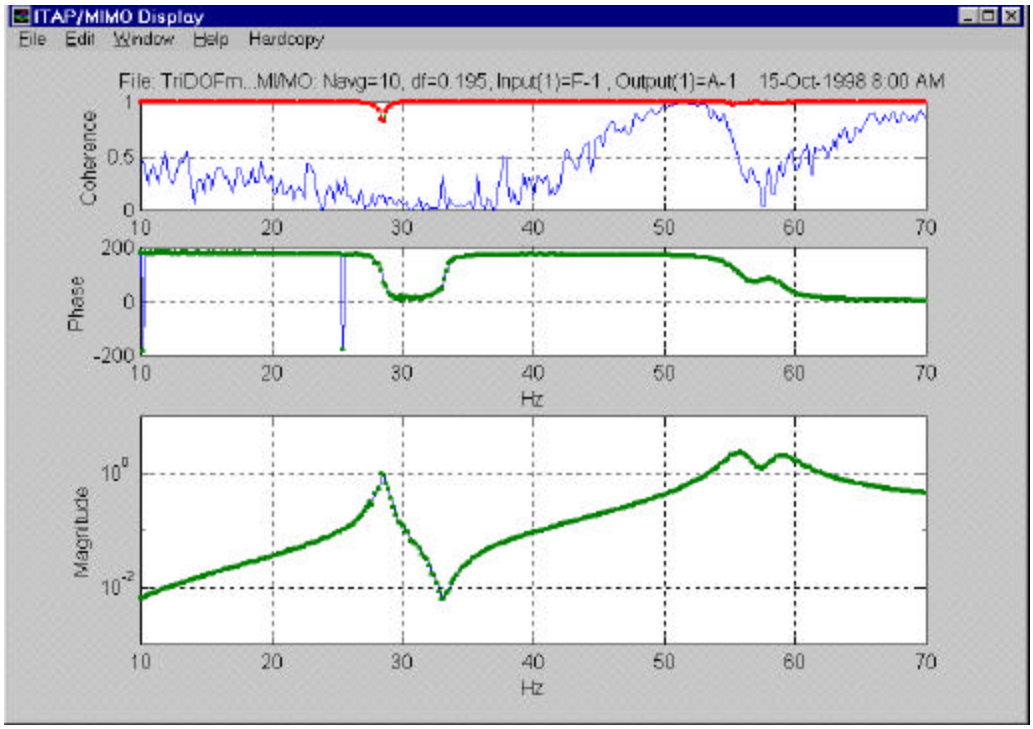
Upon pressing the "**SI/MO and MI/MO**" button, successive requests for information appear on the MATLAB command window as shown below:



Requests for the time history data file name, FFT length and overlap length, input and output channel numbers channel names appear in the command window. SI/MO calculations are launched after all of the data requests are answered. When these calculations are completed, additional parameters related to output displays are requested in the command window as shown below:

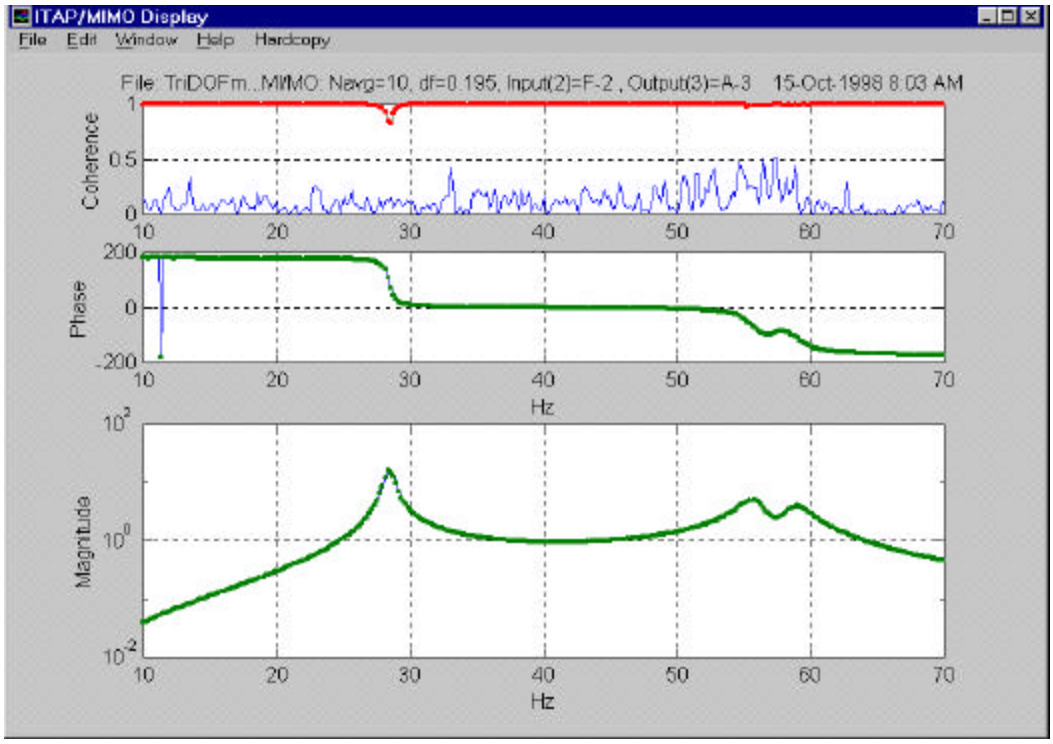


The first ITAP/MIMO FRF and (cumulative) coherence display (associated with the first output channel) then appears on the screen, as illustrated below:



Note that the coherence display indicates two coherence functions; the lower magnitude coherence function is associated with the first input (F-1) and the higher coherence is associated with the accumulated responses of both inputs (F-1 and F-2).

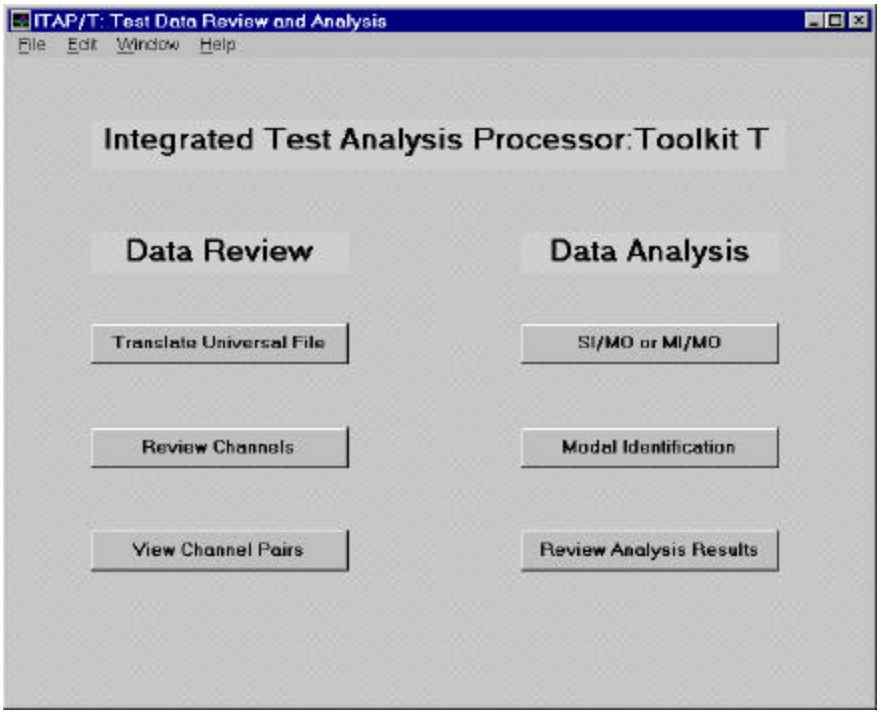
By pressing the “enter” or “return” key, successive output channel FRFs and associated coherences are displayed. Shown below is the final FRF and (cumulative) coherence display:



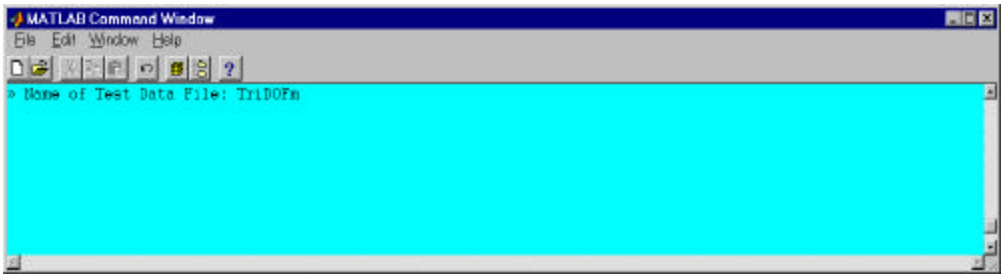
Note in the above plot that FRF and coherence data with (cumulative) coherence above the specified “minimum acceptable level=0.8” is highlighted with dots.

8.3 Review of SI/MO and MI/MO Analysis Results

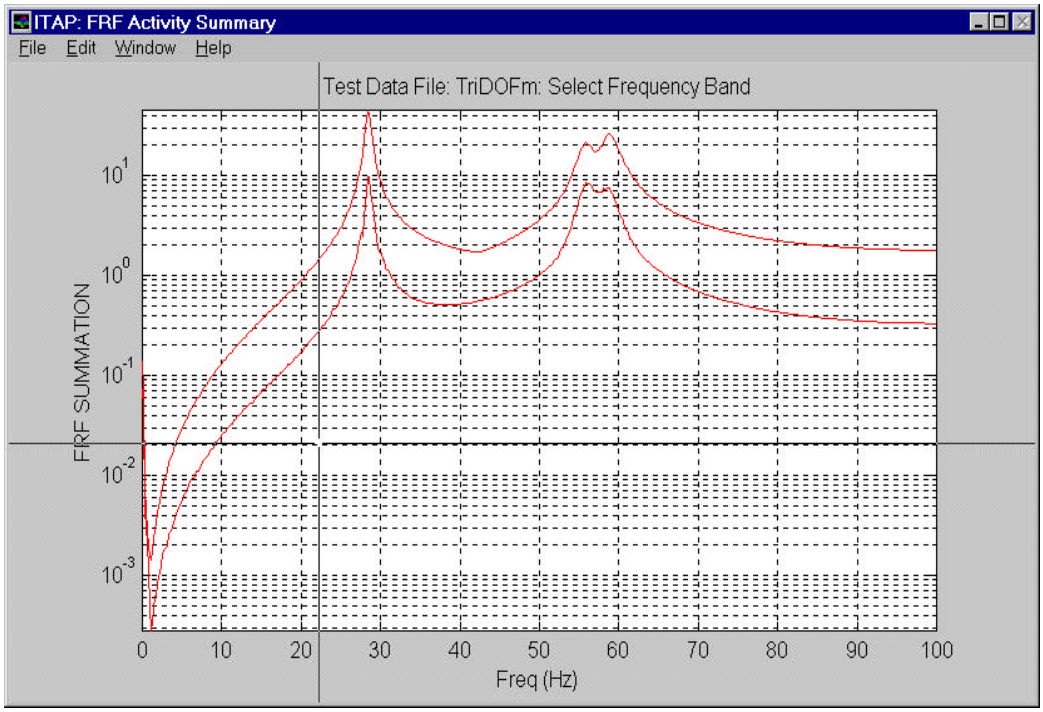
ITAP-T provides a feature which permits the user to review SI/MO and MI/MO analysis results. From the MATLAB command window, the user types "**itapt**" to initiate the **MI/MO** session. A screen with push-button choices appears as shown below:



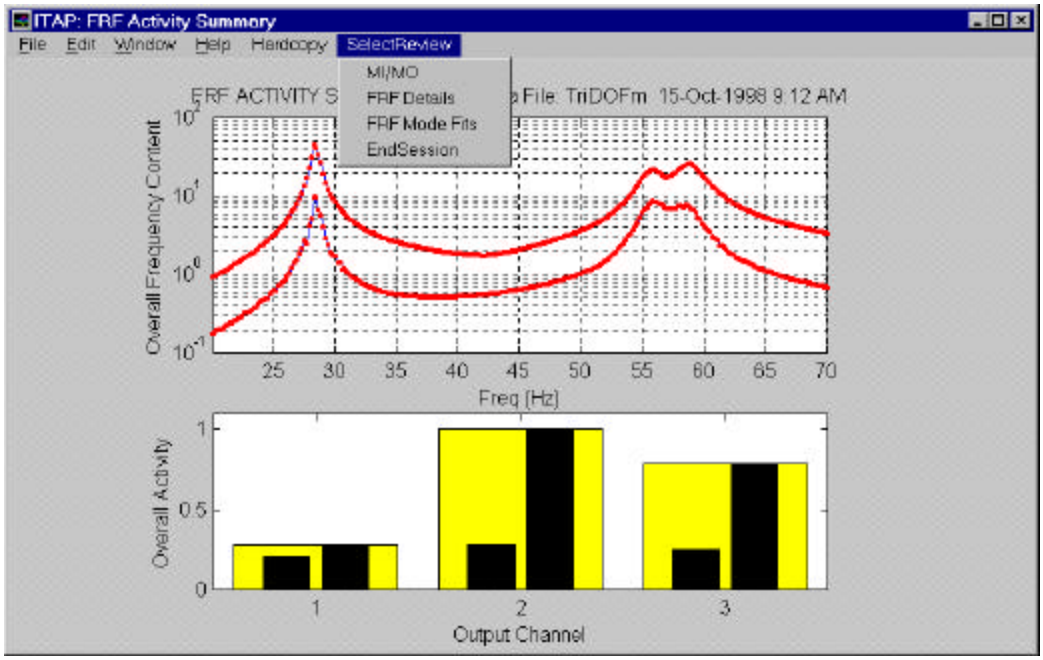
Upon pressing the "**Review Analysis Results**" button, a request for the test data file name appears on the MATLAB command window as shown below:



Upon response to this request, the following display appears, from which the user selects a frequency band of interest.



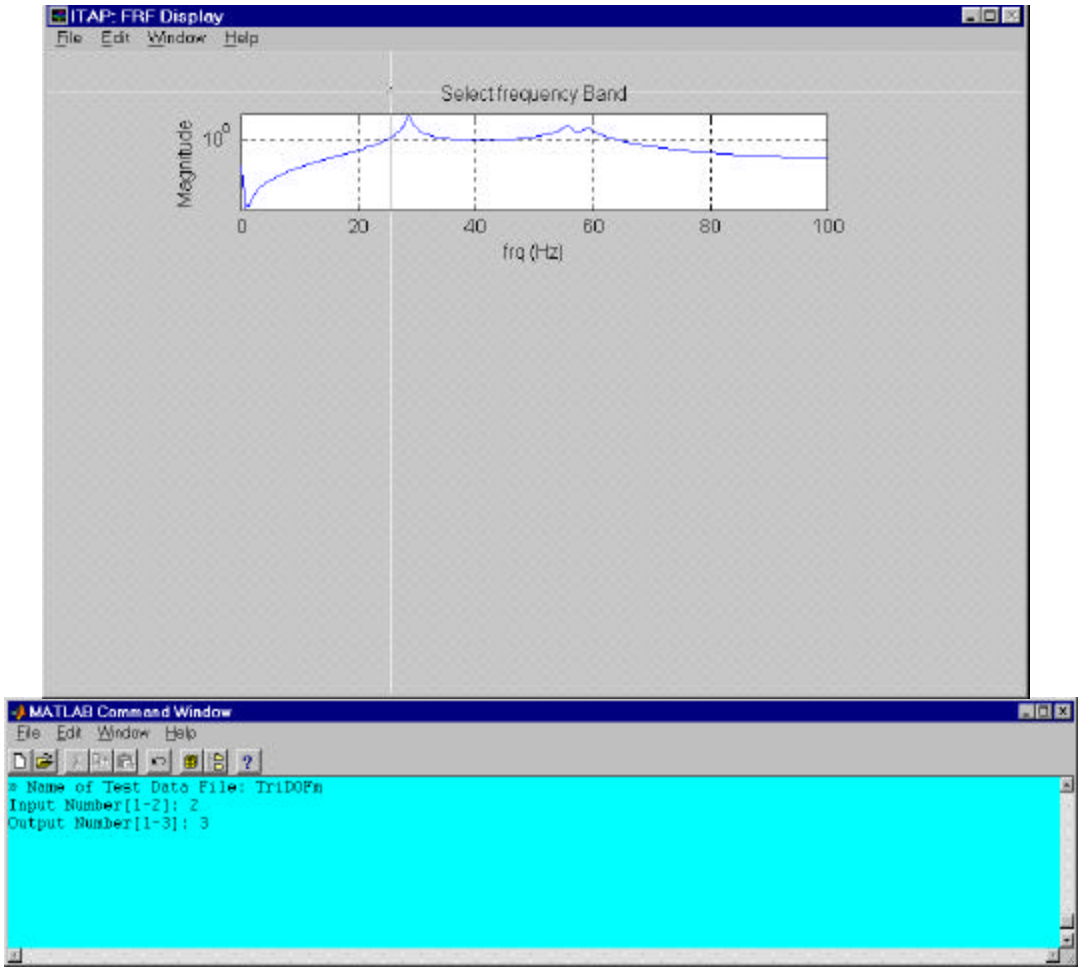
After the frequency band is selected, the following summary plot is generated with the ITAP-T function **frfsum**:



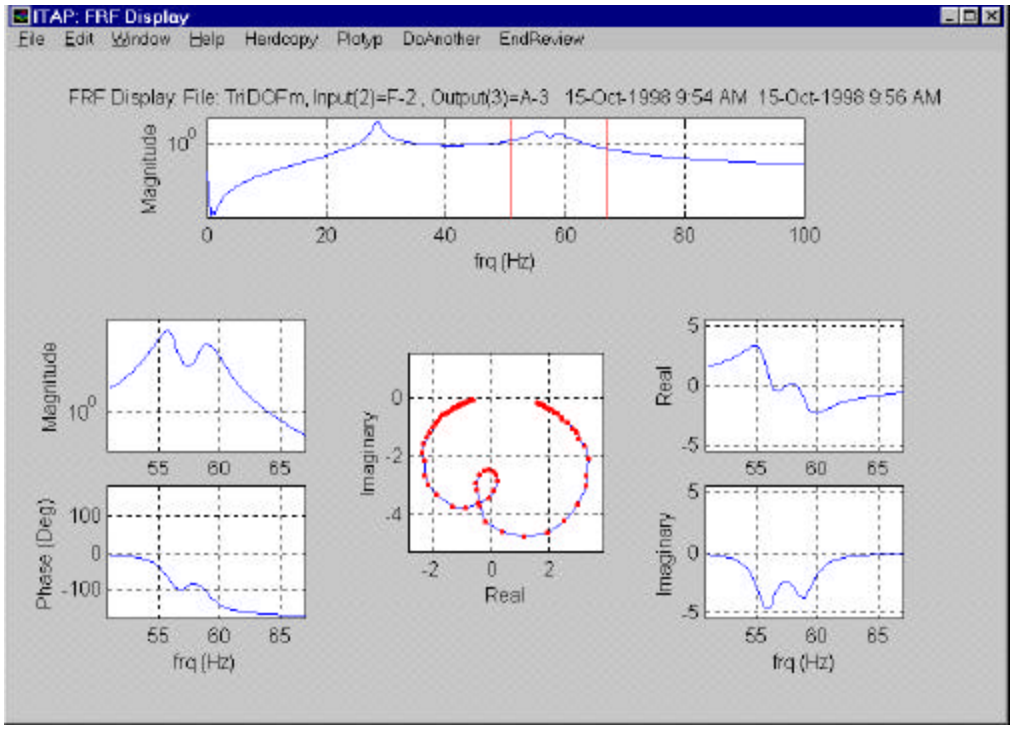
The upper plot in the display illustrates the summation of all output channel FRFs for the first input and for the first plus second inputs,

respectively. The lower bar plot provides a channel-by-channel breakdown of peak FRF magnitude over the selected frequency band; the darkened bars indicate peak contributions associated with the first input and second inputs, respectively.

By pressing the "**SelectReview**" pulldown menu button, a series of options are presented to the user. In the present situation, "**MI/MO**" and "**Review FRFs**" are the relevant options to be selected (other than "**EndSession**"). If "**MI/MO**" is selected, the ITAP-T function **mimoplt** is invoked and previously calculated FRFs and coherences are displayed for review. Sown below is an example of what occurs when the user selects the "**Review FRFs**" option:



Upon selection of a frequency band of interest, the following plot is generated by the ITAP-T routine **frfrevu**:



Additional actions, which may be taken by the user, include selection of "**Plotyp**", "**DoAnother**" or "**EndReview**" from the menu bar. "**Plotyp**" provides the user with options for alternative FRF display formats (see Chapter 10). "**DoAnother**" gives the user the option the select another FRF input-output channel pair for review. "**EndReview**", when pressed, signals an end to the review session.

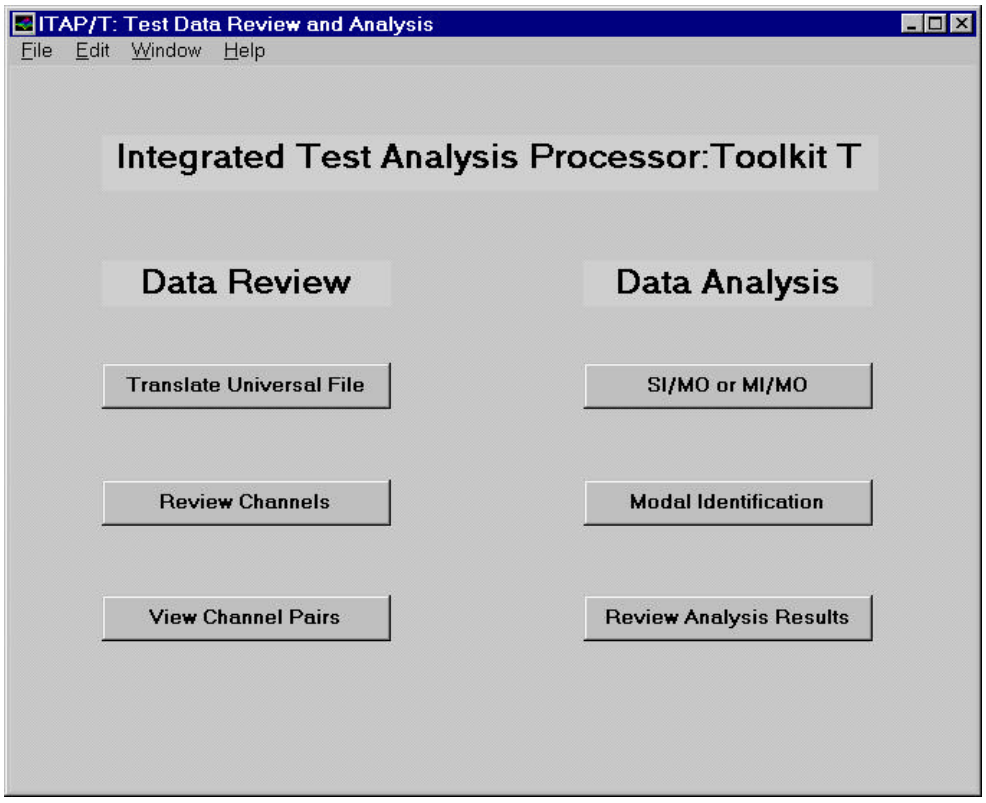
9.0 Experimental Modal Analysis

9.1 Introductory Remarks

9.2 Illustrative Example Modal Identification Sessions

9.1 Introductory Remarks

The present chapter describes features of the ITAP-T toolbox, designed to address modal parameter estimation from measured frequency response function (FRF) data (commonly known as **experimental modal analysis**). These features have been incorporated within a menu structure accessed by the user command, **itapt**, which causes the following display to appear:



The **Modal Identification** push-down button under the heading entitled **Data Analysis** is the primary function to be discussed in this chapter. **Modal Identification** initiates the modal parameter estimation process, which is based on theoretical concepts described in Section 6.8.

9.2 Illustrative Example Modal Identification Sessions

Before proceeding with modal identification, the experimental FRF data set should be thoroughly reviewed and evaluated for quality and content (see Section 8.3). Once the overall quality and content of FRF data has been confirmed, modal identification is initiated. The material presented in the following sections describes implementation and usage of modal identification methodology within the ITAP-T environment. The ITAP-T function module **modexe** coordinates all experimental modal analysis computations, output display and storage operations.

9.2.1 Overview of Modal Identification Function Modules called by modexe

The ITAP-T function module **modexe** controls the complete series of modal identification operations, which are as follows:

1. Specify the Name of the File containing FRF Data.

2. Select frequency band for modal search (from a composite display of all FRFs).

3. Calculate and display the FRF skyline (with the function module **skyline**), and select frequency bands with apparent modal activity.

4. Within each selected frequency band with apparent modal activity, calculate modal frequencies and critical damping ratios (with the function module **modexb,** which calls the function modules **sfd1** and **sfd2**).

5. Calculate system modal vectors as either real vectors (with the function module **sfd3**) or complex modal vectors (with the function module **sfd4**). Within **sfd3** and **sfd4** reconstructed FRFs are synthesized for modal identification process quality assessment.

6. For multiple input based FRF data calculate “weighted average” system modes.

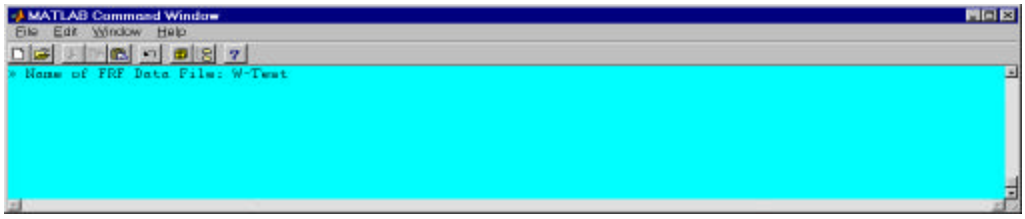
7. Plot original and reconstructed FRFs.

8. Append the Test Data File with the following matrix data:

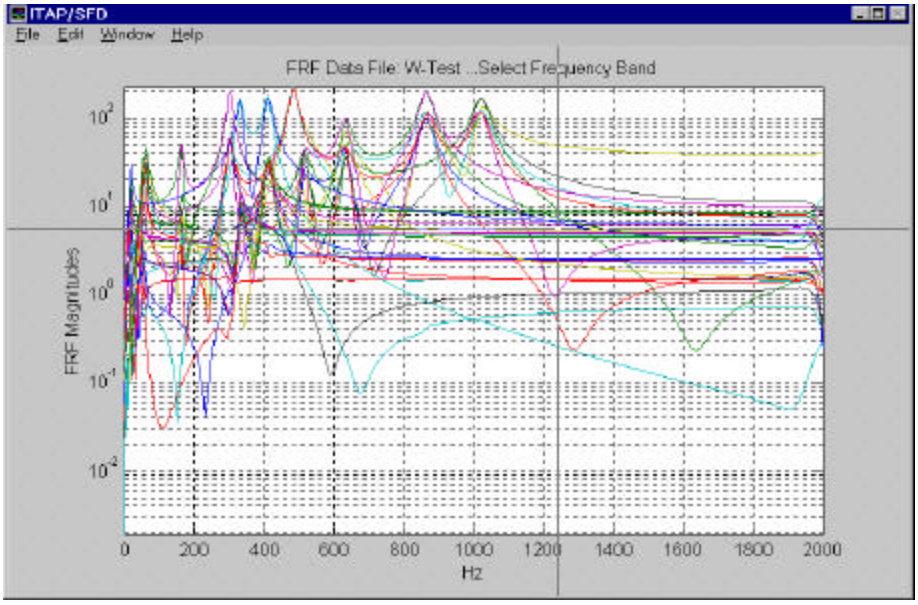
[FREQ]:	modal frequencies
[ZETA]:	modal critical damping ratios
[PHI]:	modal vectors
[PHIRAW]:	raw modal vectors
[FRFIT]:	reconstructed FRFs

9.2.2 Example “Modexe” Session (Single Input FRFs, Real Modes)

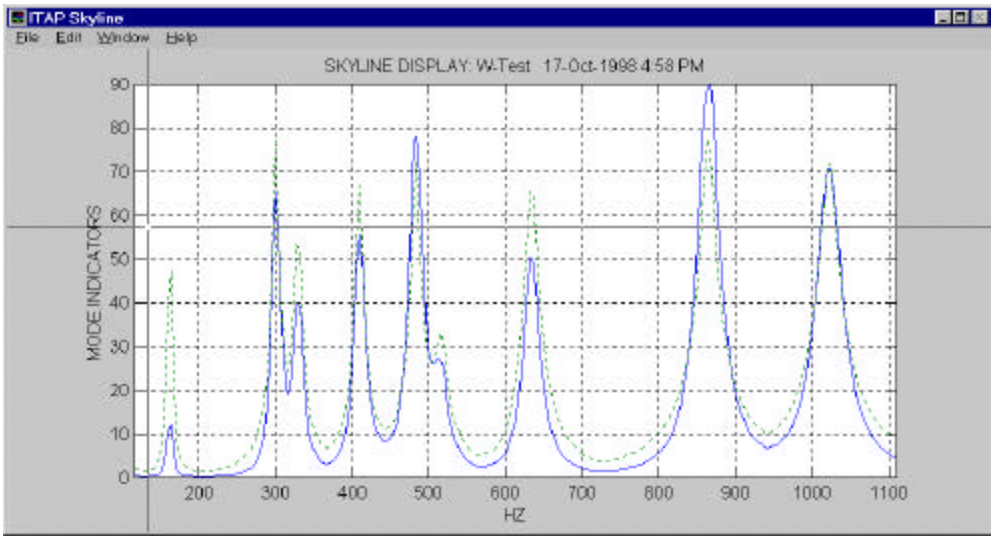
From the MATLAB command window, the user types “**itapt**” to initiate the session. Upon pressing the “**Modal Identification**” button, the name of the FRF data file is requested as shown below (with the user’s response):



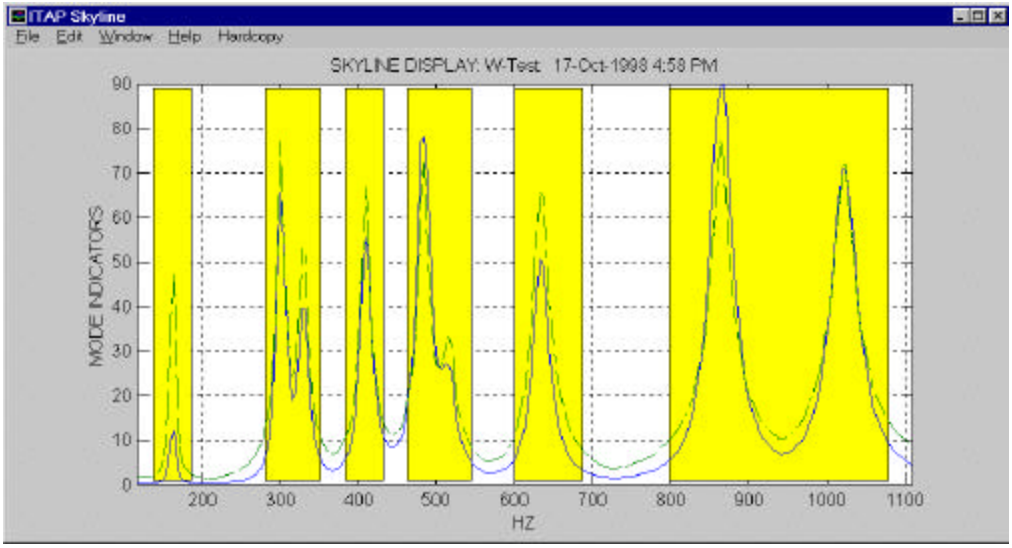
Upon pressing the return key, the following composite FRF display appears with a request to select a frequency band:



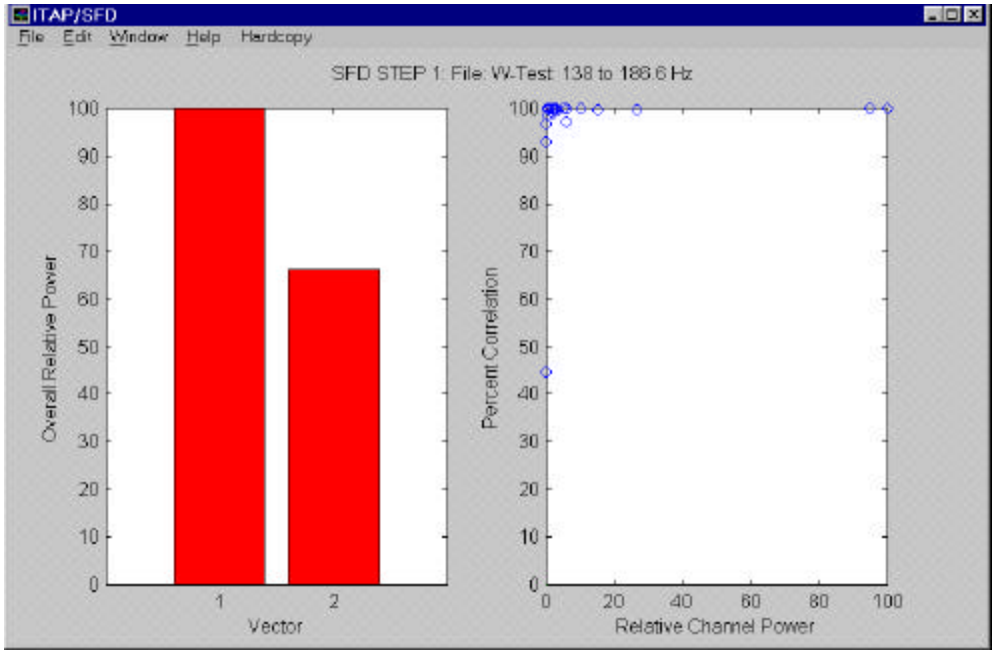
After the lower and upper limits of the frequency band are selected, the following skyline function appears:



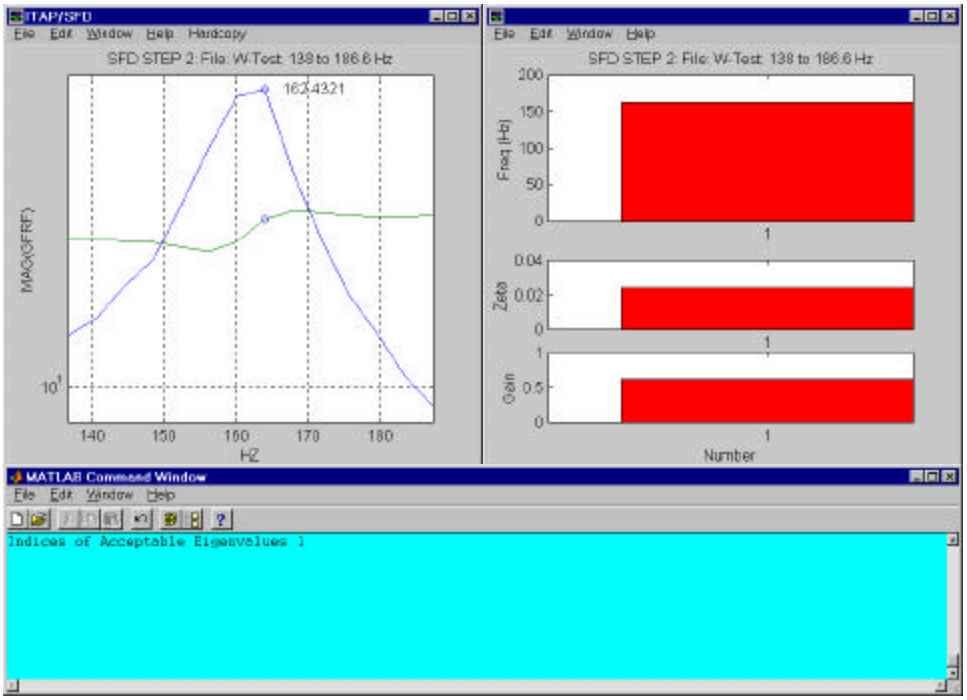
The user next selects individual frequency bands where modal activity is suspected (by clicking lower and upper bounds of each band. When all individual bands have been selected, the user presses the “return” or “enter” key. The following summary plot is then displayed:



After pressing the “return” or “enter” key, individual frequency band modal analyses are performed. The first display which appears (as shown below) Indicates that the first selected frequency band (138-186.6 Hz) has two dominant trial vectors which (with associated generalized FRFs reproduce the original FRFs to greater that 90% correlation.

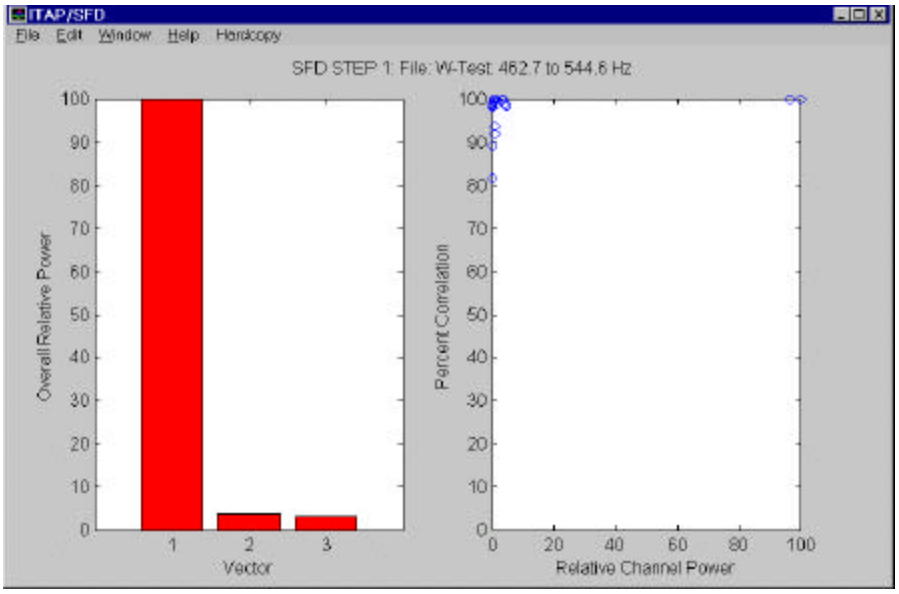


By pressing the “return” or “enter” key again, the generalized FRFs are displayed along with the identified candidate modal frequencies and critical damping ratios as follows:

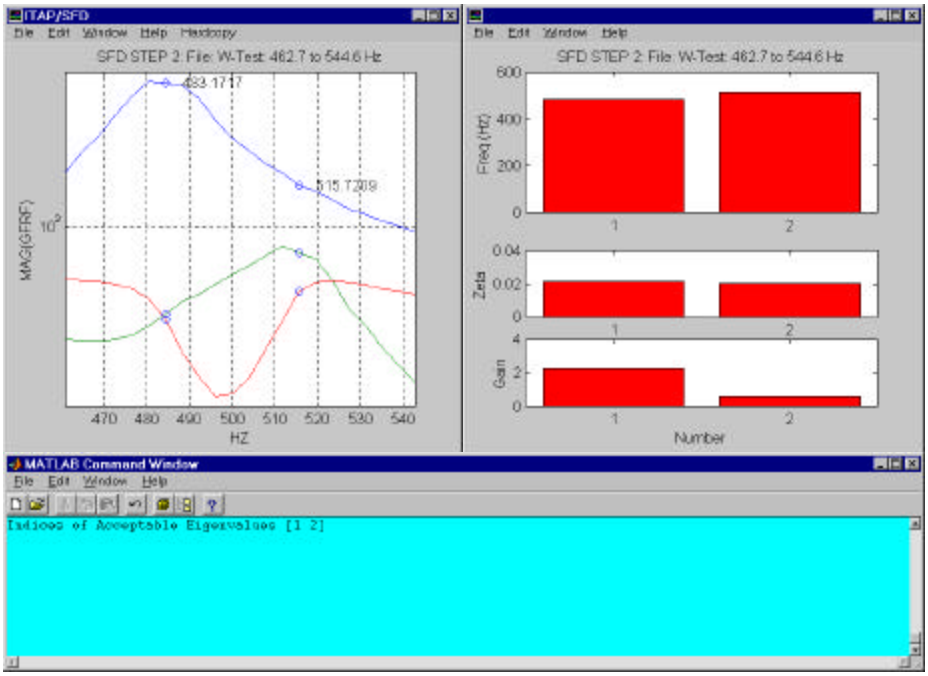


Note that although two dominant vectors were identified, only one mode was found in the frequency band. In the command window, the user is prompted to confirm which modes are acceptable.

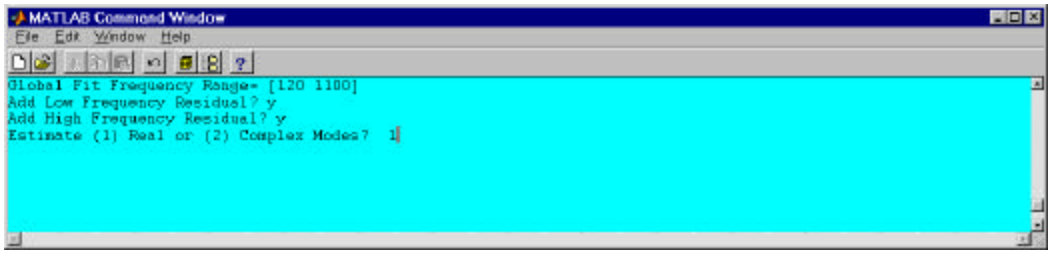
Upon pressing the “return” or “enter” key again, the process is repeated for each successive frequency band. The displays associated with the fourth frequency band are show below:



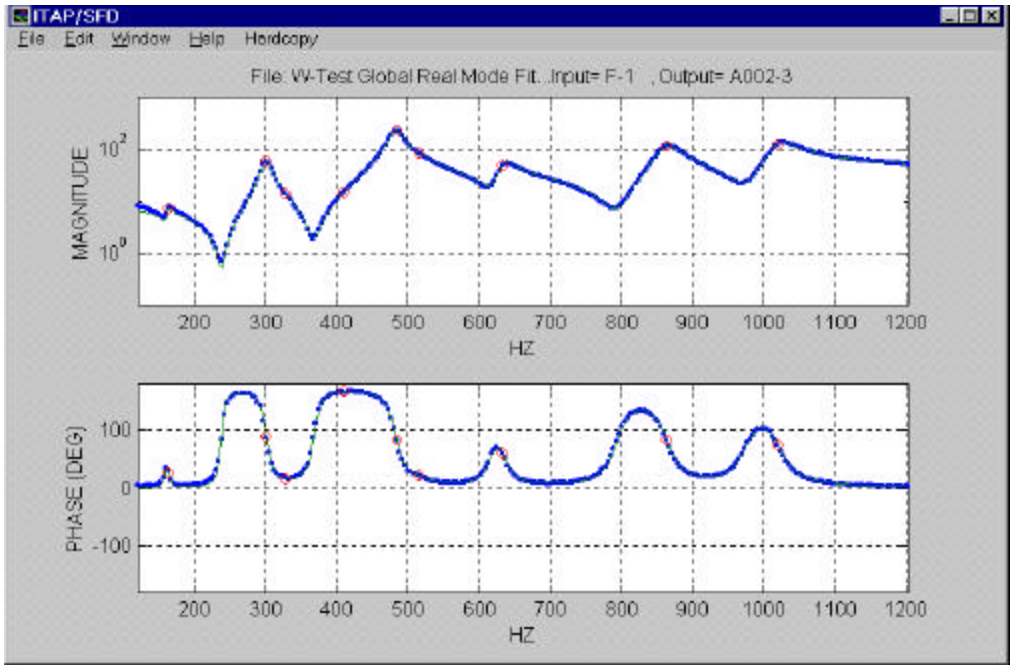
This frequency band has a strong “primary” trial vector and two “secondary” trial vectors. The next display indicates the three generalized FRFs and two candidate modes as follows:



After the final frequency band has been processed, the following request appears in the MATLAB command window:



Upon providing the requested data the “return” or “enter” key is pressed and FRF comparison displays of the following type appear:

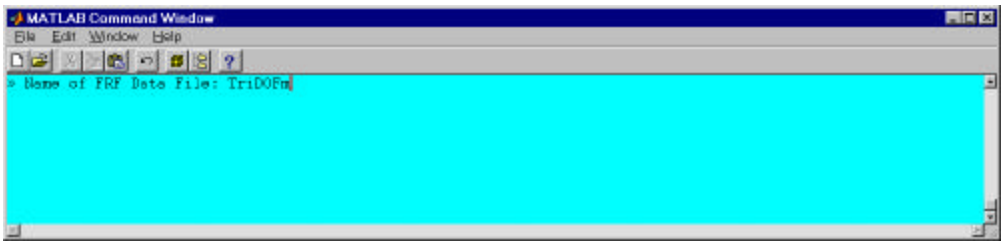


After the final FRF comparison display appears, the test data file is appended with key results of the **modexe** session.

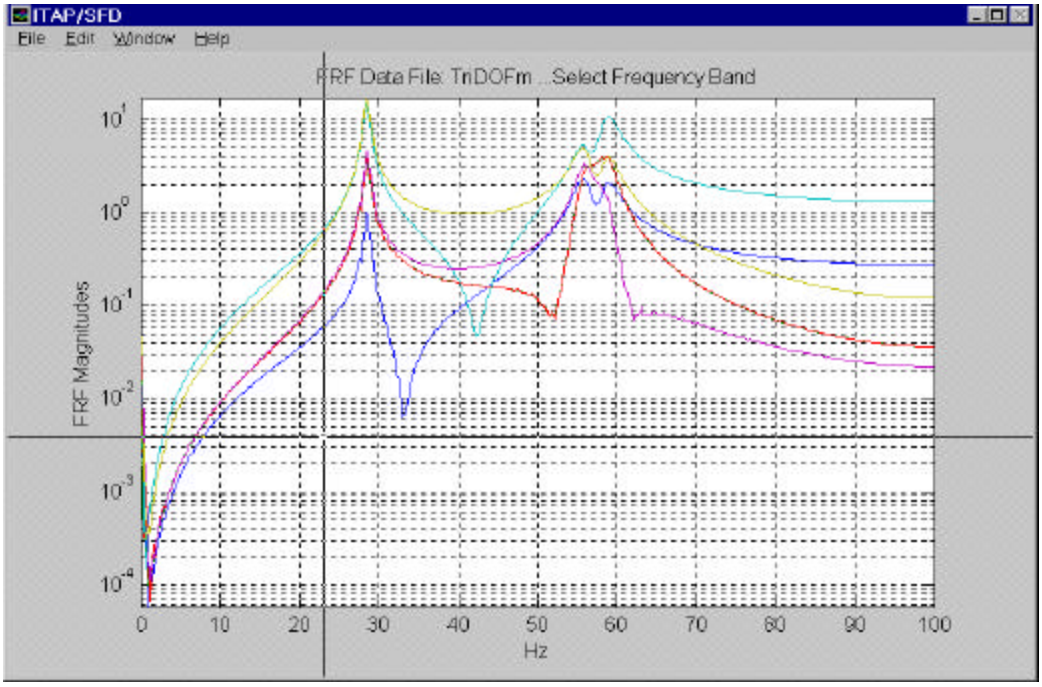
It should be noted here that results of this **modexe** session were used in the test-analysis correlation example discussed in Chapter 4.

9.2.3 Example “Modexe” Session (Multiple Input FRFs, Complex Modes)

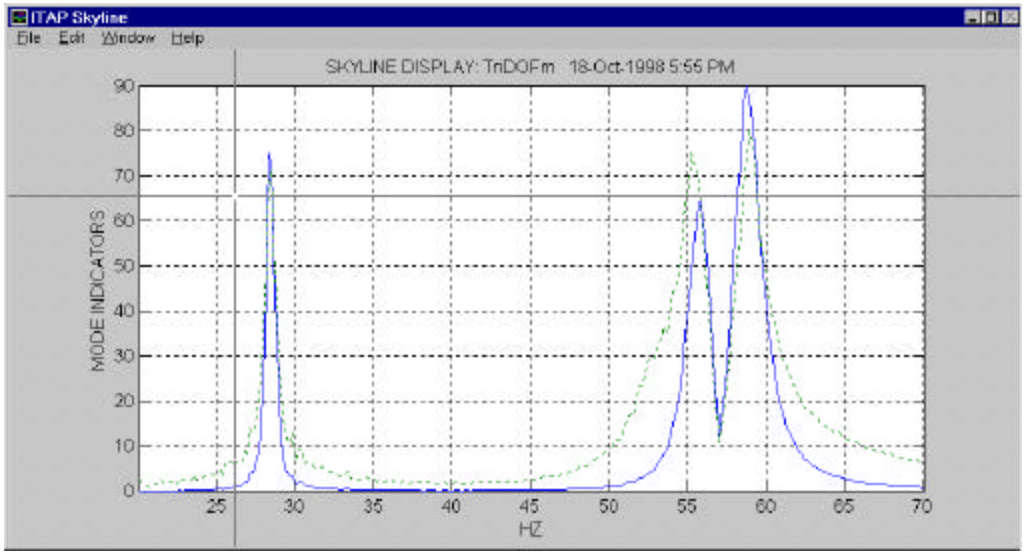
From the MATLAB command window, the user types “**itapt**” to initiate the session. Upon pressing the “**Modal Identification**” button, the name of the FRF data file is requested as shown below (with the user’s response):



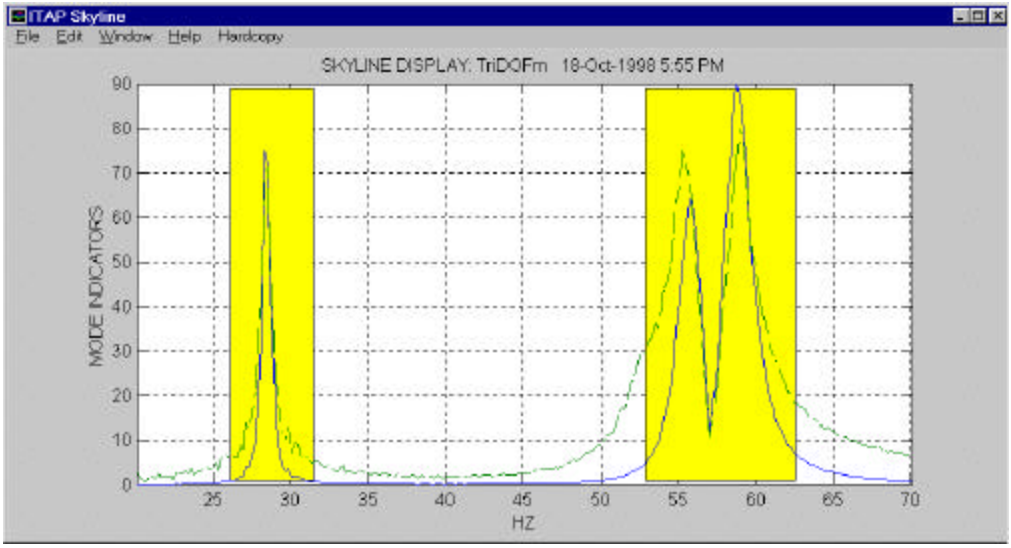
Upon pressing the return key, the following composite FRF display appears with a request to select a frequency band:



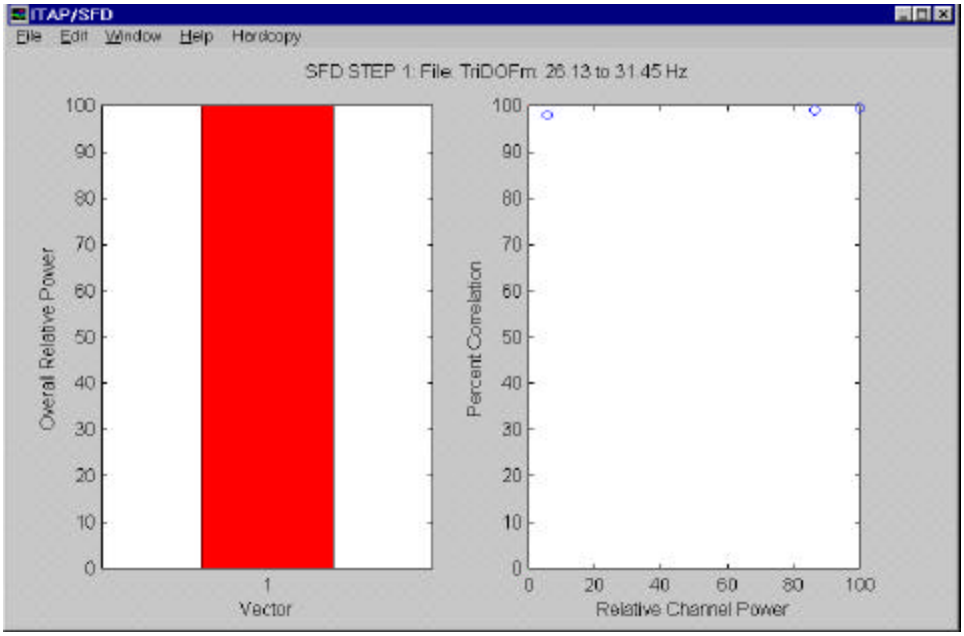
After the lower and upper limits of the frequency band are selected, the following skyline function appears:



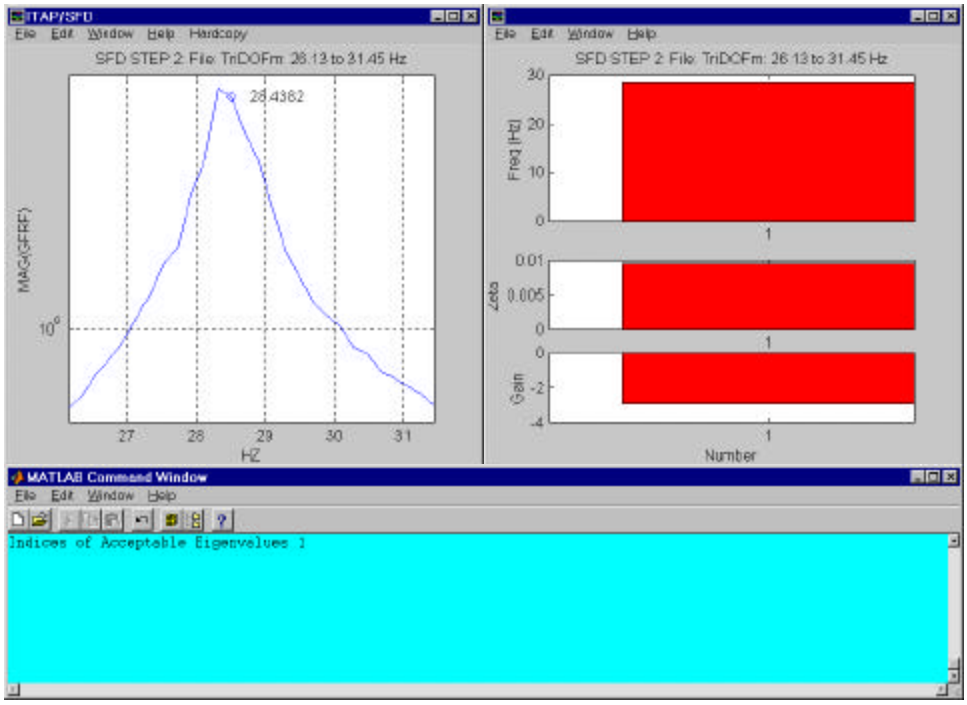
The user next selects individual frequency bands where modal activity is suspected (by clicking lower and upper bounds of each band. When all individual bands have been selected, the user presses the “return” or “enter” key. The following summary plot is then displayed:



After pressing the “return” or “enter” key, individual frequency band modal analyses are performed. The first display which appears (as shown below) Indicates that the first selected frequency band (26.13-31.45 Hz) has one dominant trial vector which (with its associated generalized FRF reproduces the original FRFs to greater that 95% correlation.

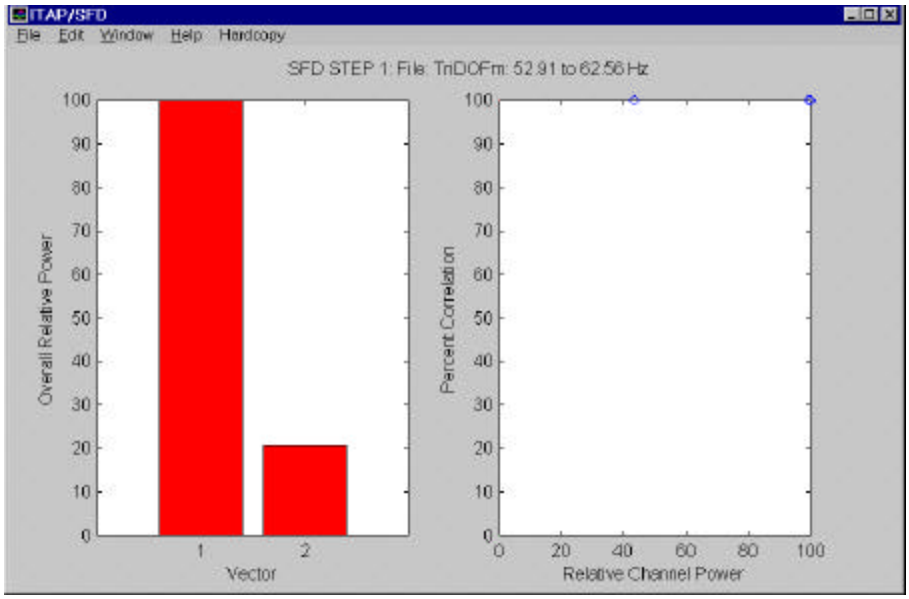


By pressing the “return” or “enter” key again, the generalized FRF is displayed along with the identified candidate modal frequency and critical damping ratio as follows:

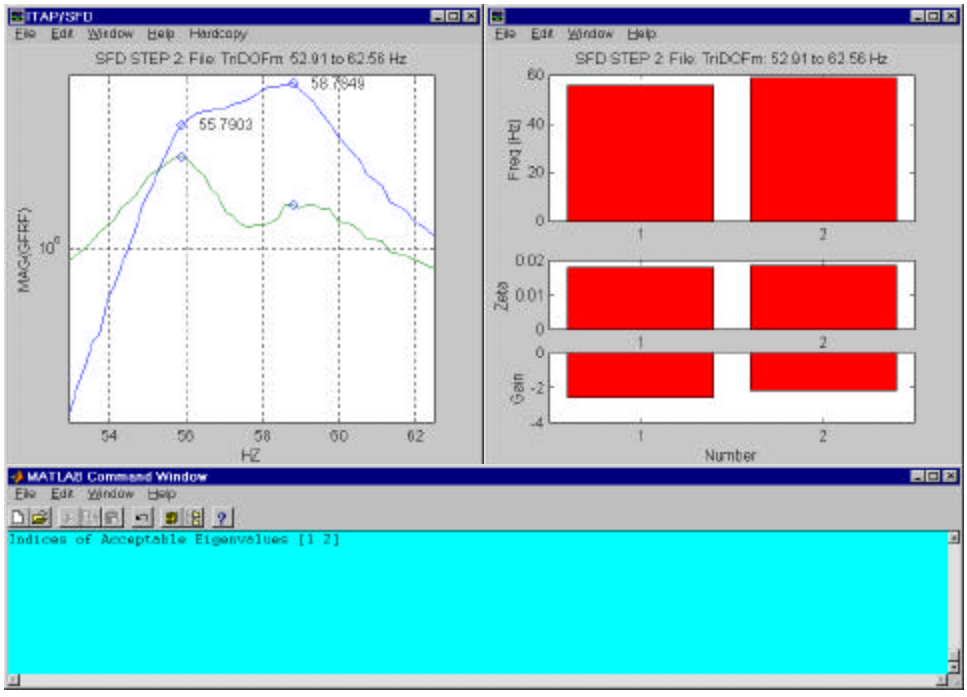


In the command window, the user is prompted to confirm that the single mode is acceptable.

Upon pressing the “return” or “enter” key again, the process is repeated for the next frequency band. The displays associated with this frequency band are show below:

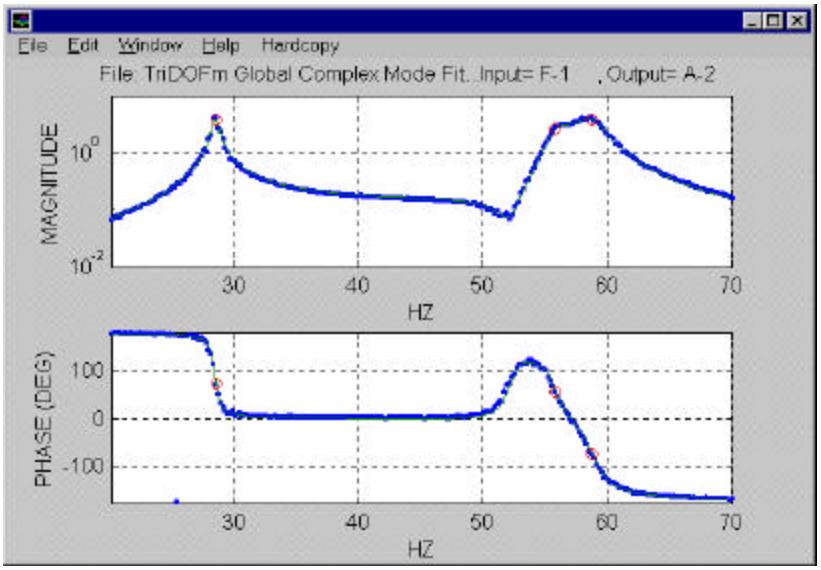


This frequency band has a strong “primary” trial vector a “secondary” trial vector. The next display indicates the two generalized FRFs and two candidate modes as follows:



After this final frequency band has been processed, the following request appears in the MATLAB command window:

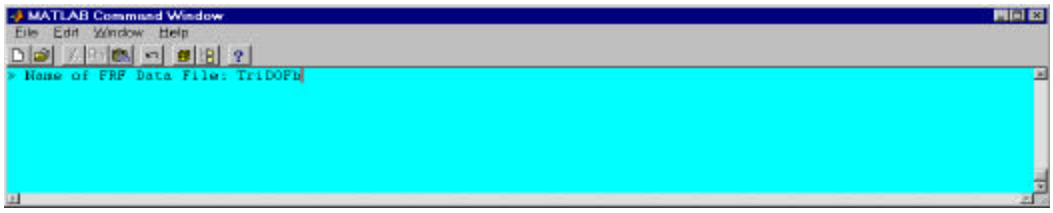
Upon providing the requested data the “return” or “enter” key is pressed and FRF comparison displays of the following type appear:



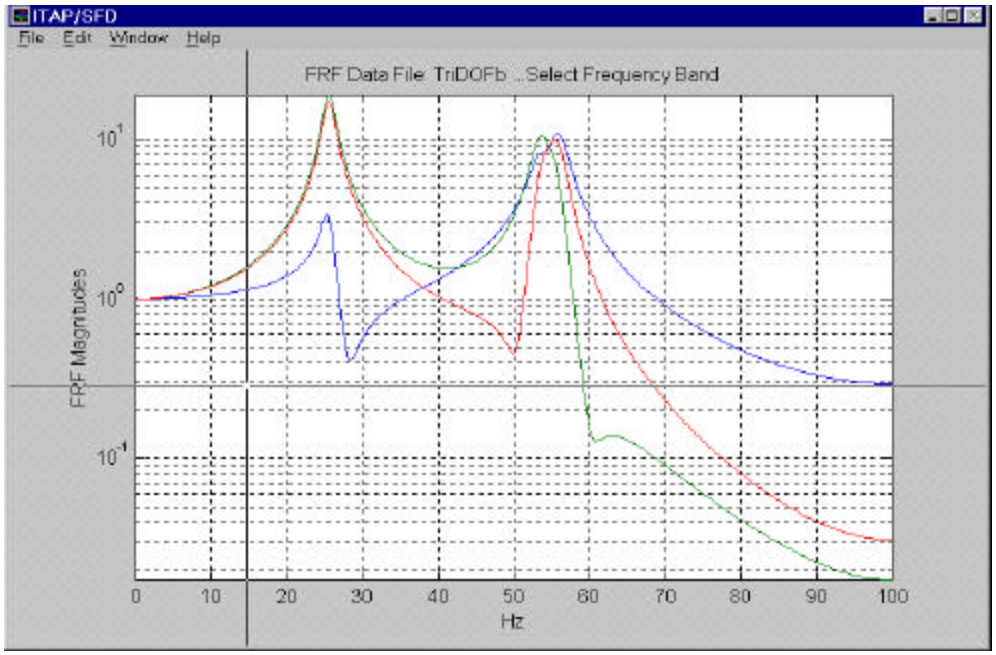
After the final FRF comparison display appears, the test data file is appended with key results of the **modexe** session.

9.2.4 Example “Modexe” Session (Base Motion Input, Complex Modes)

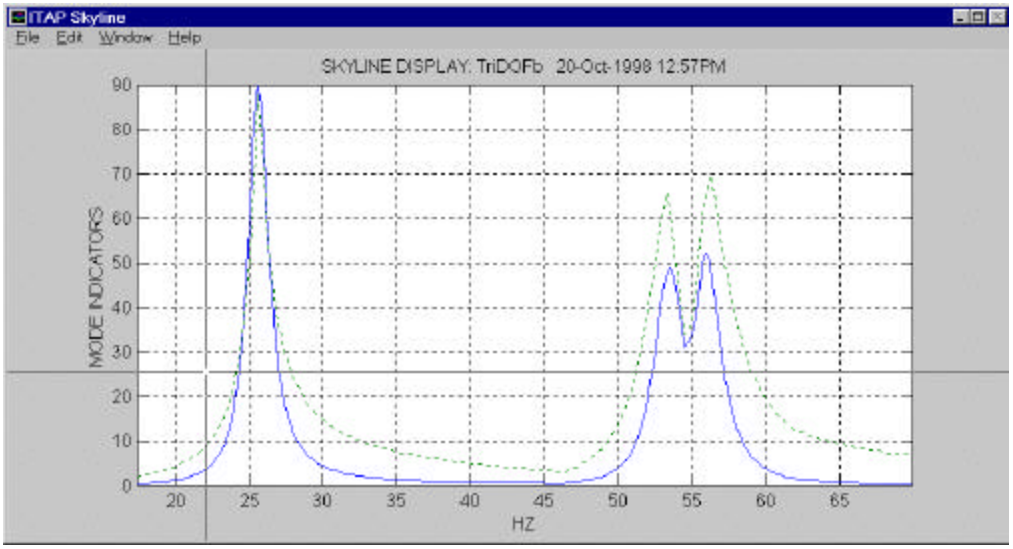
From the MATLAB command window, the user types “**itapt**” to initiate the session. Upon pressing the “**Modal Identification**” button, the name of the FRF data file is requested as shown below (with the user’s response):



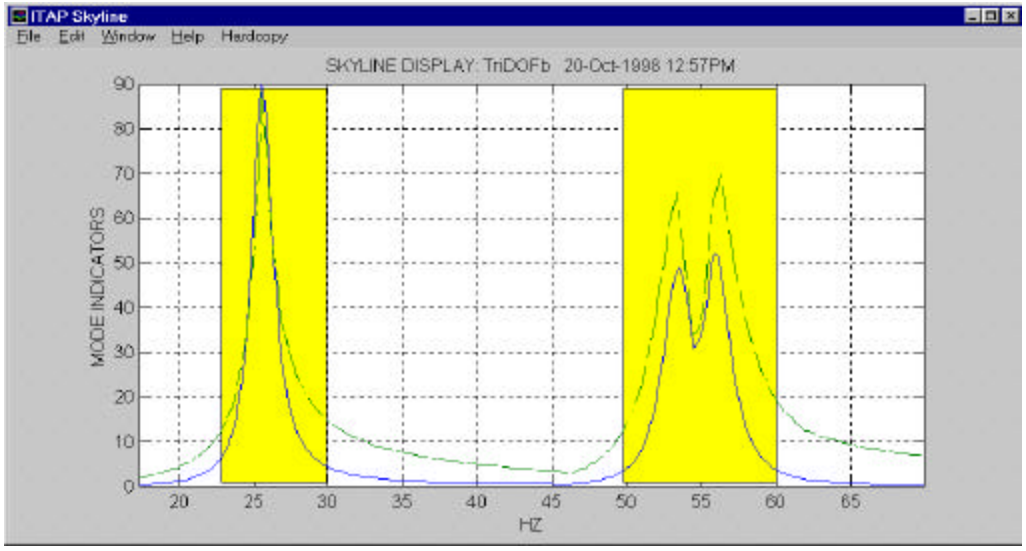
Upon pressing the return key, the following composite FRF display appears with a request to select a frequency band:



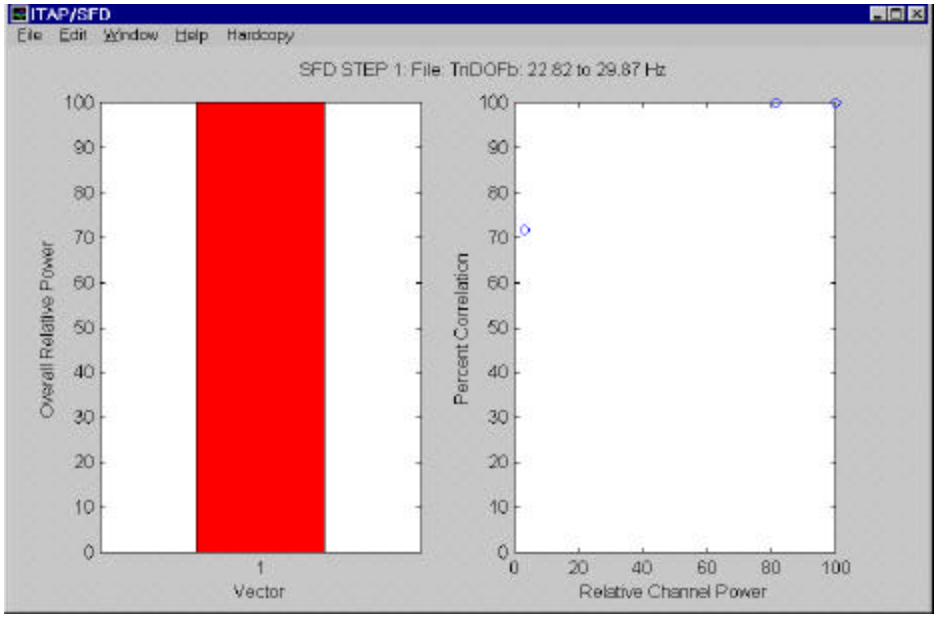
After the lower and upper limits of the frequency band are selected, the following skyline function appears:



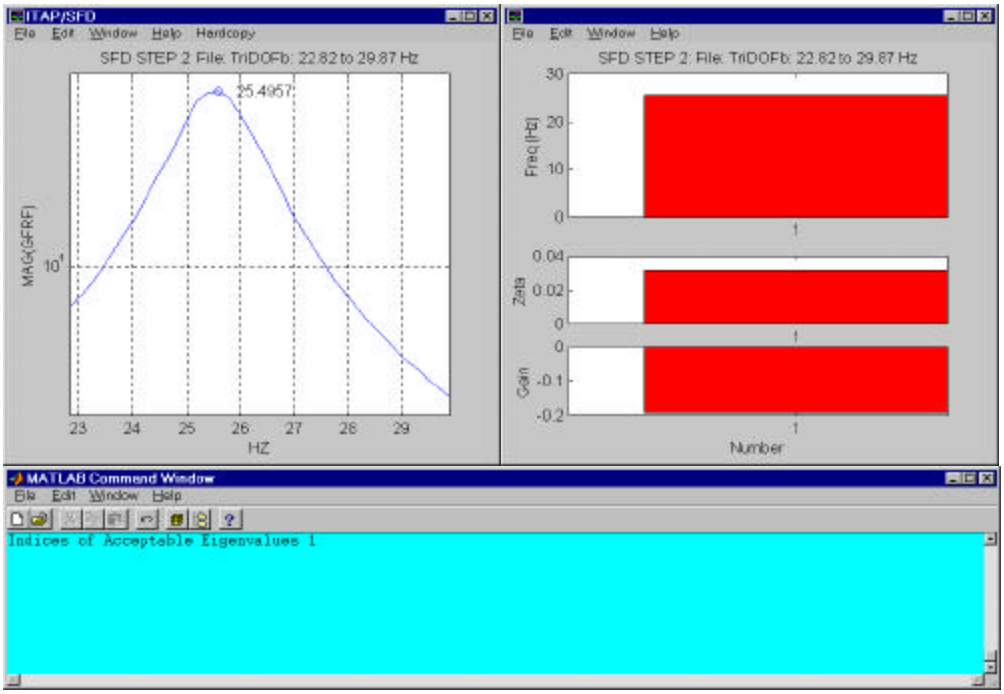
The user next selects individual frequency bands where modal activity is suspected (by clicking lower and upper bounds of each band. When all individual bands have been selected, the user presses the “return” or “enter” key. The following summary plot is then displayed:



After pressing the “return” or “enter” key, individual frequency band modal analyses are performed. The first display which appears (as shown below) indicates that the first selected frequency band (22.82-29.87 Hz) has one dominant trial vector which (with its associated generalized FRF) reproduces the original FRFs to greater that 95% correlation.

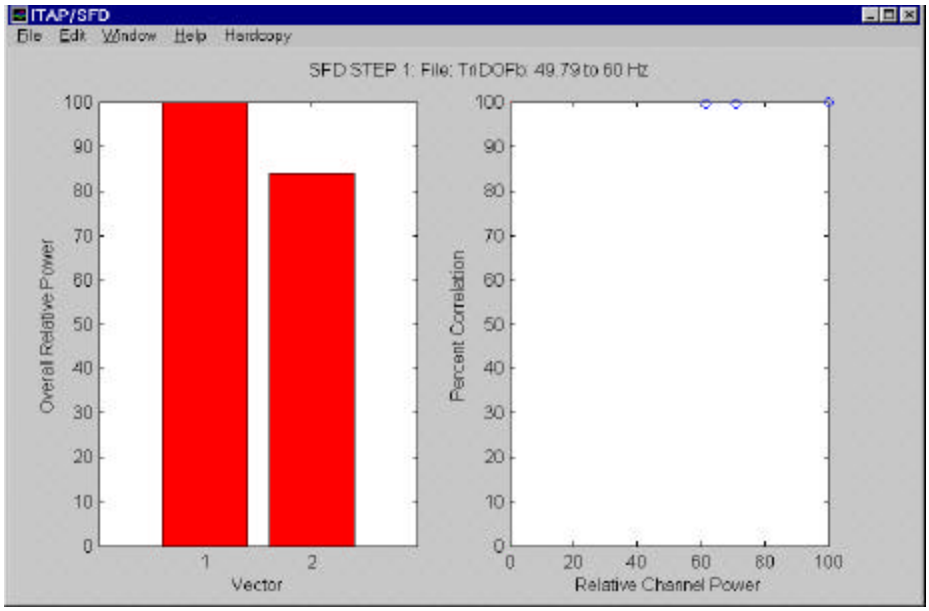


By pressing the “return” or “enter” key again, the generalized FRF is displayed along with the identified candidate modal frequency and critical damping ratio as follows:

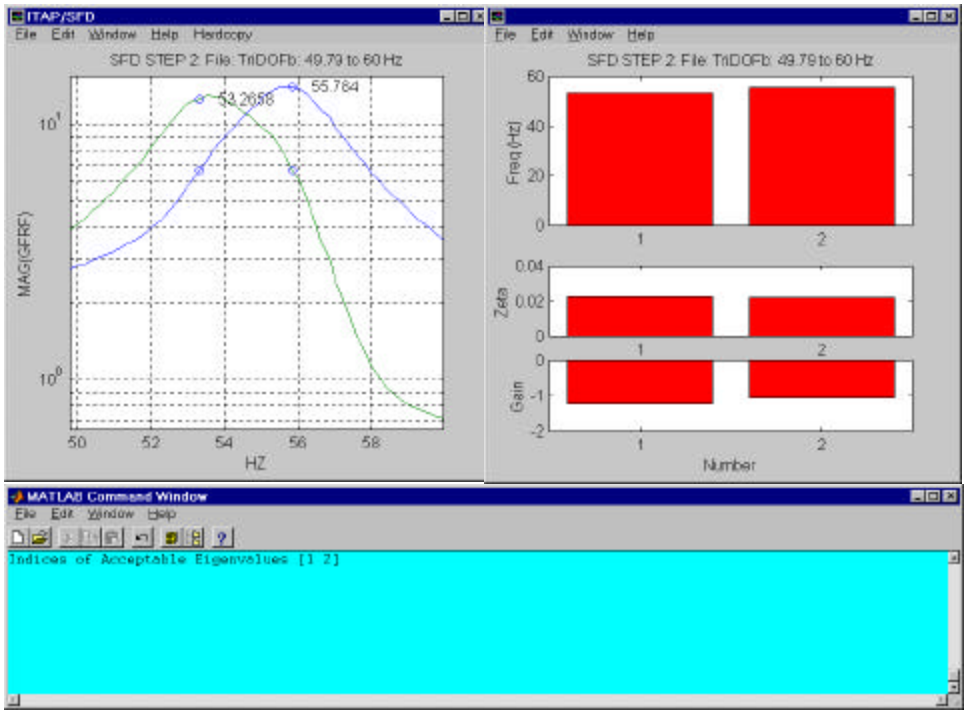


In the command window, the user is prompted to confirm that the single mode is acceptable.

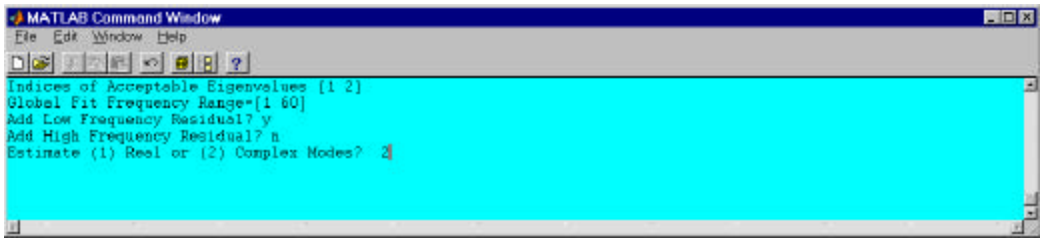
Upon pressing the “return” or “enter” key again, the process is repeated for the next frequency band. The displays associated with this frequency band are show below:



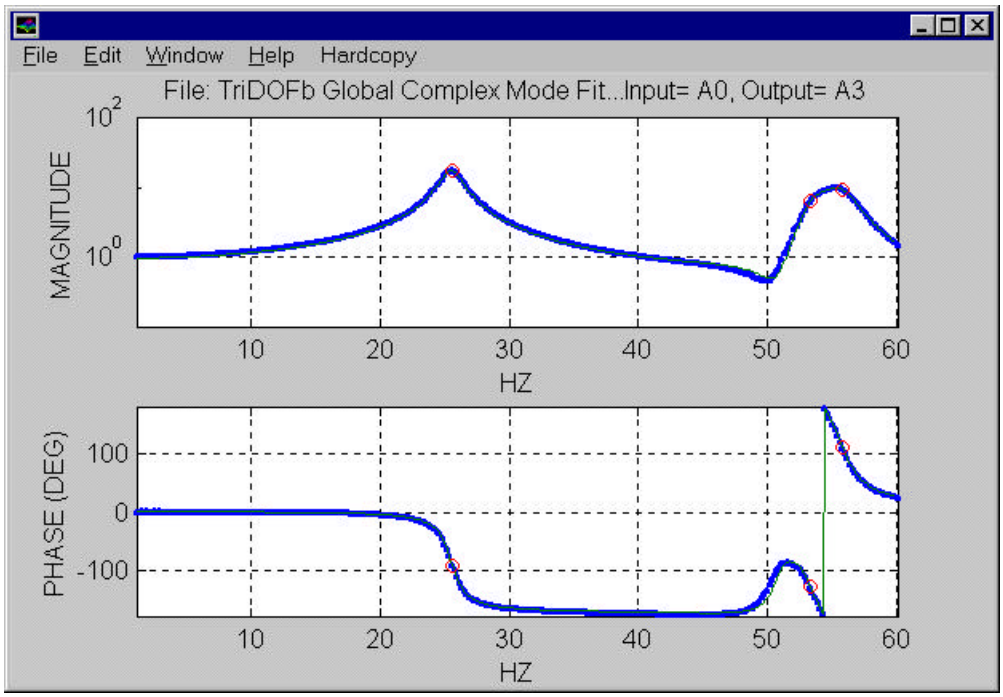
This frequency band has a strong “primary” trial vector a “secondary” trial vector. The next display indicates the two generalized FRFs and two candidate modes as follows:



After this final frequency band has been processed, the following request appears in the MATLAB command window:



Upon providing the requested data the “return” or “enter” key is pressed and FRF comparison displays of the following type appear:



After the final FRF comparison display appears, the test data file is appended with key results of the **modexe** session.

10.0 ITAP-T Function Modules

ITAP-T serves as (a) a utility for preliminary measured data evaluation, spectral and correlation analysis, and modal test data evaluation and (b) a MATLAB-based measured data analysis toolbox. In the former capacity, ITAP-T may be employed as a self-sufficient interactive utility for systematic modal test data analysis. As a toolbox, ITAP-T modules may be assembled in user defined .m files to perform a wide variety of measured data evaluation and signal processing tasks.

Measured test data may be accessed from MATLAB .mat files, from a MATLAB compatible acquisition system or translated from an external source originally written in SDRC Universal File format.

This Chapter provides a description of all user accessible ITAP-T function modules.

10.1 Overview of ITAP-T Function Modules by Category

The ITAP-T toolbox is composed of function modules, which fall into four distinct categories as grouped in the tables provided below:

Test Data Evaluation Templates and I/O Utilities

File Name	Description
frfpall	Grouped Frequency Response Function Plot Options
Frfsum	Frequency Response Array Summation Displays
Itapt	ITAP-T Test Data Analysis Menu
Mimo	MI/MO Spectral Analysis Menu
Mimoplt	MI/MO Spectral Analysis Display
Modexe	Modal Extraction from FRF Array Menu
pairevu	Preliminary Channel Pair Data Analysis
pmenu	Graphics hard copy menu
pre1	Generate Preliminary Data Channel Snapshot
Prelim	Preliminary Measured Data Evaluation
Singles	Single Channel Measured Data Operations Menu
unvread	Read (& write to .mat) universal file time history data

Single Channel Analysis Operations

File Name	Description
bfilt	Band Pass Filter a Time History Data Array
decay	Frequency and damping estimation from a free decay history
envwav	Wave Form Envelope and Harmonic Decomposition
Frfplot4	FRF Plot with key frequency tagging
Probn	Normalized Probability Density Calculation
Probt	Probability Density & Total Probability Calculation
Pwrspcf	Power Spectral Density Calculation with Editing
Pwrspec	Power Spectral Density Calculation
Randec	Random Decrement Calculations
Rdec	Random Decrement Calculation Menu
Rhistory	Generate a Random Signal History
Shock	Normalized Shock & Response Calculations
Srspec	Shock & Response Spectrum Menu

Multiple Channel Analysis Operations

File Name	Description
Frfrsp	Calculate MI/SO response history from FRFs and input histories
Mimogen	MI/MO Spectral Analysis Calculation
Misogen	MI/SO Spectral Analysis Calculation
Ranc	Singular Value Decomposition Operations
Sisogen	SI/SO Spectral Analysis Calculation
Skypsd	PSD Skyline associated with a time history data set

Experimental Modal Analysis

File Name	Description
Modexb	Modal Extraction from FRF Array Calculation
Sfd1	SFD Method Operation: FRF Array Decomposition
Sfd2	SFD Method Operation: Eigenvalue Estimation
Sfd3	SFD Method Operation: Real Mode Estimation
Sfd4	SFD Method Operation: Complex Mode Estimation
Skyline	Calculate and Display FRF Array Skyline

10.2 ITAP-T Function Module Descriptions

This section provides descriptions of user accessible ITAP-T function modules.

Purpose

Band pass filter a time history data array

Syntax

XF=bfilt(X,dt,fa,fb,n);

Description

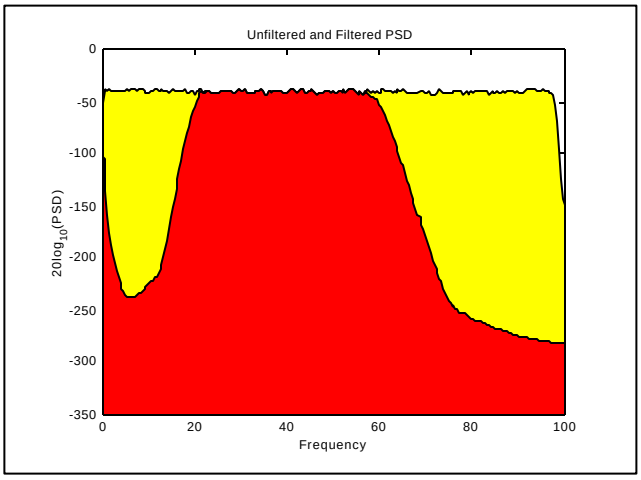
bfilt forms a band pass Butterworth filter of order "**n**" with low and high frequency limits defined by **fa** and **fb**, respectively. If fa=0, the filter is a low pass filter. If fb is equal to the Nyquist frequency, the filter is high pass. The multiiple channel data array, **X** (each column represents a channel), with sampling time, dt, is filtered resulting in the array, **XF**. This function routine utilizes the MATLAB Signal Processing Toolbox routines **butter** and **filtfilt**.

Example

Consider a time history data file with time history data channel (tdata, sampling rate, dt=.005). The following command is invoked to generate a 20-60 Hz band pass filered signal:

tdata=bfilt(tdata,dt,20,60,8);

The original and filtered signal autospectra are shown below.



Purpose

Calculate SDOF System Frequency and Damping from a Free Decay Time History Trace

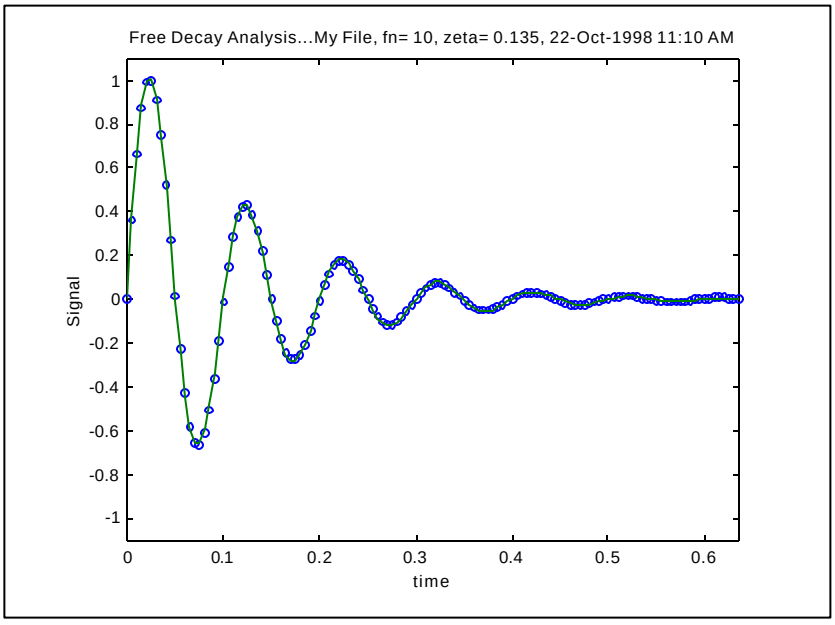
Syntax

[fn,zn,A]=decay(x,t,comment);

Description

decay performs an iterative least square error analysis on a single free decay time history trace, **x,** over the time interval, **t**. This function module estimates damped natural frequency, **fn**, critical damping ratio, **zn**, and amplitude, **A**, associated with the signal. The following display is generated by the command:

[fn,zn,A]=decay(X,t,'My File');



The curve denoted by “o” symbols represents the original data, and the solid curve represents the fitted function:

$$X_{fitted} = A \cdot e^{-(zn \cdot 2\pi \cdot f \cdot t)} \cdot \sin(2\pi \cdot fn \cdot t)$$

Purpose

Hilbert transform based envelope decomposition of a time history signal

Syntax

[env,wav,A] = envwav(x);

Description

envwav decomposes a time history signal into an envelope function (**env**), a "**wave**" function (**wav**), and a scale factor (A). The decomposition process is based the Hilbert transform and the associated Hilbert transfom envelope function, which is accessed from the MATLAB Signal Processing Toolbox). Operations performed by **envwav** are as follows:

1. Calculate the Hilbert transform, $H(t)$ of a signal x(t).

2. Form the (non-scaled) envelope function, defined as

$$E_o(t) = | x(t) + i \cdot H(t)|$$

3. Form the "wave" function as

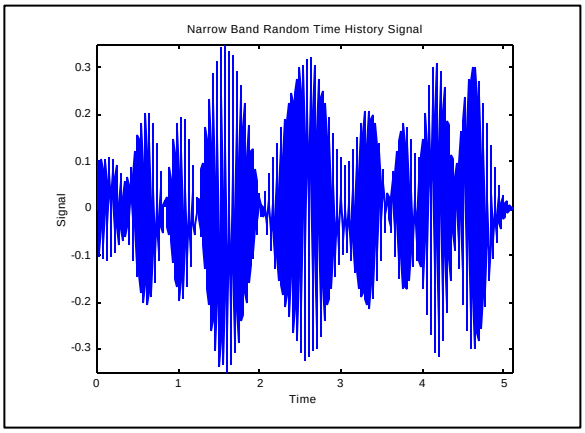
$$W(t) = x(t) / E_o(t)$$

4. Determine the scale factor and normalized envelope functions, which are defined, respectively, as:

$$A = \max(E_o(t)) \ , \ E(t) = E_o(t) / A$$

Example

Consider the time history (X versus t) depicted below:

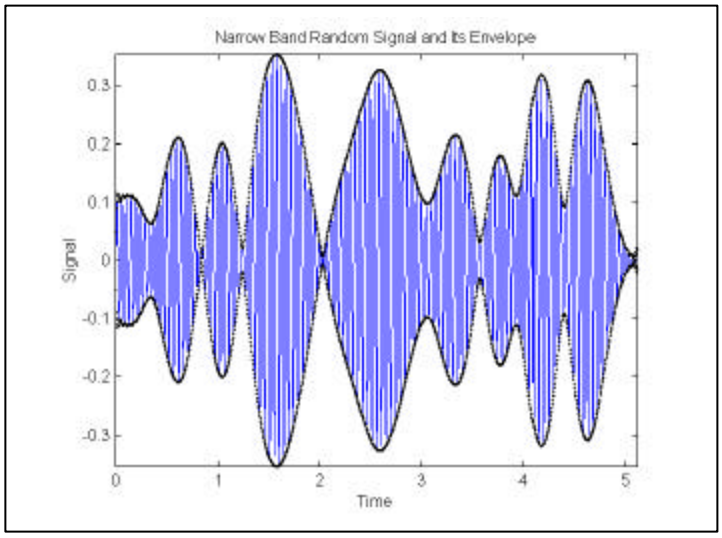


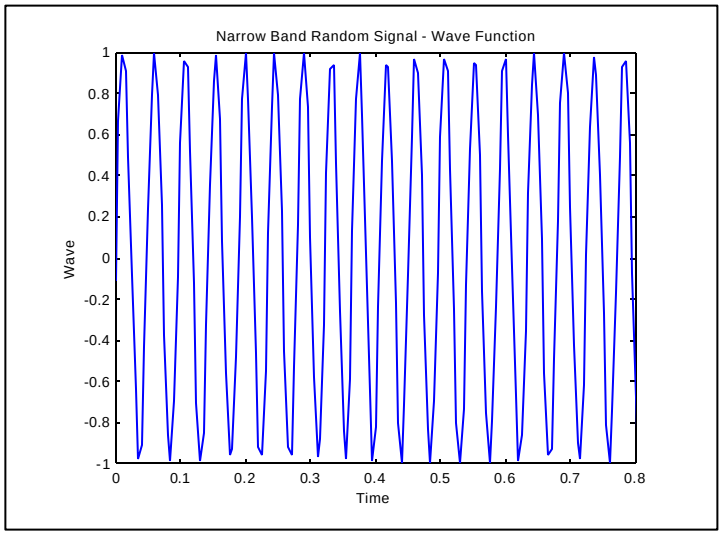
Invoke the following command on the data array “X”:

[env,wav,A]=envwav(X);

Results of this analysis are plotted below in terms of the following three curves:

(a) X versus t
(b) +A · env versus t
(c) - A · env versus t





Purpose

Generate a frequency response function plot display

Syntax

f12=frfpall(frq,H,comment,f12,opt);

Description

frfpall generates a plot display of a complex frequency response function, **H**, defined over the frequency band, **f**, where frequency is defined in terms of Hz units. The user specifies a two parameter plot frequency range, for example, **f12**=[0 100]. In addition, several plot format options (**opt**) may be specified. In particular,

if opt=0 display all plot types & select frequency band
if opt=1 display magnitude & phase
if opt=2 display real & imaginary parts
if opt=3 display real versus imaginary parts

When **opt**=0 is selected, the output function, **f12**, is the selected frequency band limits.

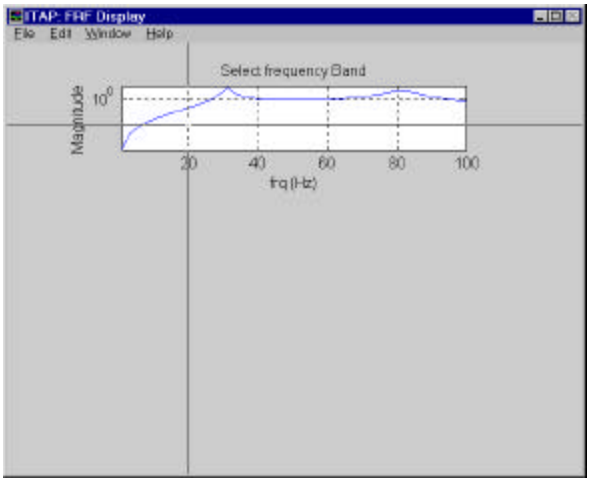
Example

Results of the example analysis from the **fresp** discussion (in Chapter 5) are used here.

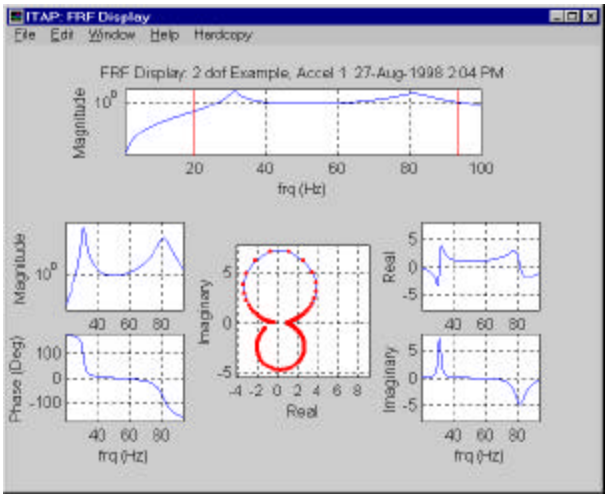
The MATLAB command

f12=frfpall(frq,A(1,:),'2 dof Example, Accel 1',[1 100],0);

is invoked, resulting in the display



The user clicks the cursor twice to define a selected frequency band, and the multiple plot window, shown below Is displayed:

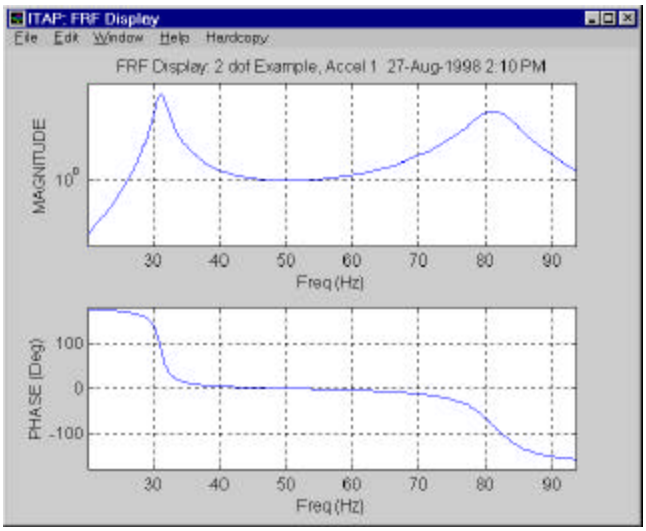


In addition, the selected frequency band, f12 = [20.25 95.50] is output for further processing.

The MATLAB command (using the selected frequency band)

frfpall(frq,A(1,:),'2 dof Example, Accel 1',f12,1);

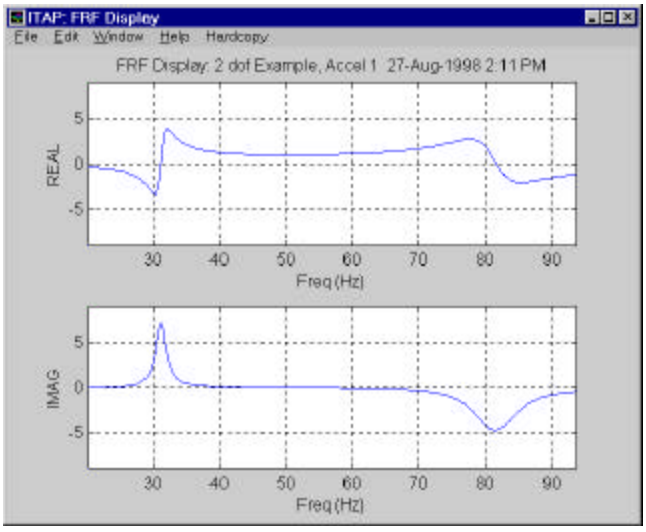
is invoked, resulting in the display



The MATLAB command (using the selected frequency band)

frfpall(frq,A(1,:),'2 dof Example, Accel 1',f12,2);

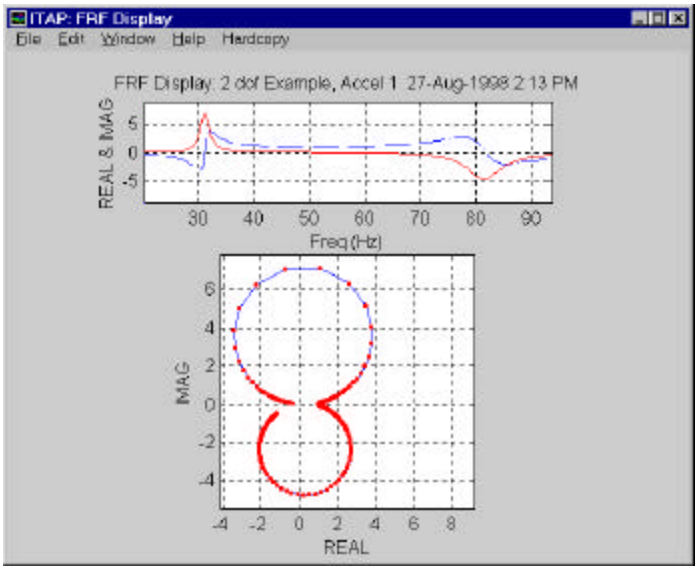
is invoked, resulting in the display



The MATLAB command (using the selected frequency band)

frfpall(frq,A(1,:),'2 dof Example, Accel 1',f12,3);

is invoked, resulting in the display



Purpose

Generate a frequency response function plot display with key frequency tagging

Syntax

frfplot4(f,H,f1,f2,comment);

Description

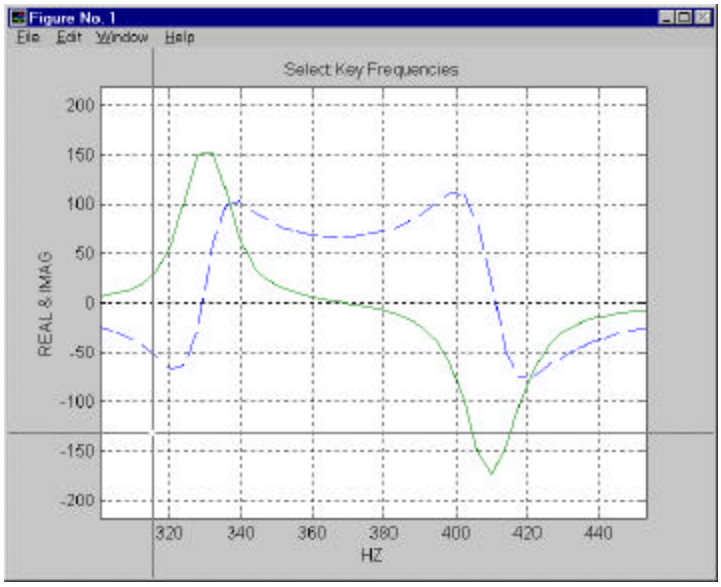
frfplot4 generates a plot display of a complex frequency response function**, H**, defined over the frequency band, **f**, where frequency is defined in terms of Hz units. The user specifies lower and upper bounds (**f1** and **f2**) for a desired plot frequency range. An initial plot of real and imaginary parts of **H** versus **f** is displayed, and the user tags a series of key frequencies. When all key frequencies have been tagged, the user presses the “return” or “enter” key and a final display (real and imaginary parts versus frequency and real versus imaginary parts) with key frequencies is formed. The alphanumeric input “**comment**” is used to form the final plot title.

Example

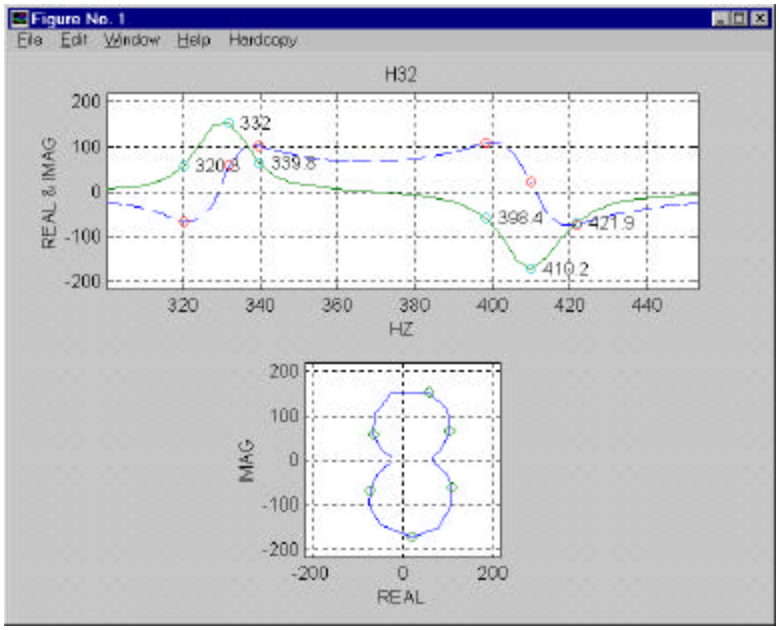
The following command is invoked for viewing of the 32nd FRF in an array, in the 300-450 Hz frequency band:

frfplot4(frq,H(32,:),300,450,'H32')

and the following initial display appears:



After “clicking” all key frequencies, the following final display is generated:



Purpose

Calculate estimated system transient response from MI/SO FRFs and excitation transients.

Syntax

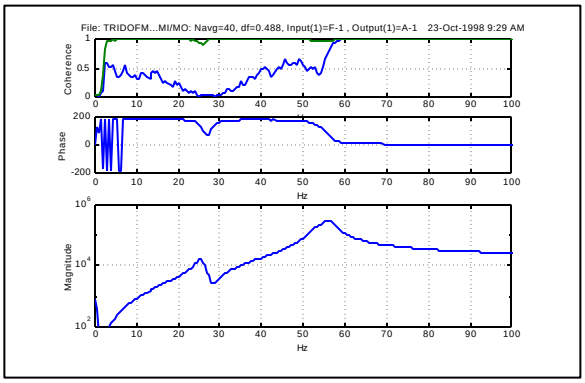
y=frfrsp(H,x);

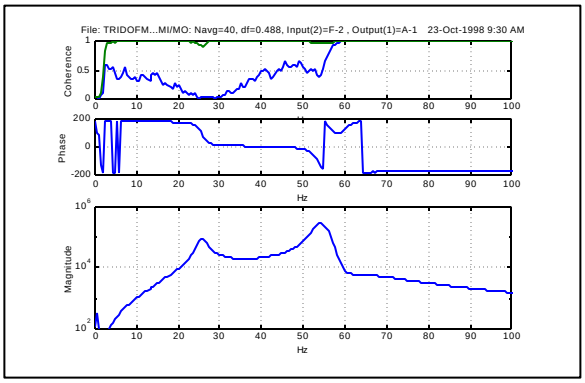
Description

frfrsp calculates MI/SO system response, **y**, due to multiple excitations, **x**. The excitation matrix, **x**, is composed of columns corresponding to the successive excitation histories. The response, **y**, is a column vector. The MI/SO frequency response functions, **H**, are described in the standard ITAP-T format (see Section 7.2.5). A fast Fourier transform (FFT) strategy is used to calculate convolutions within windows of length compatible with the FRFs. Successive windows are overlapped and spliced in a manner which suppresses window-to-window discontinuities.

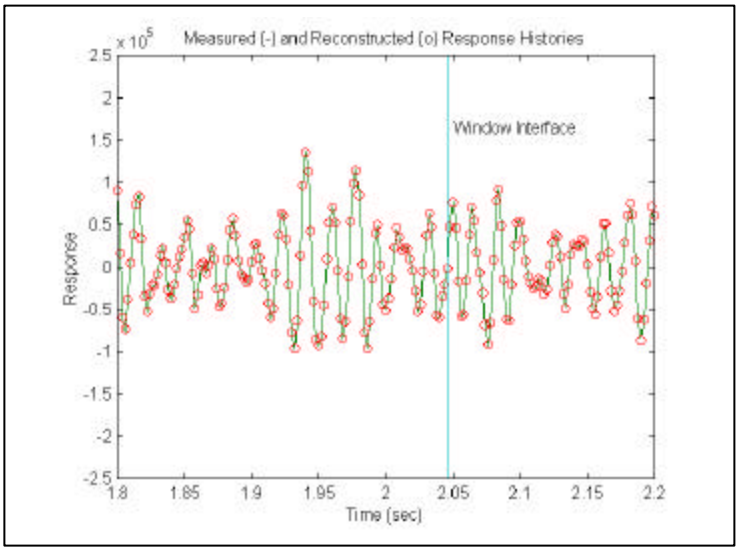
Example

Consider a two input, single output system described in terms of the following two estimated FRFs:

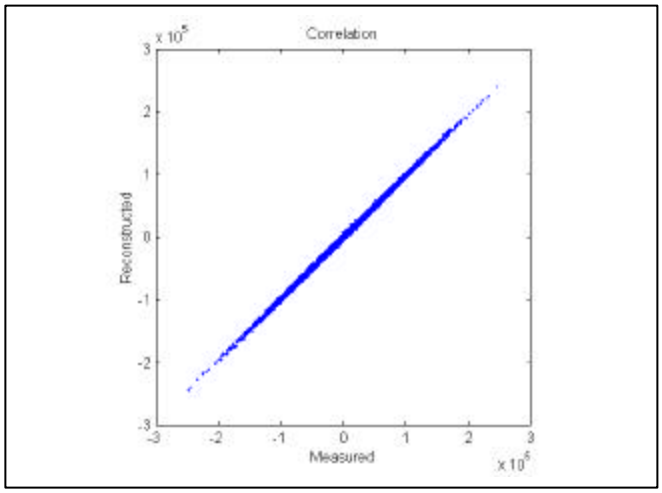




The measured data, **x** and $\mathbf{y_m}$, from which the FRFs have been estimated, are sampled at dt = 0.002. Measured response, **y**, is reconstructed by invoking the **frfrsp** command. Noting that the FRF data matrix, **H**, is described in terms of 513 frequency bins, the individual time window length is 1024. A critical check on quality of reconstructed response, **y**, is made by comparing it original measured response, $\mathbf{y_m}$, within a time zone that spans successive windows, as shown below:



General correlation of measured and reconstructed response is demonstrated by the "**y**" versus "$\mathbf{y_m}$" plot for the complete time range:



Purpose

Display a summary of FRF data

Syntax

f12=frfsum(frq,H,namin,namout,comment);

Description

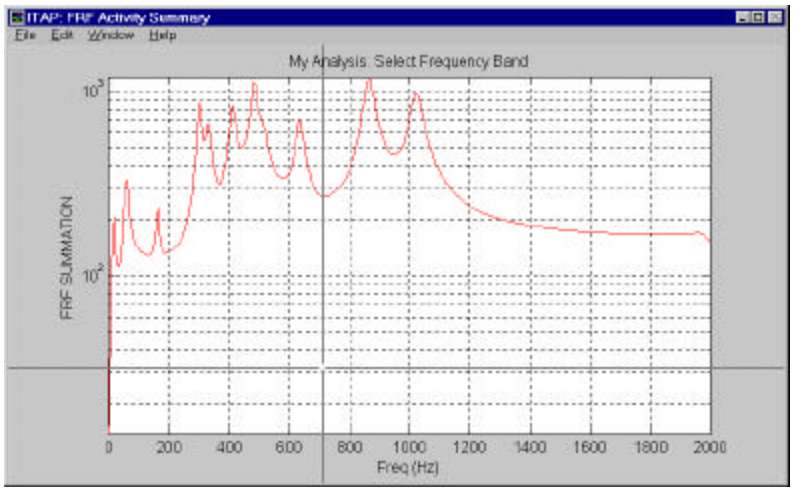
frfsum generates a summary display for a SI/MO or MI/MO FRF matrix, **H**, over the discrete frequency band, **frq**. It is assumed that the FRF data is associated with a standard ITAP-T data file where **namin** and **namout** are alphanumeric names of input and output channels. The user also specifies an alphanumeric descriptive **comment**, which is incorporated in the display title.

Examples

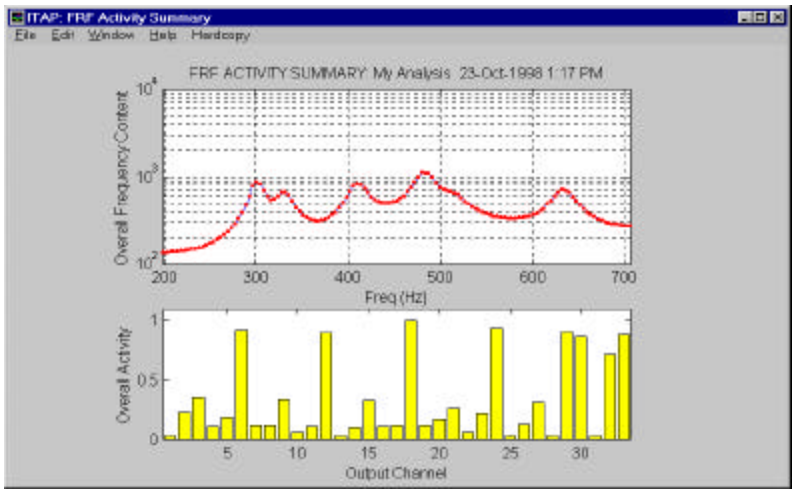
As a first example, consider the SI/MO data file, W-Test, which has been accessed. By typing the command:

f12=frfsum(frq,H,namin,namout,'My Analysis');

the following initial plot appears with a request to specify a frequency band of interest:

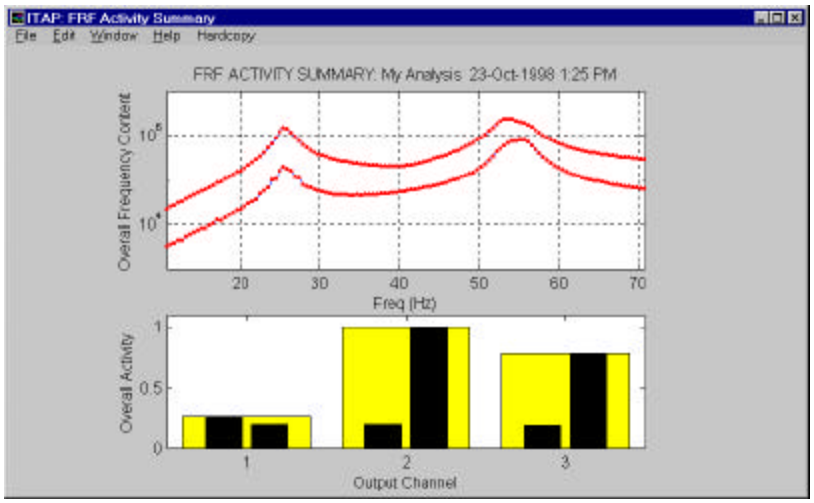


After specifying the requested data (by clicking the cursor), the following figure is displayed:



The upper plot is a display of the sum of all FRF magnitudes versus frequency and the lower plot is the peak magnitude of each FRF over the selected frequency band, **f12**, for each output channel.

When the FRF data is associated with more than a single input, a display of the following type is obtained (2 input, 3 output data set):



The upper plot is a display of the sum of all FRF magnitudes versus frequency. The lower magnitude curve represents a summation of FRFs associated with input 1 and the higher magnitude curve represents the summation of FRFs associated with inputs 1 and 2. The lower plot displays the peak magnitude of each FRF over the selected frequency

band, **f12**, for each output channel. The individual “black” bars indicate the level of contributions associated with the two inputs.

Purpose

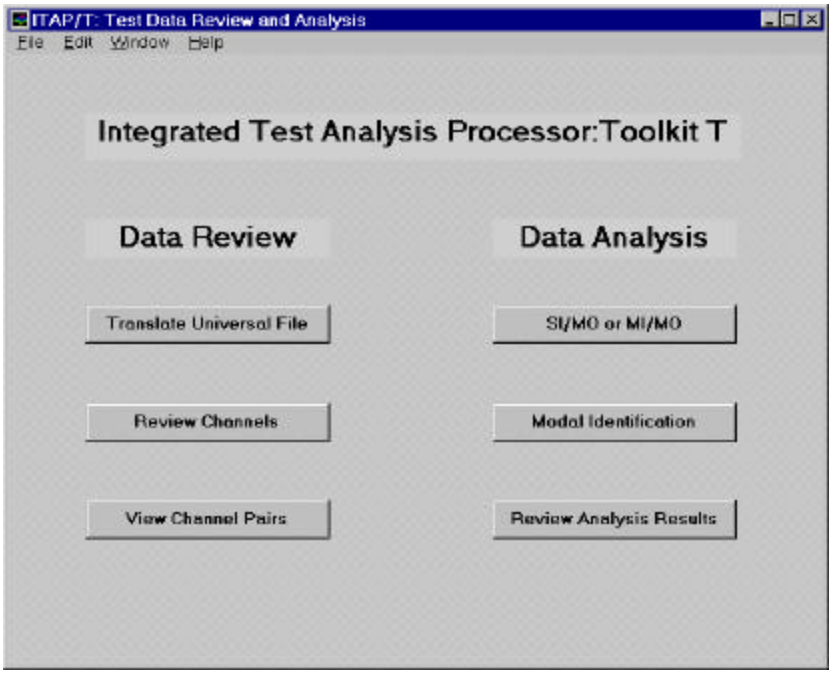
Initiate an ITAP-T session

Syntax

itapt

Description

itapt enables the main ITAP-T display for initiating a measured data analysis or test data review session. The user selects from one of the options on the display shown below:



Example

See Chapters 7-9

Purpose

Initiate an MI/MO Analysis Session

Syntax

mimo

Description

The command **"mimo"** initiates an ITAP-T MI/MO spectral and correlation analysis session. The user is prompted, in the MATLAB command window, to supply the following data:

1. Name of Time History Data File
2. FFT Analysis Bin Length
3. FFT Analysis Overlap Processing Bin Length
4. Input Channel Numbers
5. Output Channel Numbers

When all of these data entries are submitted, MI/MO computations are performed by the ITAP-T function module **mimogen** and results are displayed with the ITAP-T function module **mimoplt**.

Example

See Chapter 8

Purpose

Perform SI/MO or MI/MO Computations

Syntax

[frq,HYX,COH]=mimogen(X,Y,dt,nwin,novl);

Description

This ITAP-T function module performs SI/MO and MI/MO computations for an arbitrary number of input time history channels (columns of the matrix **X**) and an arbitrary number of output time history channels (columns of the matrix **Y**), where the data is sampled over uniform time increments, **dt**. The FFT window length, **nwin**, and overlap processing index, **novl**, are specified as input parameters. Output arrays resulting from the SI/MO or MI/MO analysis consist of a frequency vector, **frq**, FRF array, **HYX**, and ordinary or cumulative coherence array, **COH**. Details on the format of **mimogen** results are provided in Subsection 7.2.4.

Example

See Chapter 8

Reference

See Chapter 8

Purpose

Display results of SI/MO or MI/MO computations

Syntax

mimoplt;

Description

The command **"mimoplt"** interactively displays ITAP-T SI/MO or MI/MO analysis results. The user is prompted, in the MATLAB command window, to supply the following data (shown with example responses):

1. Frequency Range: [0 100]
2. Minimum Acceptable Coherence: .8
3. number of decades for frf display: 4
4. Frequency Scale (linear or log): linear

When all these data are supplied, the FRF and (ordinary or cumulative) coherence results are displayed successively. The user views successive results by pressing the "return" or "enter" key.

Example

See Chapter 8

Purpose

Perform MI/SO Computations

Syntax

[frq,HYX,COH,GYY,GXX,GYX] =
misogen(X,Y,dt,nwin,novl,iout);

Description

This ITAP-T function module performs MI/SO computations for an arbitrary number of input time history channels (columns of the matrix, **X**) and one specific output time history channel (columns vector, **Y**), where the data is sampled over uniform time increments, **dt**. The FFT window length, **nwin**, and overlap processing index, **novl**, are specified as input parameters. The integer parameter, **iout**, is used by a “waitbar” display to inform the user of computation progress during MI/MO analyses. Output arrays resulting from the MI/SO analysis consist of a frequency vector, **frq**, FRF array, **HYX**, and ordinary or cumulative coherence array, **COH**. Additional output arrays consist of the output autospectrum, **GYY**, input autospectra, **GXX**, and output/input cross-spectra, **GYX**. This function module should only be used if **GYY**, **GXX**, and/or **GYX** are desired. Otherwise, use **mimogen**.

Reference

See Chapter 6

Purpose

Estimate modal frequency and damping parameters from FRF data using the SFD method

Syntax

[FREQ,ZETA]=modexb(H,frq,f1,f2,FREQ,ZETA,namfil);

Description

The ITAP-T function module **modexb** estimates modal frequency and damping parameters from a FRF array (**H** defined in the frequency array **frq** contained in the ITAP-T data file **namfil**) with the Simultaneous Frequency Domain (SFD) method. Calculations are carried out in the frequency band defined by lower and upper bound frequencies, **f1** and **f2**, respectively. Identified modal frequencies and critical damping ratios are stored in the respective arrays **FREQ** and **ZETA**. It is implicitly assumed that the arrays, **FREQ** and **ZETA**, already exist; **modexb** appends these arrays with modal parameters identified in the specified frequency band.

Core **modexb** computations are performed by the ITAP-T function modules **SFD1** and **SFD2**. Modexb represents a major computational block within the menu driven ITAP-T function module **modexe**.

Example

See Chapter 9

Reference

See Chapter 6

Purpose

Estimate modal frequency and damping parameters and real or complex modal vectors from FRF data using the SFD method.

Syntax

modexe;

Description

The ITAP-T function module **modexe** performs a complete SFD analysis. All key operations are invoked in a menu based dialog session. **modexe** has general capabilities to handle single and multiple input FRF data arrays, applied force or base acceleration excitations, and real or complex modal estimation. Typical sessions, with illustrative examples, are provided in Chapter 9.

This routine employs the ITAP-T routine **skyline** to assist in identifying frequency bands containing apparent modal activity. Modal frequency and damping parameters are estimated with the function module **modexb**. Real mode vectors are calculated with the function module **SFD3**. Complex mode vectors are calculated with the function module **SFD4**.

The following data arrays are appended to the ITAP-T data file from which modal parameters are extracted:

Array	Description
FREQ	Modal Frequencies
ZETA	Modal Critical Damping Ratios
PHI	Modal Matrix
FRFIT	Fitted FRF array from modes
frqx	Fitted FRF frequency array
PHIRAW	Raw Modal Matrix Data

Example

See Chapter 9

Reference

See Section 6.8

Purpose

Generate a push button switch for a figure which when pushed generates a hard copy.

Syntax

pmenu

Description

The command, **pmenu**, embedded in a .m file or invoked by keystroke command generates the push button with label "**hardcopy**". The label remains on the menu bar until the figure is cleared.

Purpose

Generate a single channel time history snapshot

Syntax

pre1(X,dt,NFFT,namfil ,namch);

Description

A descriptive display of a single channel tine history array, **X**, sampled over uniform time increments **dt**, is generated. Four plots appear in the display, namely, (1) a time-frequency spectrogram, (2) time history trace, (3) autospectrum, (4) probability density. A window length, **NFFT**, is specified for spectrogram and autospectrum calculations. Alphanumeric names "**namfil**" (the data file name) and "**namch**" (the channel name) are specified for display labeling. This routine is used within the ITAP-T function module **prelim**.

Example

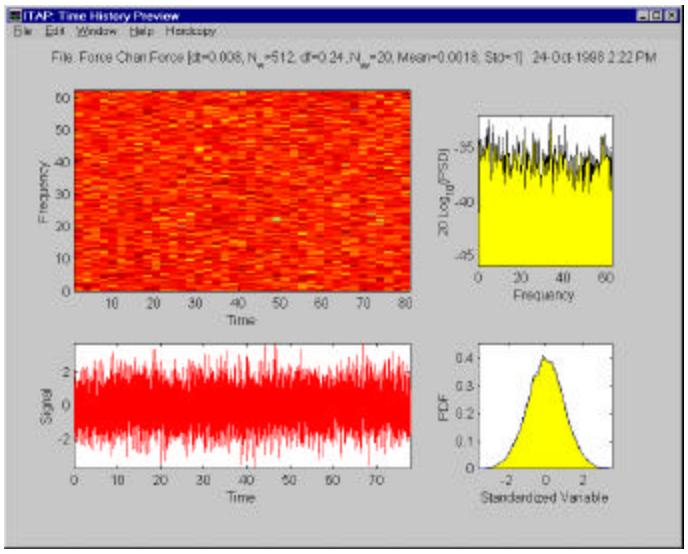
Consider the data file named "Force" containing:

Name	Size	Bytes	Class
Force	1x10240	81920	double array
dt	1x1	8	double array

In the MATLAB command window, invoke the command:

pre1(Force,dt,512,'Force','Force');

and the following display is generated:



Purpose

Preliminary measured data evaluation and analysis

Syntax

prelim;

Description

This ITAP-T function routine invokes a general preliminary measured data evaluation and analysis session. It is also invoked from the "**review channels**" option in the ITAP-T main menu screen (see **itapt**). A typical **prelim** session is described in Section 7.3.

Example

See Section 7.3

Reference

Section 6.3

Purpose

Calculate and display normalized probability density

Syntax

[bz,pz]=probn(X,dt,n,namfil,namch);

Description

This ITAP-T function routine calculates a normalized probability density for a single channel of time history data, **X**, sampled over uniform time increments, **dt**. The user specifies a number "**n**" of amplitude bins (a generally recommended value is 30) for the calculation. Alphanumeric names "**namfil**" (the data file name) and "**namch**" (the channel name) are specified for display labeling. The actual normalized probability density of the variable as well as the ideal Gaussian normalized probability density are displayed.

The output arrays, **bz** and **pz** represent the amplitude bin and probability density data, respectively.

This function routine is an option within the ITAP-T function routine **singles**.

Example

A data file named SineData contains the following arrays:

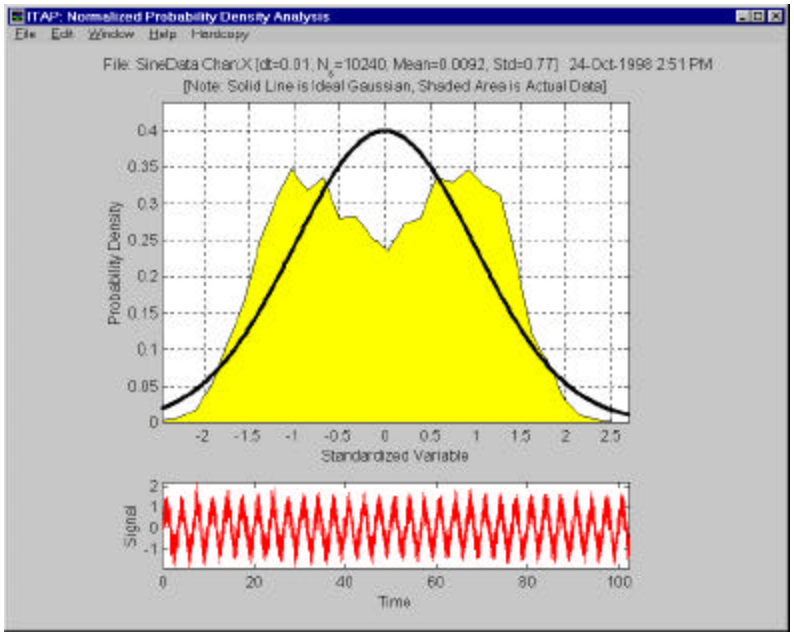
Name	Size	Bytes	Class
X	1x10240	81920	double array
dt	1x1	8	double array

The time history, **X**, consists of a sine wave with broad band random background noise.

In the MATLAB command window, invoke the command:

[bz,pz]=probn(X,dt,30,'SineData','X');

and the following display is generated



The “rabbit ear” shape of the normalized probability density indicates that a strong sinusoidal component is present.

Reference

Subsection 6.2.3

Purpose

Calculate and display normalized probability density and total probability

Syntax

[bx,px,pxt]=probt(tdata,dt,n,namfil,namch);

Description

This ITAP-T function routine calculates a normalized probability density and total probability for a single channel of time history data, **X**, sampled over uniform time increments, **dt**. The user specifies a number "**n**" of amplitude bins (a generally recommended value is 30) for the calculation. Alphanumeric names "**namfil**" (the data file name) and "**namch**" (the channel name) are specified for display labeling. The normalized probability density and total probability are displayed.

The output arrays, **bx, px** and **pxt** represent the amplitude bin, probability density, and total probability data, respectively.

This function routine is an option within the ITAP-T function routine **singles**.

Example

A data file named SineData contains the following arrays:

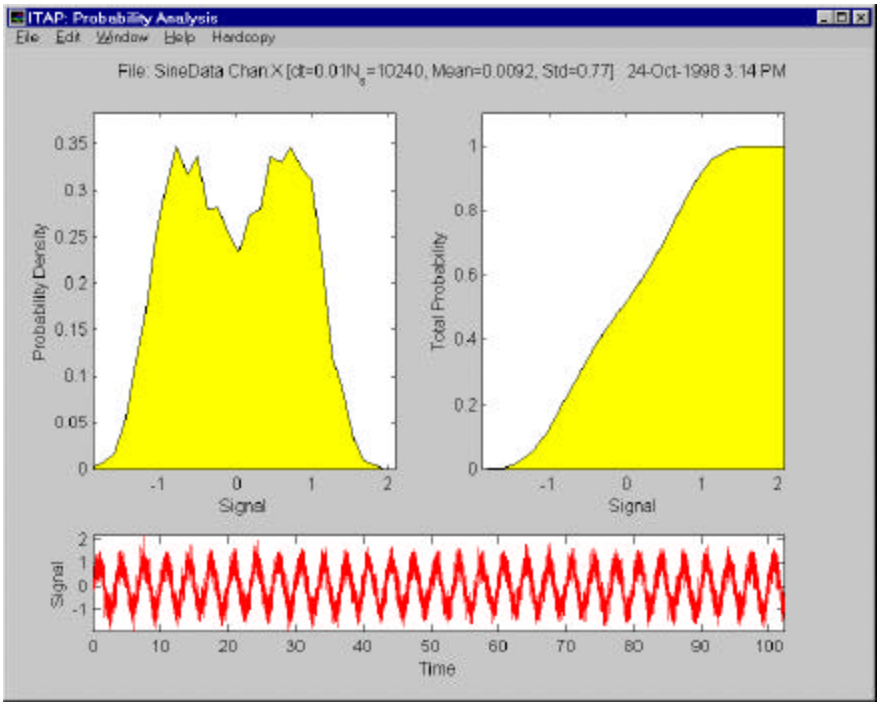
Name	Size	Bytes	Class
X	1x10240	81920	double array
dt	1x1	8	double array

The time history, **X**, consists of a sine wave with broad band random background noise.

In the MATLAB command window, invoke the command:

[bx,px, pxt]=probt(X,dt,30,'SineData','X');

and the following display is generated



Reference Subsection 6.2.4

Purpose

Calculate and display the autospectrum or power spectral density of a single channel time history array

Syntax

[freq,gxx,stx]=pwrspec(tdata,dt,NFFT,namfil,namch);

Description

This ITAP-T function routine calculates the autospectrum or power spectral density of a single channel time history array **X**, sampled over uniform time increments, **dt**. The user specifies a desired window length "**NFFT**" for the spectral calculation. Alphanumeric names "**namfil**" (the data file name) and "**namch**" (the channel name) are specified for display labeling.

The output arrays, **freq**, **gxx** and **stx** represent the frequency array, autospectrum, and tstandard deviation, respectively.

This function routine is an option within the ITAP-T function routine **singles**.

Example

A data file named Sine10 contains the following arrays:

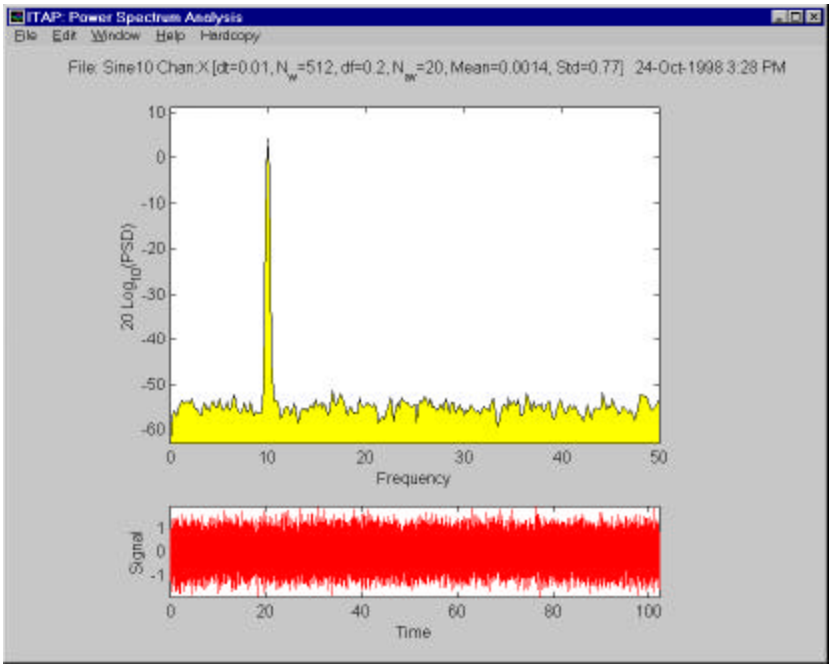
Name	Size	Bytes	Class
X	1x10240	81920	double array
dt	1x1	8	double array

The time history, **X**, consists of a 10 Hz sine wave with broad band random background noise.

In the MATLAB command window, invoke the command:

[freq,gxx,stx]=pwrspec(X,dt,512,'Sine10','X');

and the following display is generated



Note that the 10 Hz component shows up as a “spike” in the autospectrum.

Reference

Section 6.2.5

Purpose

Singular value decomposition analysis of a rectangular matrix

Syntax

[V,Z,lam,lamn,xp,yp,cor]=ranc(X,cutoff);

Description

This ITAP-T function routine performs a singular value decomposition analysis on a rectangular matrix, **X**, and calculates diagnostic data used to evaluate the operation. **ranc** is employed within the function routine **SFD1**. It has been written as a specific function routine, since it has broad application in data analysis.

Singular value decomposition is performed by calculation of the eigenvalues and eigenvectors of the smaller square matrix formed by $\mathbf{XX^T}$ or $\mathbf{X^TX}$. The eigenvalue column array, **lam**, are sorted in descending order (highest value first). A normalized eigenvalue array, **lamn**, is formed by dividing all **lam** components by the first eigenvalue. A truncated set of eigenvalues (and associated eigenvectors) is retained by including all terms satisfying the condition:

$$\mathbf{lamn}(i) \geq \mathbf{cutoff.}$$

Ultimately, the decomposed rectangular matrix is described by the product

$$\mathbf{X = V \times Z + R}$$

where **V** is a set of orthonormal dominant vectors, **Z** is a set of generalized coordinates and **R** is a residual error matrix.

The overall “power” level of each row of **Z** is equal to the eigenvalue **lam(i)** corresponding to the row index. The approximate reconstructed rectangular matrix is denoted as

$$\mathbf{Y = V \times Z}$$

The quality level to which each individual row of **X** is reconstructed by **Y** is expressed as

cor = yp/xp

where

yp = diagonal($Y \times Y^T$) , xp = diagonal($X \times X^T$).

Purpose

Calculate the random decrement array associated with a single or multiple channel time history array

Syntax

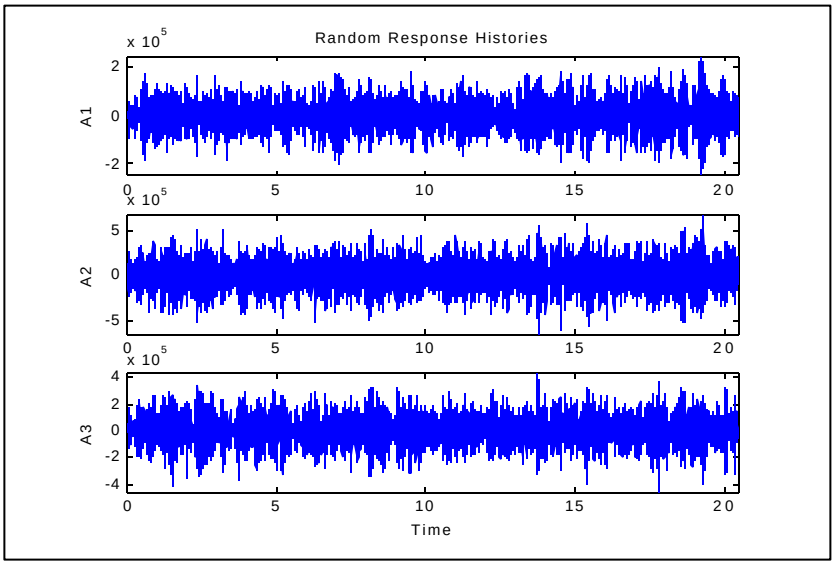
[RY,tr,Nc] = randec(Y,dt,Nw,Nr,Nd,Ns);

Description

This ITAP-T function routine calculates the random decrement signature for a single or multiple channel time history array (**Y**) sampled over uniform time increments, **dt**. The columns of **Y** represent individual time history channels. The user specifies a desired window length (**Nw)** and triggering reference channel index (**Nr**) for the random decrement array calculation. If the input parameter **Nd** is assigned a non-zero value, linear trends are removed from all data channels. The random decrement array **RY** is defined over a time window, **tr**, which consists of **Nw** time bins. The input parameter **Ns** defines an exponential smoothing window, $e^{-Ns \times tr}$, which artificially "damps" the raw random decrement columns. The averaging process used to define the (reference channel) random decrement signature employs a "positive zero crossing" time trigger. Time bin numbers at which positive zero crossings occur are stored in the output array **Nc**.

Example

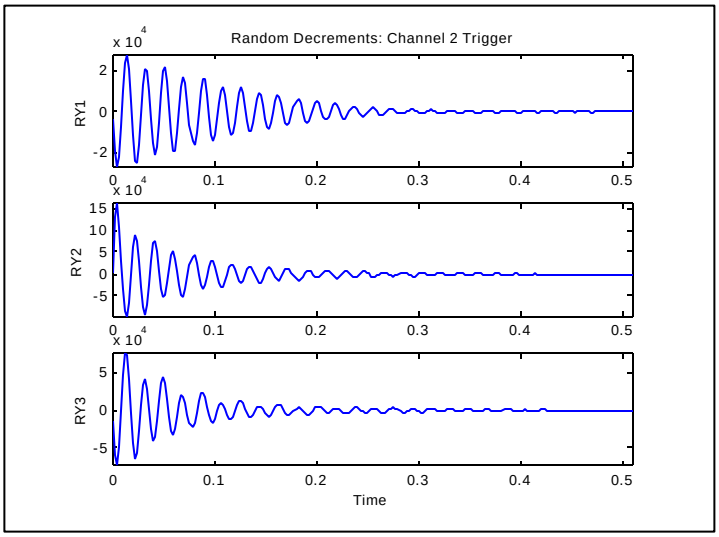
Consider a three channel (10240 X 3) random response array, **Y**, which is sampled at uniform time increments, **dt**=.002. Time histories of the channels are displayed below:



In the MATLAB command window, invoke the command:

[RY,tr,Nc]=randec(Y,dt,256,2,1,3);

The three channels of the random decrement signature array are displayed below:



Reference

See Subsection 6.2.7

Purpose

Calculate and display the random decrement associated with a single time history record and its Fourier transform

Syntax

RY,tr,RYF,F]=rdec(Y,dt,Nw,namfil,namch);

Description

The ITAP-T function routine **rdec** calculates the random decrement signature (**RY**) of a single channel time history array **Y**, sampled over uniform time increments, **dt**. The user specifies a desired window length (**Nw**) for the random decrement calculation, which sets the time band, **tr**, of the random decrement signature. Alphanumeric names "**namfil**" (the data file name) and "**namch**" (the channel name) are specified for display labeling.

The output arrays, **RY**, **tr**, **RYF** and **F** represent the random decrement signature, its time band, the Fourier transform of **RY**, and the frequency band of **RYF**, respectively.

rdec performs random decrement calculations by calling the ITAP-T function routine **randec** with Nd=1 and Ns=3 (see the description of randec).

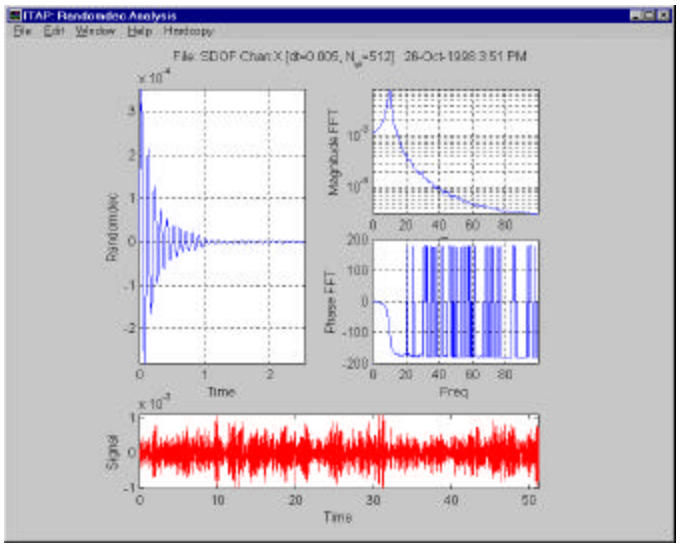
This function routine is an option within the ITAP-T function routine **singles**.

Example

Random decrement analysis of channel 2 of a time history data file (named SDOF) is performed by invoking the following statement in the MATLAB command window:

[RY,tr,RYF,F]=rdec(tdata(:,2),dt,512,'SDOF','X');

The following display is generated:



Purpose

Generate a random time history whose autospectrum closely follows a specified distribution

Syntax

[X,t]=rhistory(frq,gxx,dt,nt,nfilt);

Description

The ITAP-T function routine **rhistory** generates a random time history, **X**, over the time array, **t** (uniformly sampled at **dt** increment, of length **nt**), which closely follows the specified autospectrum, **gxx**, over the frequency array, **frq**. The time history, X, is defined by passing a white noise signal through a digital filter. The filter is designed, utilizing the MATLAB Signal Processing Toolbox "**yulewalk**" function module of order **nfilt**, designed to produce imitate the specified autospectrum.

Example

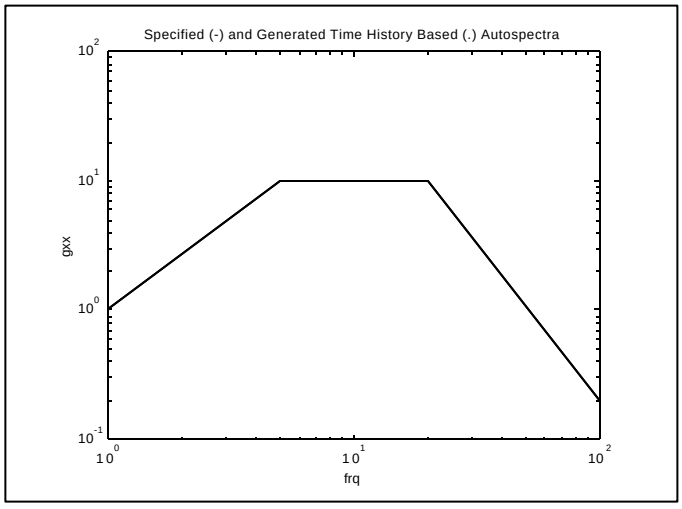
Consider the following specified autospectrum:

frq=[1 5 20 100]; gxx=[1 10 10 0.2];

From the MATLAB command window, invoke the following command to generate the time history, **X**:

X=rhistory(frq,gxx,dt,20480,48);

The autospectrum of the generated time history, compared to the specified autospectrum is illustrated below:



Purpose

Perform SFD Step 1 calculations to identify dominant vectors and generalized FRFs from a FRF data array

Syntax

[V,GFRF,frqx,lam,lamn,xp,yp,cor]
=sfd1(H,frq,f1,f2,tol,comment);

Description

The ITAP-T function module **sfd1** (which is one of the function routines called by **modexb**) calculates dominant vectors and generalized FRFs from a FRF data array. Key computations are performed within **sfd1** by the ITAP-T function module **ranc**. The theoretical foundation for **sfd1** calculations is described in Subsection 6.8.5.

This function module offers the advanced ITAP-T user maximum flexibility for performing SFD computations. The input and output variables are defined in the table below:

Variable	Description
H	FRF Data Array
Frq	FRF Data frequency array
f1, f2	Lower and Upper Bounds of Analysis Frequency Band
Tol	Vector selection Tolerance Parameter
Comment	Descriptive Comment for Display Title
V	Trial Vector Array
GFRF	Generalized FRF Array
frqx	Generalized FRF frequency Array
lam	Dominant Eigenvalues from “ranc” Analysis
lamn	Normalized Dominant Eigenvalues
xp	Original FRF Array “Power” Values
yp	Reconstructed FRF Array “Power” Values
cor	FRF Reconstruction Correlation Coefficients

Reference

See Subsection 6.8.5

Purpose

Perform SFD Step 2 calculations to identify natural frequency and modal damping from generalized FRFs .

Syntax

[FREQ,ZETA,MGAIN]=sfd2(GFRF,frqx,comment);

Description

The ITAP-T function module **sfd2** (which is one of the function routines called by **modexb**) calculates natural frequency and modal damping from generalized FRF data. The theoretical foundation for **sfd2** calculations is described in Subsection 6.8.6.

This function module offers the advanced ITAP-T user maximum flexibility in performing SFD computations. The input and output variables are defined in the table below:

Variable	Description
GFRF	Generalized FRF Array
frqx	Generalized FRF frequency array
comment	Descriptive Comment for Display Title
FREQ	Array of Modal Frequencies
ZETA	Array of Modal Damping Parameters
MGAIN	Array of Complex Modal Gains

Reference

See Subsection 6.8.6

Purpose

Perform SFD Step 3 calculations to estimate real modal vectors and calculate reconstructed FRFs from identified modal parameters.

Syntax

[PHI,COR,FRFIT,frqx]
=sfd3(H,frq,f1,f2,FREQ,ZETA,comment)

Description

The ITAP-T function module **sfd3** (which is one of the function routines called by **modexe**) calculates real modal vectors and reconstructed FRFs from identified modal parameters.

The theoretical foundation for **sfd3** calculations is described in Subsection 6.8.7.

This function module offers the advanced ITAP-T user maximum flexibility in performing SFD computations. The input and output variables are defined in the table below:

Variable	Description
H	FRF Data Array
frq	FRF Data frequency array
f1, f2	Lower and Upper Bounds of Analysis Frequency Band
FREQ	Array of Modal Frequencies
ZETA	Array of Modal Damping Parameters
comment	Descriptive Comment for Display Title
PHI	Real Modal Matrix
COR	Correlation Coefficients for Modal Fitting of FRF Array
FRFIT	Reconstructed FRF Array
frqx	Reconstructed FRF frequency array

Reference

See Section 6.8.7

Purpose

Perform SFD Step 3 calculations to estimate complex modal vectors and calculate reconstructed FRFs from identified modal parameters.

Syntax

[PHI,COR,FRFIT,frqx]
=sfd4(H,frq,f1,f2,FREQ,ZETA,comment)

Description

The ITAP-T function module **sfd4** (which is one of the function routines called by **modexe**) calculates complex modal vectors and reconstructed FRFs from identified modal parameters.

The theoretical foundation for **sfd4** calculations is described in Subsection 6.8.7.

This function module offers the advanced ITAP-T user maximum flexibility in performing SFD computations. The input and output variables are defined in the table below:

Variable	Description
H	FRF Data Array
frq	FRF Data frequency array
f1, f2	Lower and Upper Bounds of Analysis Frequency Band
FREQ	Array of Modal Frequencies
ZETA	Array of Modal Damping Parameters
comment	Descriptive Comment for Display Title
PHI	Complex Modal Matrix
COR	Null Matrix (not calculated in SFD4)
FRFIT	Reconstructed FRF Array
frqx	Reconstructed FRF frequency array

Reference

See Subsection 6.8.7

Purpose

Compute the normalized response or shock spectrum associated with a force or acceleration history

Syntax

```
[freq,amx,amn,vmx,vmn,xmx,xmn] =
                    shock(F,dt,df,fmax,zeta,n1,ns);
```

Description

shock calculates a normalized response or shock spectrum, which is associated with a specified excitation time history, **F**, which is sampled in uniform time increments, **dt**. Various types of response and shock spectra are calculated at frequencies, **freq**, (increment **df** to maximum frequency, **fmax**) consisting of maximum and minimum absolute acceleration (**amx, amn**), maximum and minimum velocity (**vmx, vmn**), and maximum and minimum displacement (**xmx, xmn**) spectra. A reference SDOF system damping value, **zeta**, is specified for the calculations. The input parameter, **n1**, is used to select an initial condition (0 for static equilibrium, 1 for zero displacement and velocity). The input parameter, **ns**, is used to indicate whether a response (ns=0) or shock (ns=1) is desired.

The response or shock spectrum is calculated as a unit amplitude normalized quantity by setting the peak input value to unity. If non-normalized spectra are desired, multiply the normalized spectra by the peak absolute amplitude of the excitation, F.

Example

Consider the single cycle force history, F(t), defined as

$F = \sin(4\pi t)$ for $t \leq 0.5$ seconds

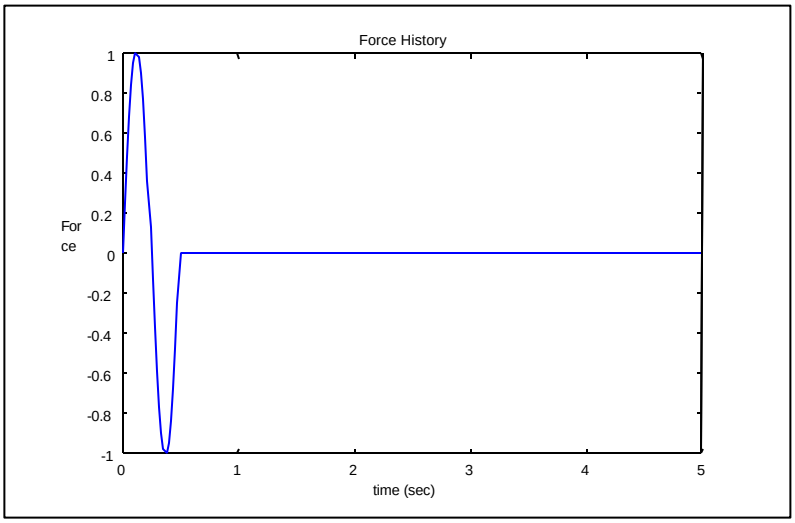
$F = 0$ for $t > 0.5$ seconds,

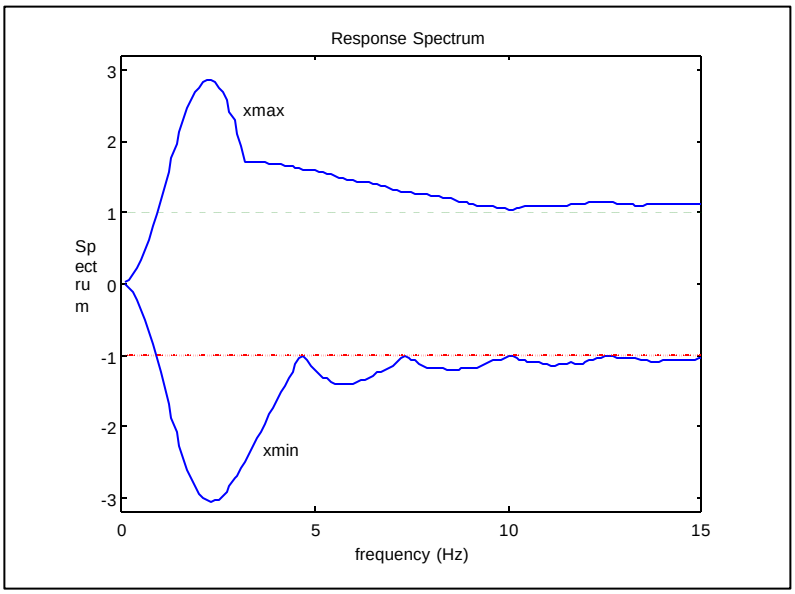
over uniform time increments, dt=0.02 seconds. Invoke the command:

```
[ff,amax,amin,vmax,vmin,xmax,xmin]=
                    shock(F',dt,0.1,15,dt,1,0);
```

Note that F has been defined as a row array. Therefore, since shock requires a column excitation history, F' is specified in the command.

The force history and normalized displacement response spectra are illustrated below:





Reference: See Subsection 1.2.4

Purpose

Perform single channel signal processing operations requests.

Syntax

singles(X,dt,NFFT,nfil,nch);

Description

The ITAP-T function routine **singles** performs a variety of single channel signal processing operations on a time history array, **X**, sampled at uniform intervals, **dt**. An initial data quality evaluation operation is performed by a call to **prelim**. **NFFT** defines the window length for spectrogram and autospectrum calculations. Alphanumeric names "**nfil**" (the data file name) and "**nch**" (the channel name) are specified for display labeling.

The user is offered a choice of additional signal processing and display options, namely, (1) Autospectrum (**pwrspec**), (2) Probability Density (**probn**) , (3) Total Probability (**probt**), (4) Shock Spectrum (**srspec**), and (5) Random Decrement (**rdec**).

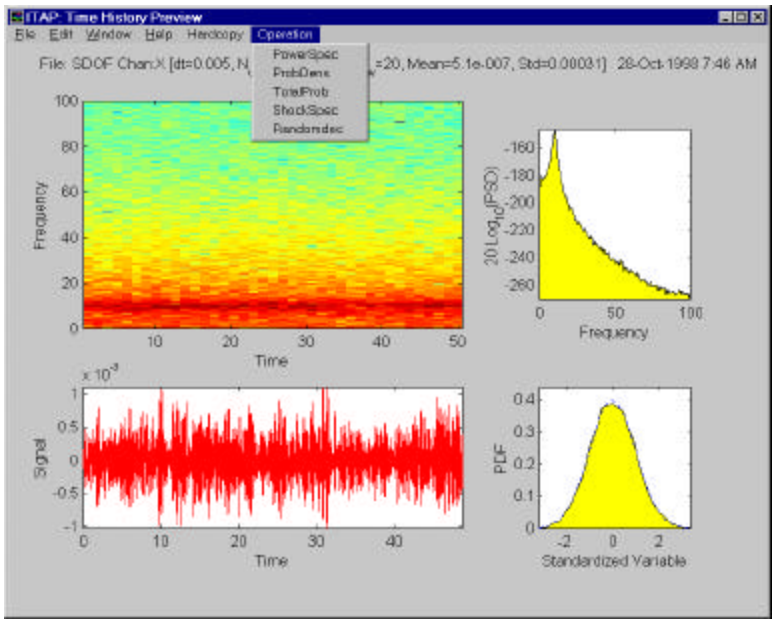
Example

Single channel analysis of channel 2 of a time history data file (named SDOF) is performed by invoking the following statements in the MATLAB command window:

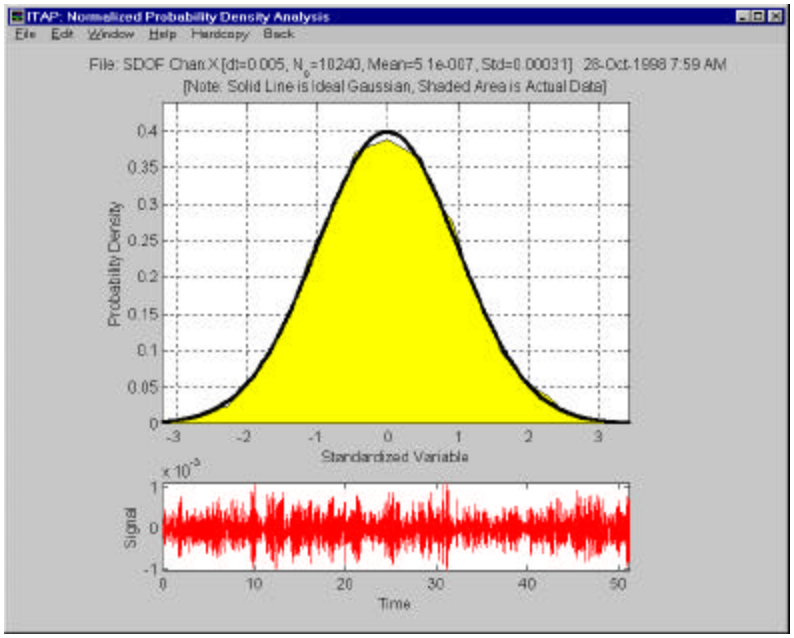
Load SDOF;
nfil='SDOF';nch=2;X=tdata(:,2);nch=namch(2,:);
singles(X,dt,512,nfil,nch);

[Note: The ITAP-T function module singles was originally written to operate within the module prelim. Explicit variable names must be used in the function call statement, as indicated above].

The following display is generated:



In the above display, the “**Operation**” menu is activated showing the additional data analysis choices. By clicking the selection named “**ProbDens**”, the following (**probn**) display is generated:



By clicking on the menu option “back”, this figure is closed. The original display appears allowing the user to choose another signal processing option.

Purpose

Perform SI/SO analysis computations

Syntax

```
[frq,HYX,COH,GYY,GXX,GYX] =
                sisogen(X,Y,dt,nwin,novl);
```

Description

This ITAP-T function module performs SI/SO computations for one specific input time history channel (columns of the matrix, **X**) and one specific output time history channel (columns vector, **Y**), where the data is sampled over uniform time increments, **dt**. The FFT window length, **nwin**, and overlap processing index, **novl**, are specified as input parameters. Output arrays resulting from the MI/SO analysis consist of a frequency vector, **frq**, FRF array, **HYX**, and ordinary coherence array, **COH**. Additional output arrays consist of the output autospectrum, **GYY**, input autospectra, **GXX**, and output/input cross-spectra, **GYX**. This function module should only be used if **GYY**, **GXX**, and/or **GYX** are desired. Otherwise, use **mimogen**.

This module performs computations using the MATLAB Signal Processing Toolbox function **spectrum**.

Reference

See Chapter 6.

Purpose

Calculate and display a skyline modal indicator function

Syntax

[SKY,frqx,FRNG]=skyline(H,frq,f1,f2,comment);

Description

The ITAP-T function module **skyline** (which is one of the function routines called by **modexe**) calculates and displays the complex skyline modal indicator function, **SKY**, from the FRF data array, **H**, defined at frequencies in the array **f**. The computations are performed in the frequency band bounded by **f1** ≤ f ≤ **f2**, which are used to define the skyline function frequency array, **frqx**. The alphanumeric variable **comment** is used as part of the display title.

Upon invoking the **skyline** command, the skyline function is displayed. The user then selects frequency bands within which there appears to be modal activity. The selected frequency band limits are stored in the output array, **FRNG**. A final display is generated with the selected frequency bands highlighted.

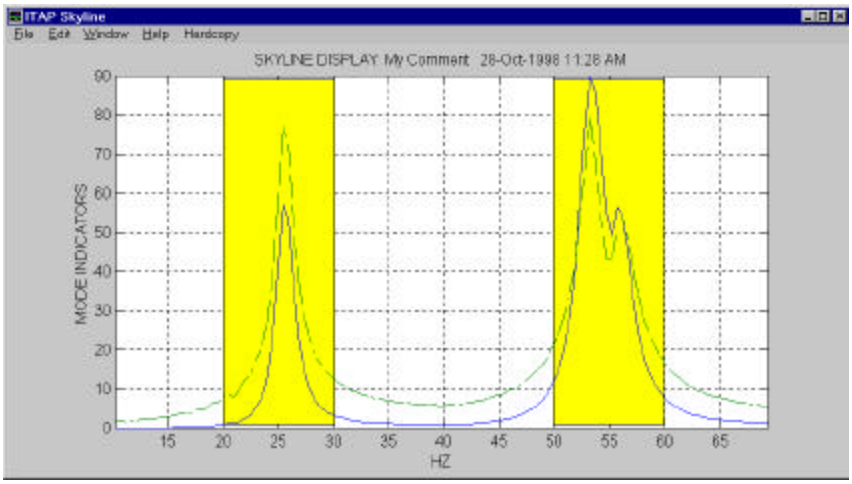
The theoretical foundation for **skyline** calculations is described in Subsection 6.8.4.

Example

Skyline analysis of a data file (TriDOFm) containing FRF data is initiated by the statements in the MATLAB command window:

load TriDOFm;
[SKY,frqx,FRNG]=skyline(H,frq,10,70,'My Comment');

The following display is generated (after selection of frequency bands of interest):



The output array FRNG is:

```
FRNG =

   20.0045   30.0030
   49.9174   59.9159
```

Reference

Subsection 6.8.4

Purpose

Calculate and display a skyline based on the sum of autospectra

Syntax

[SKY,frq,FRNG]=skypsd(tdata,dt,NFFT,f1,f2,comment);

Description

The ITAP-T function module **skypsd** calculates and displays a skyline function, **SKY**, which is composed of the sum of autospectra associated with a multi-channel time history array, tdata, sampled over uniform time increments, **dt**. The autospectra are calculated using a window length, **NFFT**. The computations are performed in the frequency band bounded by **f1** ≤ f ≤ **f2**, which are used to define the skyline function frequency array, **frq**. The alphanumeric variable **comment** is used as part of the display title.

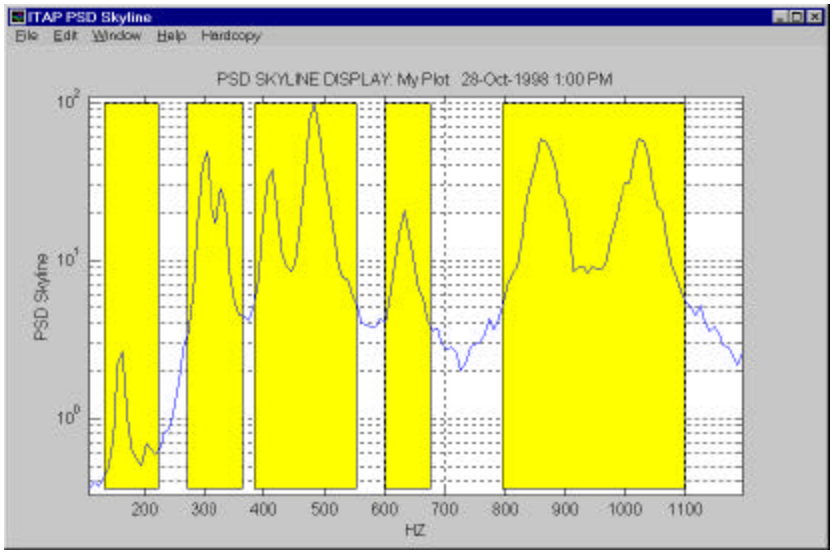
Upon invoking the **skypsd** command, the skyline function is displayed. The user then selects frequency bands within which there appears to be modal activity. The selected frequency band limits are stored in the output array, **FRNG**. A final display is generated with the selected frequency bands highlighted.

Example

Autospectrum based skyline analysis of a data file (TriDOFm) containing time history data is initiated by the statements in the MATLAB command window:

```
load W-Test;
skypsd(tdata(:,[2:34]),dt,512,100,1200,'My Plot');
```

The following display is generated (after selection of frequency bands of interest):



The output array FRNG is:

```
FRNG =

  1.3421e+002  2.2360e+002
  2.7160e+002  3.6431e+002
  3.8417e+002  5.5302e+002
  6.0103e+002  6.7717e+002
  7.9636e+002  1.0993e+003
```

Purpose

Calculate and display the normalized “shock response spectrum” associated with a single time history record.

Syntax

[freq,SRS]=srspec(X,dt,fmax,namfil,namch);

Description

The ITAP-T function routine **srspec** calculates the normalized “shock response spectrum” (actually a response spectrum), **SRS**, associated with a selected time slice of a single channel time history array **X**, sampled over uniform time increments, **dt**. The user specifies the maximum frequency, **fmax**, for which the **SRS** is to be calculated. The 128 frequencies, **freq**, at which **SRS** is computed are bounded by $0 \leq \text{freq}(i) \leq$ **fmax**. Alphanumeric names “**namfil**” (the data file name) and “**namch**” (the channel name) are specified for display labeling.

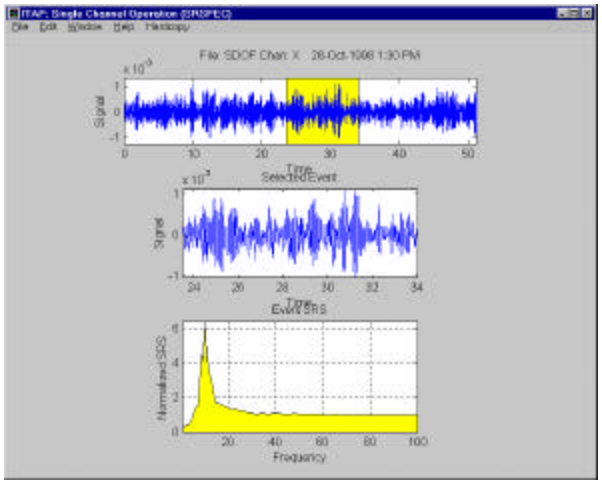
This function routine, which uses the function module **shock**, is an option within the ITAP-T function routine **singles**.

Example

Analysis of channel 2 of a time history data file (named SDOF) is performed by invoking the following statement in the MATLAB command window:

[freq,SRS]=srspec(tdata(:,2),dt,100,'SDOF','X');

The following display is generated (after responding to a request for a selected time segment):



Reference

Subsection 1.2.4

Purpose

Read time history data from a SDRC Universal File. Save to a standard ITAP-T time history data file.

Syntax

unvread;

Description

The ITAP-T function routine **unvread** scans a SDRC Universal File for basic measured data arrays. The arrays, which are translated to standard **ITAP-T names** and format, are (1) data channel time histories (**tdata**), (2) sampling rate (**dt**), and channel names (**namch**). In addition, the original file name ”**namfil.unv**” is used to generate the name of the ITAP-T data file “**namfil.mat**”.

The user is prompted to specify the name of the SDRC Universal File “**namfil**” after invoking **unvread** in the MATLAB command window.