Name: SOLUTIONS **ME308 Prerequisites Self-Quiz**

1. Determine the velocity of a block that has an acceleration, *a*, of:

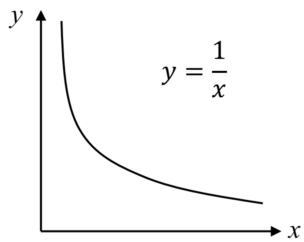
   $a(t) = c_1 t + c_2$

   where *t* is time and c_1 and c_2 are constants. The initial velocity of the block is $U\,(t = 0) = U_0$.

$$a = \frac{dU}{dt} \Rightarrow dU = a\,dt \Rightarrow \int_{U=U_o}^{U=U} dU = \int_{t=0}^{t=t} \left(c_1 t + c_2\right) dt$$

$$\boxed{\therefore U - U_0 = \tfrac{1}{2} c_1 t^2 + c_2 t}$$

2. Consider the curve shown below:



   a. Determine the slope of the curve at $x = 5$.

$$\frac{dy}{dx} = -\frac{1}{x^2} \quad \Rightarrow \quad \boxed{\left.\frac{dy}{dx}\right|_{x=5} = -\frac{1}{25}}$$

   b. Determine the area under the curve from $x = 2$ to $x = 5$.

$$\boxed{\text{area} = \int_{x=2}^{x=5} \frac{dx}{x} = \ln\left(\frac{5}{2}\right)}$$

3. Determine the following, assuming *a*, *b*, *c*, and z_0 are constants.

   a. $\nabla\left(at + bx^2 + by + c\ln z/z_0\right) =$

$$\nabla\left(at + bx^2 + by + c\ln z/z_0\right) = \left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right)\left(at + bx^2 + by + c\ln z/z_0\right)$$

$$\boxed{\therefore \nabla\left(at + bx^2 + by + c\ln z/z_0\right) = 2bx\hat{\mathbf{i}} + b\hat{\mathbf{j}} + \frac{c}{z}\hat{\mathbf{k}}}$$

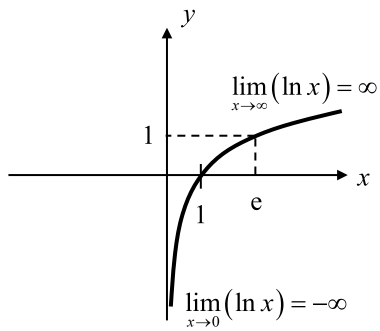
   b. $\nabla \cdot \left(ay\hat{\mathbf{i}} + bxy\hat{\mathbf{j}} + c\hat{\mathbf{k}}\right) =$

$$\nabla \cdot \left(ay\hat{\mathbf{i}} + bxy\hat{\mathbf{j}} + c\hat{\mathbf{k}}\right) = \left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}\right) \cdot \left(ay\hat{\mathbf{i}} + bxy\hat{\mathbf{j}} + c\hat{\mathbf{k}}\right)$$

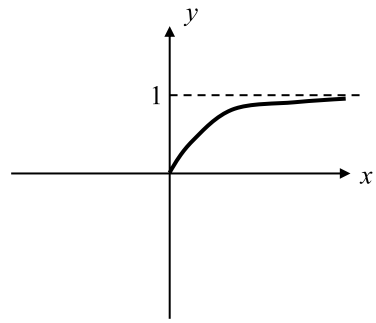
$$\boxed{\therefore \nabla \cdot \left(ay\hat{\mathbf{i}} + bxy\hat{\mathbf{j}} + c\hat{\mathbf{k}}\right) = bx}$$

4. Sketch the following curves:

a. $y = \ln x$ for $x > 0$



b. $y = 1 - \exp(-x)$ for $x \geq 0$



5. Solve the following equation subject to the condition $x(t = 1) = x_0$:

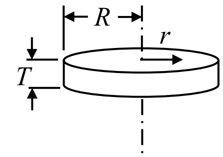
$$t\frac{dx}{dt} - x = 0$$

$$\Rightarrow t\frac{dx}{dt} = x \Rightarrow \int_{x=x_0}^{x=x} \frac{dx}{x} = \int_{t=1}^{t=t} \frac{dt}{t} \Rightarrow \ln\left(\frac{x}{x_0}\right) = \ln\left(\frac{t}{1}\right)$$

$$\boxed{\therefore \frac{x}{x_0} = \frac{t}{1}}$$

6. The density, ρ, of a circular plate of radius, *R*, and thickness, *T*, varies with the radial distance from the plate center, *r*, as:

$$\rho(r) = \rho_0 + \rho_1 \frac{r}{R}$$

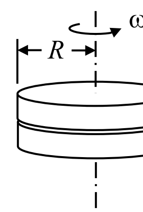


where ρ_0 and ρ_1 are constants. Determine the mass of the plate in terms of ρ_0, ρ_1 *R*, and *T*.

$$M = \int_V \rho dV = \int_{r=0}^{r=R} \left(\rho_0 + \rho_1 \frac{r}{R}\right)(2\pi r dr T) = \rho_0 \pi R^2 T + \frac{2\pi\rho_1 T}{R} \int_{r=0}^{r=R} r^2 dr$$

$$\boxed{\therefore M = \rho_0 \pi R^2 T + \tfrac{2}{3} \rho_1 \pi R^2 T}$$

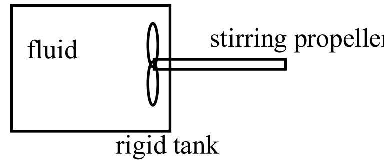
7. Two circular plates of radius, *R*, are brought into contact as shown in the figure. The top plate rotates with angular velocity, ω, and the bottom plate is fixed. The pressure between the two plates is uniform and constant with a value of p_0 and the friction coefficient between the two plates is μ. Determine:
   a. the torque acting on the top plate due to the bottom plate, and
   b. the power required to keep the top plate spinning at a constant angular velocity, ω.



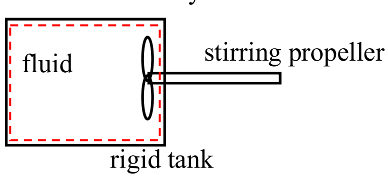
$$T = \int_{r=0}^{r=R} r dF = \int_{r=0}^{r=R} r\mu p_0 2\pi r dr = 2\pi\mu p_0 \int_{r=0}^{r=R} r^2 dr = \frac{2\pi}{3}\mu p_0 R^3$$

$$P = \omega T = \frac{2\pi}{3}\mu p_0 R^3 \omega$$

8. A rigid tank contains a hot fluid that is cooled while being stirred. Initially the internal energy of the fluid is 800 kJ. During the cooling process, the fluid loses 500 kJ of heat and the stirring propeller does 100 kJ of work on the fluid. What is the final internal energy of the fluid?



Apply the 1st Law of Thermodynamics to the control volume shown in the following figure.



$$\Delta E_{\text{CV}} = E_{f,\text{CV}} - E_{i,\text{CV}} = Q_{\substack{\text{added}\\ \text{to CV}}} + W_{\text{on CV}}$$

$$E_{f,\text{CV}} = E_{i,\text{CV}} + Q_{\substack{\text{added}\\ \text{to CV}}} + W_{\text{on CV}} = 800\text{ kJ} - 500\text{ kJ} + 100\text{ kJ}$$

$$\boxed{\therefore E_{f,\text{CV}} = 400\text{ kJ}}$$