## The Reynolds Transport Theorem





 $B_{in}(t+\delta t)$ 

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CV



$$B_{CV}(t+\delta t) = B_{sys}(t+\delta t) - B_{out}(t+\delta t) + B_{in}(t+\delta t)$$

$$\frac{B_{CV}(t+\delta t) - B_{sys}(t)}{\delta t} = \frac{B_{sys}(t+\delta t) - B_{sys}(t)}{\delta t} - \frac{B_{out}(t+\delta t)}{\delta t} + \frac{B_{in}(t+\delta t)}{\delta t}$$

$$\frac{D}{Db} \left[ \prod_{sys} B_{sys}(t+\delta t) - B_{sys}(t) - \frac{B_{out}(t+\delta t) - B_{out}(t)}{\delta t} + \frac{B_{in}(t+\delta t) - B_{in}(t)}{\delta t} \right]$$

$$\lim_{\delta t \to 0} \frac{B_{sys}(t+\delta t) - B_{sys}(t)}{\delta t} - \lim_{\delta t \to 0} \frac{B_{out}(t+\delta t) - B_{out}(t)}{\delta t} + \lim_{\delta t \to 0} \frac{B_{in}(t+\delta t) - B_{in}(t)}{\delta t}$$

$$\Rightarrow \frac{dB_{CV}}{dt} = \frac{DB_{sys}}{Dt} - \frac{dB_{out}}{dt} + \frac{dB_{in}}{dt}$$

$$\mathbf{u}_{rel} \cdot \hat{\mathbf{n}}$$
small piece of control surface, *dA*, with outward point normal vector,  $\hat{\mathbf{n}}$ 

$$dV = (\mathbf{u}_{rel} \cdot \hat{\mathbf{n}}) \delta t dA = (\mathbf{u}_{rel} \cdot d\mathbf{A}) \delta t$$

$$dQ = \mathbf{u}_{rel} \cdot d\mathbf{A}$$

$$\frac{d(B_{\text{out}} - B_{\text{in}})}{dt} = \int_{CS} \beta \rho dQ = \int_{CS} \beta (\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A})$$

$$\underbrace{\frac{D}{Dt}\left[\int_{V_{\text{sys}}}\beta\rho \, dV\right]}_{\text{rate of increase of }B \text{ within the system}} = \underbrace{\frac{d}{dt}\left[\int_{CV}\beta\rho \, dV\right]}_{\text{rate of increase of }B \text{ within the CV}} + \underbrace{\int_{CS}\beta(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A})}_{\text{net rate at which }B \text{ leaves the CV through the CS}}$$