12.8. System Operating Point

How do we select a pump for a given system? Analyze the system to determine the shaft head required to give a specified volumetric flow rate. Compare this equation to a pump performance curve (H - Q curve) to determine if the pump operates efficiently at this Q. If so, then the choice of pump is appropriate.

For example, consider the system shown in Figure 12.31. Apply the Extended Bernoulli Equation from point 1 to point 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \tag{12.45}$$

where $p_1 = p_2 = p_{atm}$ and $\bar{V}_1 \approx \bar{V}_2 \approx 0$. Thus, the head required from the pump is,

$$H_{S,12}=(z_2-z_1)+H_{L,12}. \tag{12.46}$$

Recall that,

$$H_{L,12}=\sum_i K_i\frac{\bar{V}_i^2}{2g}=\sum_i K_i\frac{Q_i^2}{2gA_i^2}, \tag{12.47}$$

so that,

$$H_{S,12}\approx(z_2-z_1)+cQ^2, \tag{12.48}$$

where c is a constant that incorporates the loss coefficients and area ratios, and an "$\approx$" is used since the loss coefficient may depend on the flow speeds.

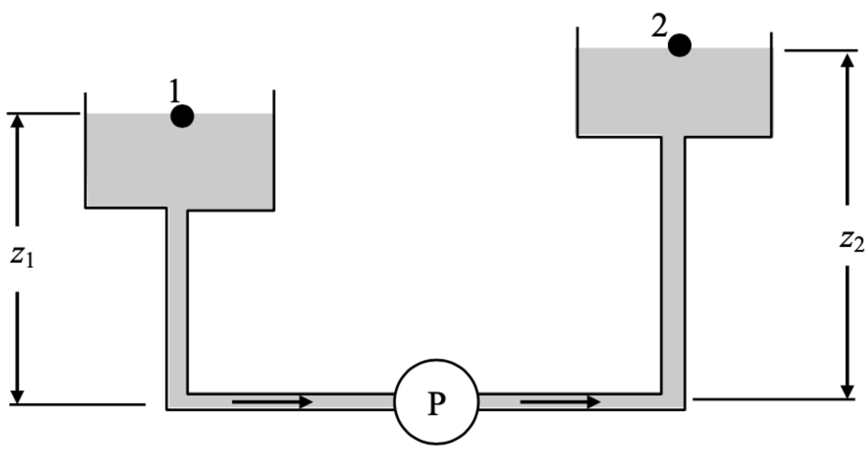


Figure 12.31. The system used in the operating point example.

The flow rate at which the system operates is at the intersection of the system head curve with the pump performance curve, as shown in Figure 12.32.

Notes:

(1) Ideally we would want the operating point to occur near the Best Efficiency Point for the pump.
(2) For laminar flow,

$$K_{\text{major}}=\frac{64}{\text{Re}}\left(\frac{L}{D}\right)=\frac{64\nu}{\bar{V}D}\frac{L}{D}=\frac{c}{Q}, \tag{12.49}$$

where c is a constant. Thus,

$$H_L\sim Q, \tag{12.50}$$

$$\implies \text{system curve is:}\quad H_S=c_1+c_2Q\quad\text{(a line instead of a parabola).} \tag{12.51}$$

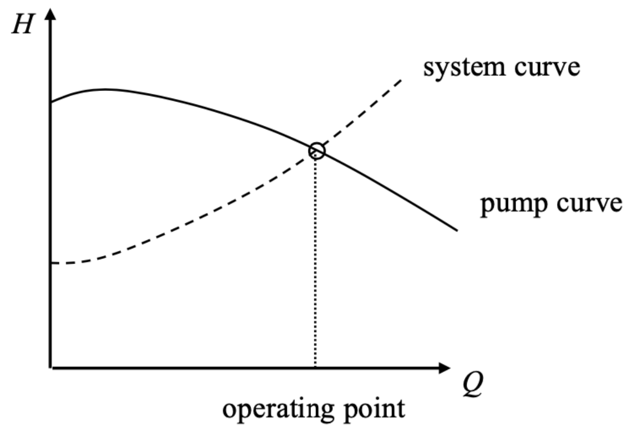


Figure 12.32. The head (H) required for the system to operate at a given flow rate (Q), i.e., the system head curve, and the head rise generated by the pump (H) at a given flow rate (Q), i.e., the pump performance curve. The flow rate at which the two curves intersect is the system operating flow rate.

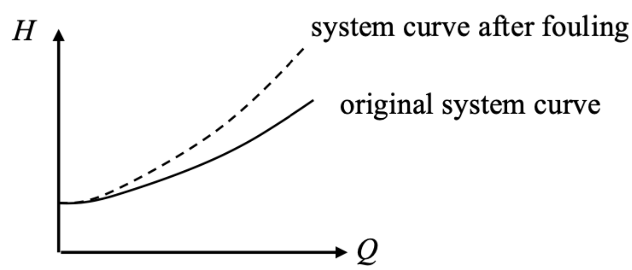


Figure 12.33. An illustration of the original system head curve and the system head curve after fouling.

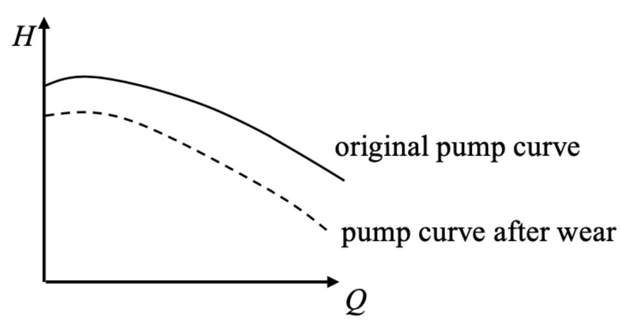


Figure 12.34. An illustration of the original pump head curve and the pump head curve after wear.

(3) The system curve may change over time due to fouling of the pipes and other factors $\Longrightarrow$ increased losses $\Longrightarrow$ the system curve becomes steeper, as shown in Figure 12.33. The pump curve may also change due to wear on the bearings, impeller, etc., as shown in Figure 12.34.

(4) Stability issues become significant when the pump has a flat or falling performance curve, which is defined as a performance curve in which H decreases as Q decreases (Figure 12.35). For example, Figure 12.36 shows the system curve intersecting the pump curve at two different flow rates. The flow rate in the falling portion of the pump curve (the left point in the figure) is unstable since a slight perturbation results in the flow rate diverging way from the point. The operating point in the rising portion of the pump curve (right point in the figure), however, is stable since conditions resulting from a small perturbation from this point will drive the flow rate back to the operating point.

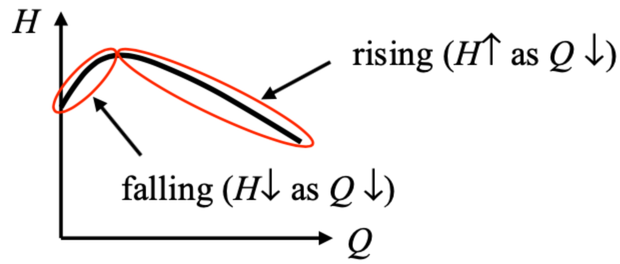


Figure 12.35. An illustration of a "falling" pump performance curve.

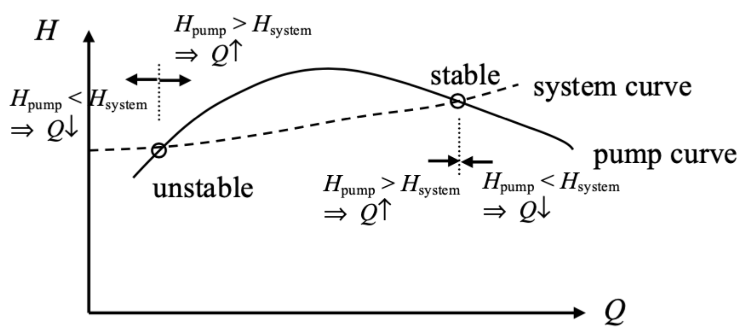


Figure 12.36. An illustration showing stable and unstable operating points.

Figure 12.37 shows a more complex pump curve with two rising sections and one falling section. The operating points in the rising sections are stable while the operating point in the falling portion is unstable. Usually this type of situation is undesirable since in engineering we typically prefer to have an unambiguous, stable operating point rather than the possibility that the operating point might suddenly change if a sufficiently large perturbation occurs.

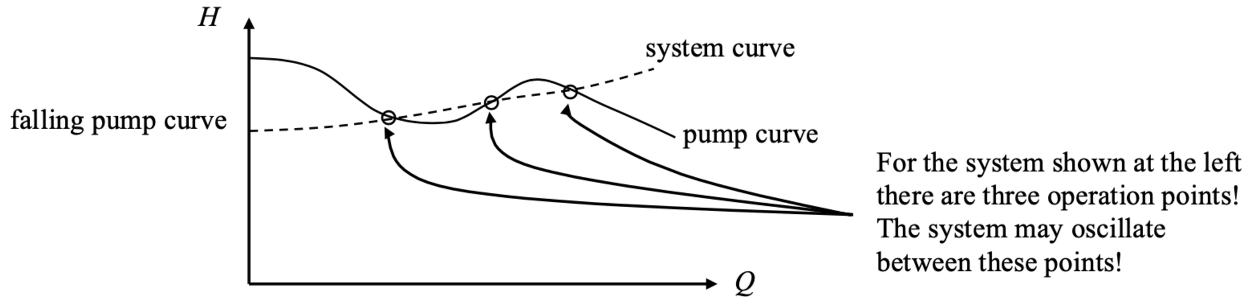


Figure 12.37. An illustration showing a pump curve resulting in two stable operating points and one unstable operating point.

Figure 12.38 shows a situation in which the system and pump curves remain close to each other for a range of flow rates. This condition is also undesirable since a perturbation from the operating point will take a long time to come back to equilibrium and, as a result, the flow rate will drift over a range of values. Instead, it is better to have a situation in which the system and pump curves intersect with a large angle between the curves so there's a large potential (i.e., head difference) driving the system back into equilibrium if there's a perturbation.

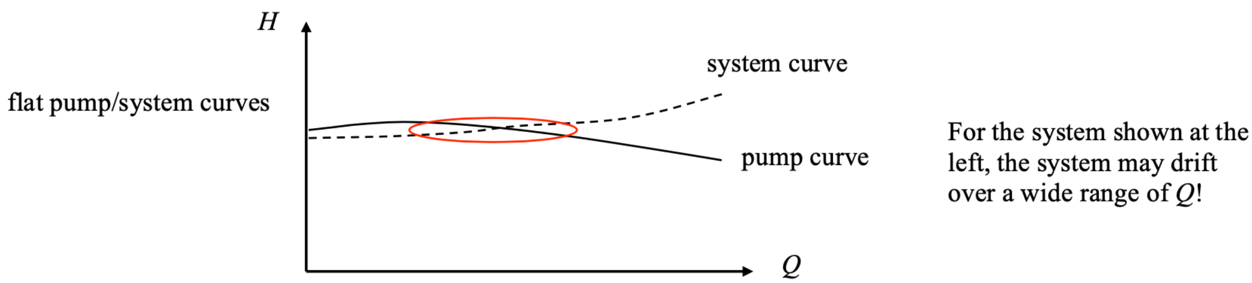
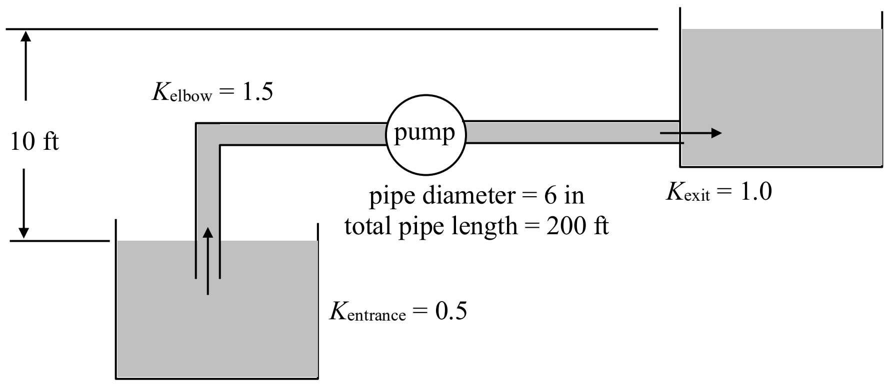
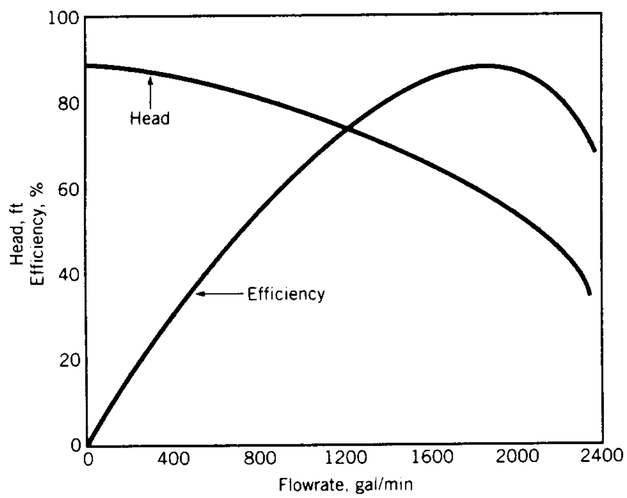


Figure 12.38. An illustration demonstrating that the operating flow rate for a flat pump performance curve can vary considerably.

Water is to be pumped from one large open tank to a second large open tank. The pipe diameter throughout is 6 in. and the total length of the pipe between the pipe entrance and exit is 200 ft. Minor loss coefficients for the entrance, exit, and the elbow are shown on the figure and the friction factor can be assumed constant and equal to 0.02. A certain centrifugal pump having the performance characteristics shown is suggested as a good pump for this flow system.

a. With this pump, what would be the flow rate between the tanks?
b. Do you think this pump would be a good choice?

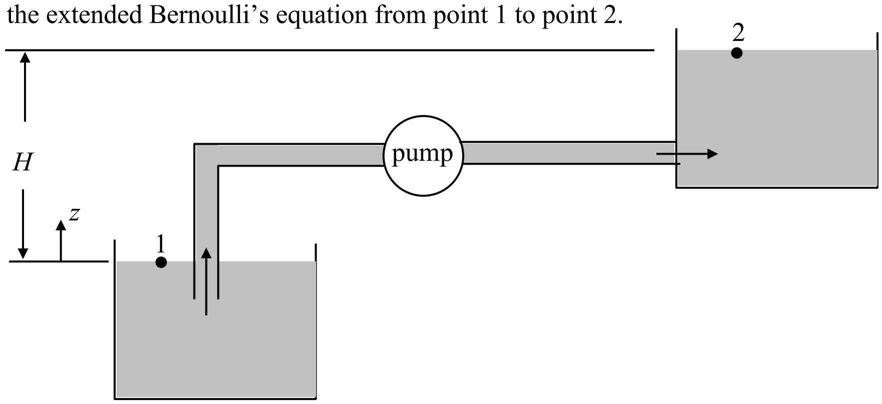




(b)

SOLUTION:

Apply the extended Bernoulli's equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \quad (1)$$

where

$p_2 = p_1 = p_{atm}$ (free surface)

$\bar{V}_2 \approx \bar{V}_1 \approx 0$ (large tanks)

$z_2 - z_1 = H$

$$H_L=\frac{\bar{V}^2}{2g}\left[f\left(\frac{L}{D}\right)+K_{entrance}+K_{exit}+K_{elbow}\right] \text{ (where } \bar{V} \text{ is the mean velocity in the pipe)} \quad (2)$$

Note that the mean pipe velocity can be expressed in terms of the volumetric flow rate.

$$\bar{V}=\frac{Q}{\pi D^2/4}$$

Substitute and simplify.

$$H_S=H+\frac{8Q^2}{\pi^2 g D^4}\left[f\left(\frac{L}{D}\right)+K_{entrance}+K_{exit}+K_{elbow}\right] \quad (3)$$

For the given problem:

H	=	10 ft	
g	=	32.2 ft/s^2	
f	=	0.02	
D	=	6 in = 0.5 ft	
L	=	200 ft	(Note: $K_{major} = f(L/D) = 8.0$)
$K_{entrance}$	=	0.5	
K_{exit}	=	1.0	
K_{elbow}	=	1.5	

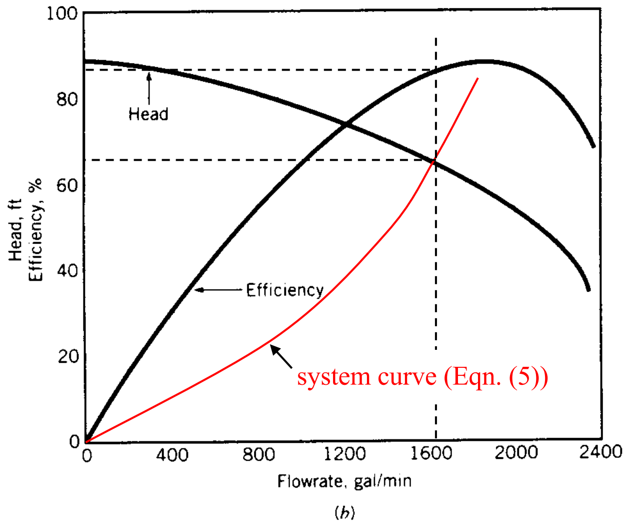
$\Rightarrow$ $H_S = (10 + 4.43Q^2)$ ft Note that $[Q] = \text{ft}^3/\text{s}$. (4)

This is the head that must be added to the fluid by the pump in order to move the fluid at the volumetric flow rate Q.

With $[Q]$ = gpm, Eqn. (4) becomes:

$\boxed{H_S = (10 + 2.25 * 10^{-5} Q^2) \text{ ft}}$ Note that $[Q]$ = gpm. (5)

Plot Eqn. (5) on the pump performance curve to determine the operating point.



From the figure we observe that the operating point occurs at:

$\boxed{Q \approx 1600 \text{ gpm}}$

corresponding to a head rise and efficiency of

$H \approx 67$ ft

$\eta \approx 84\%$

The operating efficiency is close to the optimal efficiency of 86% so this is a good pump to use.

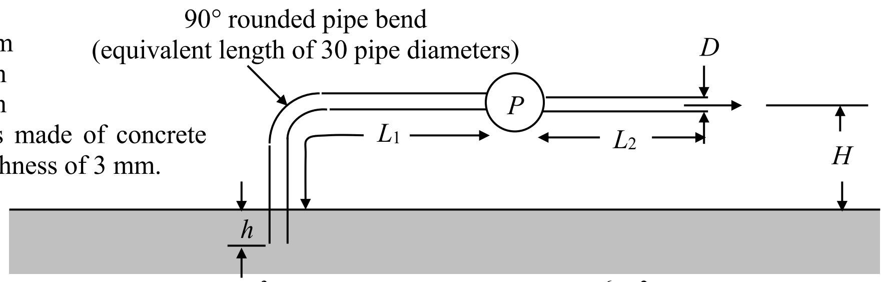
The power required to operate this pump is,

$$\dot{W} = \frac{\rho Q g H}{\eta} = \frac{\left(62.4\ \frac{\text{lb}_\text{f}}{\text{ft}^3}\right)\left(1600\ \frac{\text{gal}}{\text{min}}\right)\left(\frac{\text{min}}{60\text{ s}}\right)\left(\frac{\text{ft}^3}{7.48\text{ gal}}\right)\left(66.5\text{ ft}\right)\left(\frac{\text{hp}}{550\ \text{ft.lb}_\text{f}/\text{s}}\right)}{0.84}$$

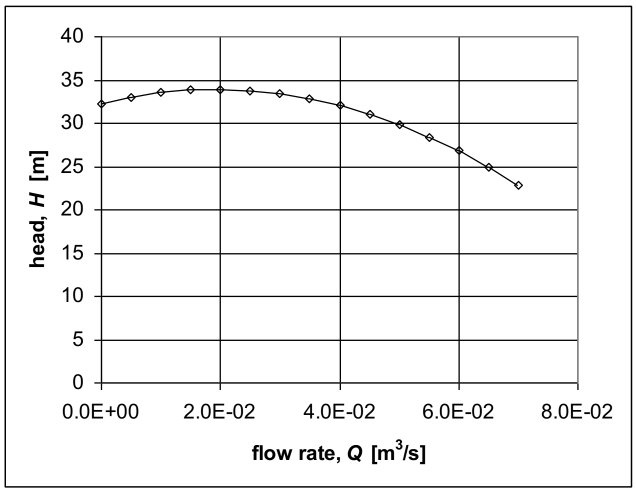
$\boxed{\dot{W} = 32.0 \text{ hp}}$

Consider the pipe/pump system shown in the figure below.

h = 0.5 m
H = 2 m
D = 0.2 m
L_1 = 10 m
L_2 = 20 m
The pipe is made of concrete with a roughness of 3 mm.



water with density of 1000 kg/m^3, kinematic viscosity of 1.0×10^{-6} m^2/s, and vapor pressure of 2.34 kPa



The pump performance head curve is approximated as:

$$H = (3.23 \times 10^1 \text{ m}) + (1.65 \times 10^2 \text{ s/m}^2)Q - (4.82 \times 10^3 \text{ s}^2/\text{m}^5)Q^2$$

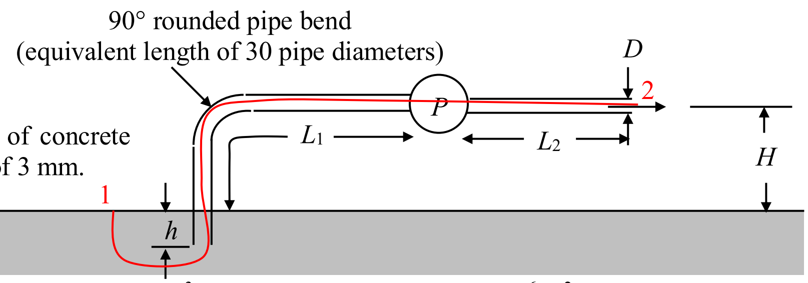
where $[H]$ = m and $[Q]$ = m^3/s.

a. Determine the system head curve for the pipe system.
b. Determine the operating point for the system.
c. How will the flow rate within the pipe change over time if the pipe carries "hard" water and lime deposits form on the interior pipe walls? Explain your answer. You should assume that the deposits do not significantly affect the pipe diameter.
d. Calculate the net positive suction head available at the pump inlet.
e. If we wanted to add a valve to control the flow rate in the pipe, would it be better to put the valve upstream or downstream of the pump? Explain your answer.

SOLUTION:

Apply the Extended Bernoulli Equation from points 1 to 2.

h = 0.5 m
H = 2 m
D = 0.2 m
L_1 = 10 m
L_2 = 20 m
The pipe is made of concrete with a roughness of 3 mm.



water with density of 1000 kg/m^3, kinematic viscosity of 1.0*10^{-6} m^2/s, and vapor pressure of 2.34 kPa

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \qquad (1)$$

where

$$p_1=p_2=p_{atm} \qquad (2)$$

$$\bar{V}_1\approx 0 \quad \text{and} \quad \bar{V}_2=\frac{4Q}{\pi D^2} \quad \text{(Also assume turbulent flow, } \alpha_2\approx 1.) \qquad (3)$$

$$z_2-z_1=H \qquad (4)$$

$$H_L=\left(K_{major}+K_{inlet}+K_{elbow}\right)\frac{\bar{V}_2^2}{2g} \qquad (5)$$

Solve for H_S.

$$H_S=H+\left[1+f\left(\frac{L}{D}\right)+K_{inlet}+f\left(\frac{L_e}{D}\right)\right]\frac{8Q^2}{\pi^2 gD^4} \qquad (6)$$

Here,

H = 2 m
L = L_1+L_2 = 30 m
D = 0.2 m
K_{inlet} = 0.78 (re-entrant inlet)
L_e/D = 30
g = 9.81 m/s^2

The relative roughness is:

$$e/D=(3*10^{-3}\text{ m})/(0.2\text{ m})=0.015 \qquad (7)$$

Assume the flow Reynolds number is large enough so that it is in the fully rough zone and the friction factor is independent of the Reynolds number.

$$e/D=0.015 \text{ in fully rough zone (Re} > 70{,}000) \Rightarrow f\approx 0.044 \qquad (8)$$

Substitute and simplify.

$$H_S=2\text{ m}+\left[1+\underbrace{6.6}_{\text{major losses}}+\underbrace{0.78+1.32}_{\text{minor losses}}\right]\left(5.16*10^1\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2 \qquad (9)$$

(Note that the minor losses are not negligible compared to the major loss.)

$$\boxed{H_S=2\text{ m}+\left(5.01*10^2\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2} \quad \text{(This is the system head curve.)} \qquad (10)$$

The operating point occurs where the system and pump curves intersect.

$$\underbrace{2\text{ m}+\left(5.01*10^{2}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2}_{\text{system curve}}=\underbrace{\left(3.23*10^{1}\text{ m}\right)+\left(1.65*10^{2}\ \tfrac{\text{s}}{\text{m}^2}\right)Q-\left(4.82*10^{3}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2}_{\text{pump curve}} \quad (11)$$

$$\left(5.32*10^{3}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2-\left(1.65*10^{2}\ \tfrac{\text{s}}{\text{m}^2}\right)Q-\left(3.03*10^{1}\text{ m}\right)=0 \quad (12)$$

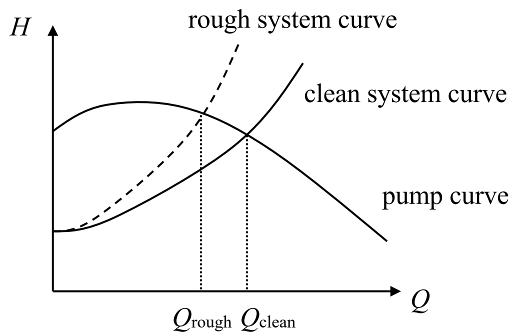
$$\boxed{Q=9.26*10^{-2}\ \tfrac{\text{m}^3}{\text{s}}} \quad (13)$$

Verify the Reynolds number assumption.

$$\bar{V}_2=\frac{4Q}{\pi D^2}=\frac{4\left(9.26*10^{-3}\ \frac{\text{m}^3}{\text{s}}\right)}{\pi\left(0.2\text{ m}\right)^2}=2.95\ \tfrac{\text{m}}{\text{s}} \quad (14)$$

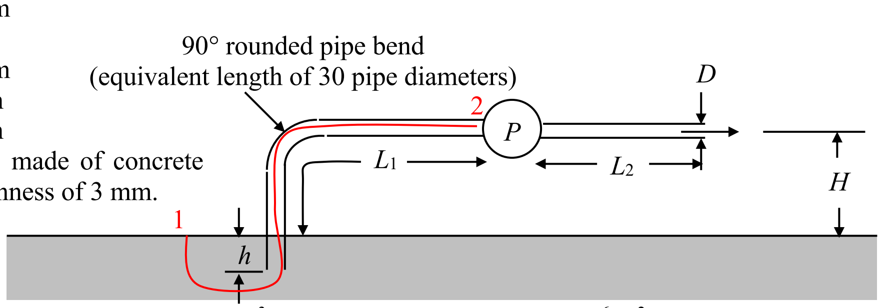
$$\text{Re}=\frac{\bar{V}_2 D}{\nu}=\frac{\left(2.95\ \frac{\text{m}}{\text{s}}\right)\left(0.2\text{ m}\right)}{\left(1*10^{-6}\ \frac{\text{m}^2}{\text{s}}\right)}=590{,}000 \Rightarrow \text{The assumption of fully turbulent flow is ok!} \quad (15)$$

As lime deposits collect, the relative roughness will increase resulting in an increase in the friction factor. Thus, the system curve will steepen over time and the operating flow rate will decrease.



Apply the Extended Bernoulli Equation from points 1 to 2 in the figure below.

h = 0.5 m
H = 2 m
D = 0.2 m
L_1 = 10 m
L_2 = 20 m
The pipe is made of concrete with a roughness of 3 mm.



water with density of 1000 kg/m³, kinematic viscosity of $1.0*10^{-6}$ m²/s, and vapor pressure of 2.34 kPa

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \quad (16)$$

where

$p_1 = p_{atm}$ (17)

$\bar{V}_1 \approx 0$ (The flow has been shown to be turbulent $\Rightarrow \alpha_2 \approx 1$.) (18)

$z_2 - z_1 = H$ (19)

$$H_L=\left(K_{\text{major}}+K_{\text{inlet}}+K_{\text{elbow}}\right)\frac{\bar{V}_2^2}{2g} \quad (20)$$

$H_S = 0$ (There is no pump between points 1 and 2.) (21)

Re-arrange to put in terms of *NPSHA*.

$$NPSHA\equiv\left(\frac{p}{\rho g}+\frac{\bar{V}^2}{2g}\right)_2-\frac{p_v}{\rho g}=\frac{p_1-p_v}{\rho g}-H-\left[f\left(\frac{L_1}{D}\right)+K_{\text{inlet}}+f\left(\frac{L_e}{D}\right)\right]\frac{8Q^2}{\pi^2 gD^4} \quad (22)$$

(Note that the major loss is based on L_1.)

Here,

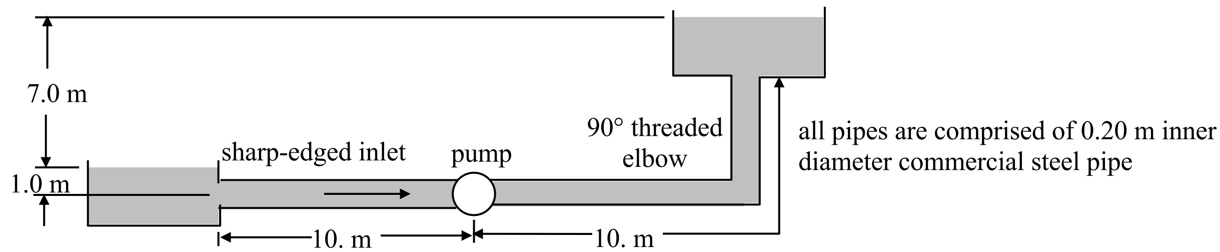
p_1 = p_{atm} = 101 kPa (abs)
p_v = 2.34 kPa (abs)
ρ = 1000 kg/m^3
g = 9.81 m/s^2
H = 2 m
f ≈ 0.044 (from previous work)
L_1 = 10 m
D = 0.2 m
K_{inlet} = 0.78 (re-entrant inlet)
L_e/D = 30
Q = 9.26*10^{-2} m^3/s

Substitute and simplify.

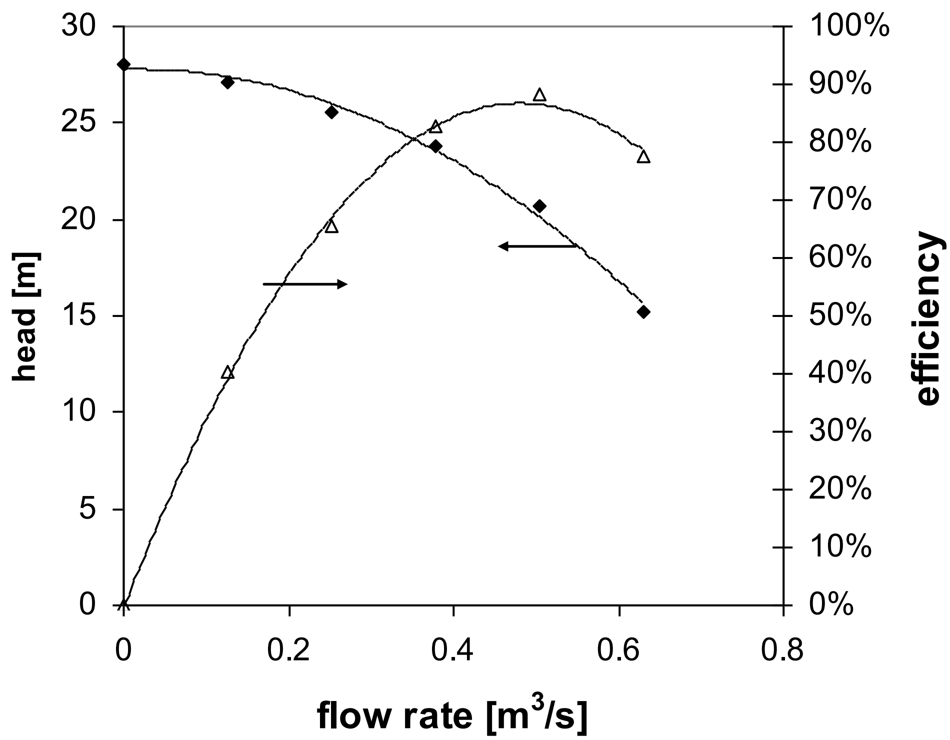
$\boxed{NPSHA = 6.16\text{ m}}$ (23)

We would be better off putting the valve downstream of the pump so that the NPSHA remains as large as possible to avoid cavitation in the pump.

Consider the pipe system shown in the figure below. The fluid to be pumped is water with a density of 1.0E3 kg/m^3, a kinematic viscosity of 1.0E-6 m^2/s, and a vapor pressure of 2.3E3 Pa.



The pump used in this system has the performance plot shown below.



Curve fits to the pump performance data are given below:

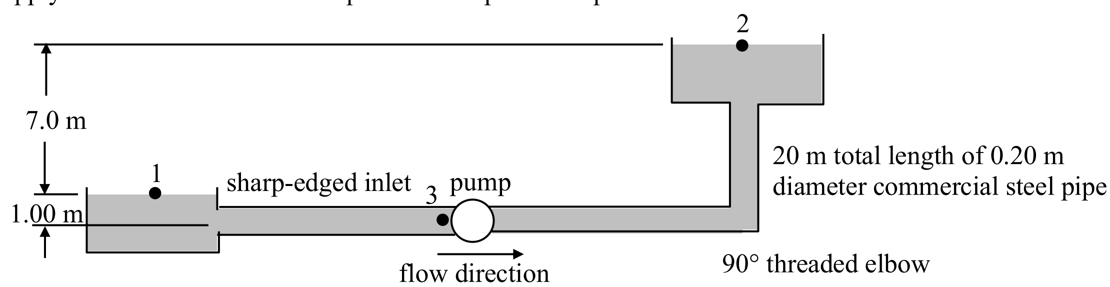
H [m] = (-3.25E1 s^2/m^5) Q^2 + (1.23E0 s/m^2) Q + (2.78E1 m)

η_P = (-3.74E0 s^2/m^6) Q^2 + (3.60E0 s/m^3) Q

a. Determine the operating volumetric flow rate of the system.
b. Is the given pump a good choice for this system? Explain your answer.
c. Determine the NPSHA to the pump for the flow rate determined in part (a).
d. Give one specific modification to the pipe system that could be employed to decrease the likelihood that cavitation will occur in the pump.

SOLUTION:

Apply the Extended Bernoulli Equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12} \quad (1)$$

where

$p_1 = p_2 = p_{\text{atm}}$ (free surfaces exposed to the atmosphere)

$\bar{V}_1 \approx \bar{V}_2 \approx 0$ (large tanks)

$z_2 - z_1 = 7.00\text{E}0 \text{ m}$ (given)

$$H_{L,12}=f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}+K_{\substack{\text{sharp-edged}\\\text{entrance}}}\frac{\bar{V}^2}{2g}+K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}\frac{\bar{V}^2}{2g}+K_{\text{exit}}\frac{\bar{V}^2}{2g}$$

$$=\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{sharp-edged}\\\text{entrance}}}+K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}+K_{\text{exit}}\right]\frac{\bar{V}^2}{2g}$$

$$=\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{sharp-edged}\\\text{entrance}}}+K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}+K_{\text{exit}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}$$

Re-arrange Eqn. (1) to solve for $H_{S,12}$.

$$H_{S,12}=(z_2-z_1)+\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{sharp-edged}\\\text{entrance}}}+K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}+K_{\text{exit}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2} \quad (2)$$

Assume that the flow is in the fully rough zone where the friction factor is independent of the Reynolds number. The pipe roughness, ε, is $\varepsilon = 0.0450E\text{-}3$ m so that the relative roughness is:

$$\frac{\varepsilon}{D}=\frac{4.50\text{E-5 m}}{2.00\text{E-2 m}}=2.25\text{E-4}$$

From the Moody chart in the fully rough zone:

$f =$ 1.41E-2

Substitute in the given data into Eqn. (2):

z_2-z_1	=	7.00E0 m
L	=	2.00E1 m
D	=	2.00E-1 m
K_{inlet}	=	5.00E-1
K_{elbow}	=	1.50E0
K_{exit}	=	1.00E0
g	=	9.81E0 m/s^2

$$H_{S,12} = \left(7.00\text{E}0\text{ m}\right) + \left(2.28\text{E}2\ {}^{\text{s}^2}\!/_{\text{m}^5}\right)Q^2 \tag{3}$$

Equate the system head curve (Eqn. (3)) to the given curve fit for the pump head curve to solve for the operating point flow rate.

$$\left(\text{-3.25E1 s}^2/\text{m}^5\right)Q^2 + \left(1.23\text{E0 s/m}^2\right)Q + \left(2.78\text{E1 m}\right) = \left(7.00\text{E0 m}\right) + \left(2.28\text{E2}\ {}^{\text{s}^2}\!/_{\text{m}^5}\right)Q^2$$

$$\left(\text{-2.60E2 s}^2/\text{m}^5\right)Q^2 + \left(1.23\text{E0 s/m}^2\right)Q + \left(2.08\text{E1 m}\right) = 0$$

$$\boxed{\therefore Q = 2.85\text{E-1 m}^3/\text{s}}$$

Check that the Reynolds number is in the fully rough zone as assumed.

$$\bar{V} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{2.85\text{E-1 m}^3/\text{s}}{\frac{\pi}{4}\left(2.0\text{E-2 m}\right)} = 9.08\text{ m/s}$$

$$\text{Re} = \frac{\bar{V}D}{\nu} = \frac{\left(9.08\text{E0 m/s}\right)\left(2.0\text{E-1 m}\right)}{\left(1.0\text{E-6 m}^2/\text{s}\right)} = 1.82\text{E6}$$

The Reynolds number and the relative roughness put the flow in the fully rough zone so the assumption was a good one.

The efficiency is determined using the given curve fit for the efficiency and the calculated volumetric flow rate.

$$\eta_P = 72\%$$

This efficiency is not near the Best Efficiency Point for the pump (BEP = 90%) so this is <u>not a good pump to use for this application</u>.

The NPSHA to the pump is found by applying the Extended Bernoulli Equation between points 1 and 3 and utilizing the definition of NPSH.

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_3 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_{L,13} + H_{S,13} \tag{4}$$

where

$p_1 = p_{\text{atm}}$ (free surface exposed to the atmosphere)

$\bar{V}_1 \approx 0$ (large tank)

$z_1 - z_3 = 1.00\text{E0 m}$ (given)

$\alpha_3 \approx 1$ (turbulent flow based on the Reynolds number calculated previously)

$H_{S,13} = 0$

$$H_{S,13} = \left[f\left(\frac{L_{13}}{D}\right) + K_{\substack{\text{sharp-edged}\\\text{entrance}}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}$$

Substitute into the definition of NPSH.

$$NPSH = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g}\right)_s - \frac{p_v}{\rho g}$$

$$NPSHA = \left(\frac{p_{\text{atm}} - p_v}{\rho g}\right) + \left(z_1 - z_3\right) - \left[f\left(\frac{L_{13}}{D}\right) + K_{\substack{\text{sharp-edged}\\ \text{entrance}}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}$$

Using the given data:

p_{atm}	=	1.01E5 Pa
p_v	=	2.30E3 Pa
ρ	=	1.00E3 kg/m^3
g	=	9.81E0 m/s^2
z_1-z_3	=	1.00E0 m
f	=	1.41E-1 (found previously)
L_{13}	=	1.0E1 m
D	=	2.0E-1 m
K_{inlet}	=	5.0E-1
Q	=	2.85E-1 m^3/s (found previously)
$\Rightarrow$		NPSHA = 6.01E0 m

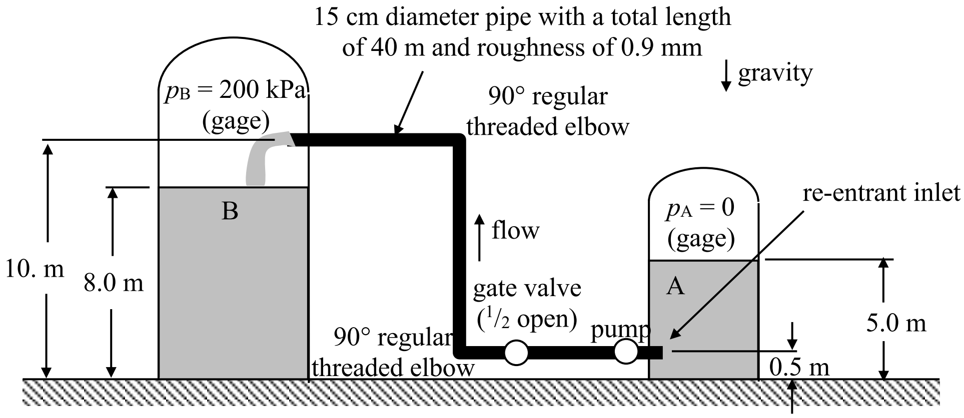
In order to avoid cavitating the pump, we would need to make sure that NPSHA > NPSHR for the pump.

If NPSHA < NPSHR, then the following could be easily implemented to increase NPSHA:

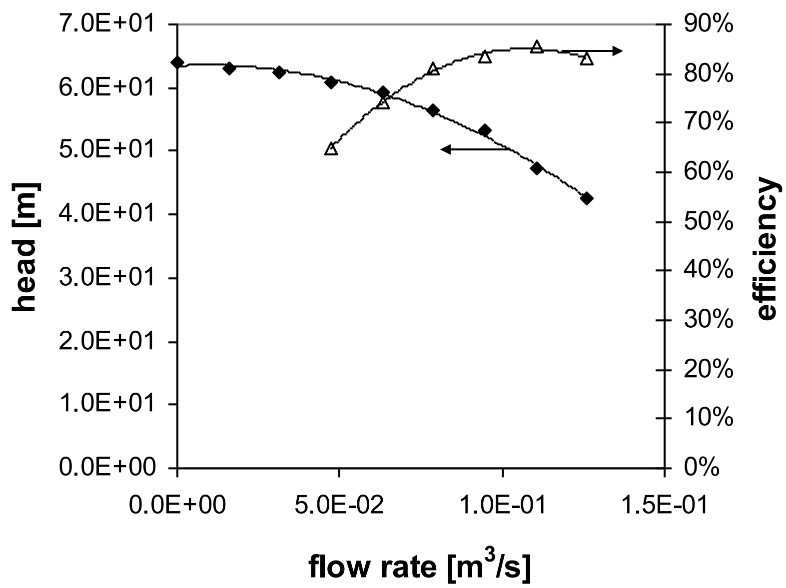
1. Decrease the elevation of the pump inlet so that $z_1 - z_3$ increases.
2. Decrease the losses from 1 to 3 by:
   a. decreasing the pipe length from 1 to 3 and
   b. using a rounded inlet into the pipe.

Note that increasing the pipe diameter from 1 to 3 or changing the pipe material from 1 to 3 might be difficult to implement and would also change the system operating point. Although they would be difficult to implement, increasing the pressure in tank 1 or decreasing the flow temperature to decrease the vapor pressure would also act to increase NPSHA.

Consider the pipe/pump system shown below in which water (with a density of 1.0E3 kg/m^3 and dynamic viscosity of 1.3E-3 Pa·s) is pumped from tank A to tank B.



The pump to be used in the system has the following pump performance curve.



Curve fits to the pump performance data are given below:

H [m] = (-1.5E3 s^2/m^5) Q^2 + (2.8E1 s/m^2) Q + (6.3E1 m)

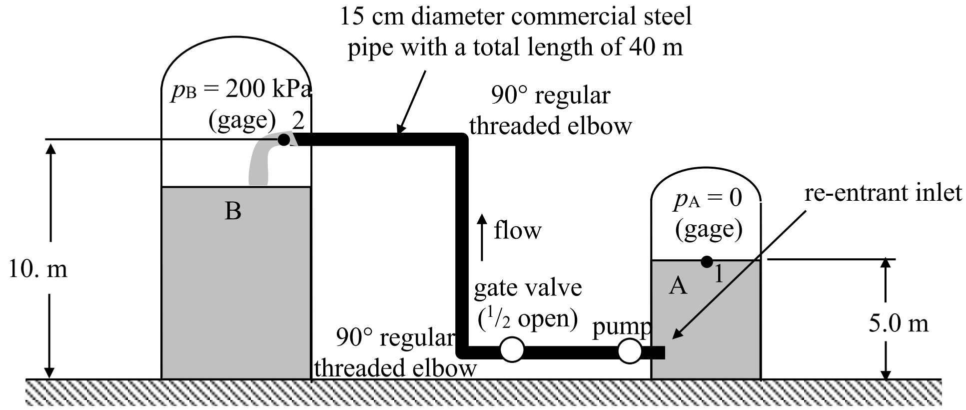
η_P = (-5.6E1 s^2/m^6) Q^2 + (1.2E1 s/m^3) Q + (2.1E-1)

a. Determine the operating point for the system.

b. Is the given pump efficient for this application? Explain your answer.

c. Do you anticipate that cavitation in the pump will be an issue? Explain your answer.

SOLUTION:



Apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the end of the pipe leading into tank B (point 2).

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12} \quad (1)$$

where

$p_1 = p_A = 0$ (gage) and $p_2 = p_B = 2.0\text{E}5$ Pa (gage) (given)

$\bar{V}_1 \approx 0$ (large tank)

$\bar{V}_2 = \bar{V}_P = \dfrac{Q}{\frac{\pi}{4}D^2}$ and $\alpha_2 \approx 1$ (assuming turbulent flow)

$z_1 = 5.0\text{E}0$ m and $z_2 = 1.0\text{E}1$ m (given)

$$H_{L,12} = f\left(\frac{L}{D}\right)\frac{\bar{V}_P^2}{2g}+K_{\substack{\text{re-entrant}\\\text{inlet}}}\frac{\bar{V}_P^2}{2g}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}\frac{\bar{V}_P^2}{2g}+2K_{\substack{90^\circ\ \text{threaded}\\\text{elbow}}}\frac{\bar{V}_P^2}{2g}$$

$$H_{L,12} = \left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\\text{inlet}}}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}+2K_{\substack{90^\circ\ \text{threaded}\\\text{elbow}}}\right]\frac{\bar{V}_P^2}{2g} \quad (2)$$

(Note that there are no exit losses at point 2.)

The friction factor, *f*, is determined from the Moody chart using the Reynolds number in the pipe, Re, and the relative roughness, ε/D. Since the Reynolds number is unknown at this point (since the flow rate and hence velocity are unknown), assume that the flow occurs in the fully rough zone. The pipe has a roughness of 0.9 mm. Hence:

$$\frac{\varepsilon}{D} = \frac{(9.0\text{E-4 m})}{(1.5\text{E-1 m})} = 6.0\text{E-3}$$

$$f = 3.2\text{E-2}$$

Hence, the major loss coefficient for the system is:

$$K_{\text{major}} = f\left(\frac{L}{D}\right) = (3.2\text{E-2})\left(\frac{4.0\text{E1 m}}{1.5\text{E-1 m}}\right) = 8.6\text{E0}$$

The minor loss coefficients are found from minor loss tables to be:

$$K_{\substack{\text{re-entrant}\\ \text{inlet}}} = 8.0\text{E-1}$$

$$K_{\substack{\text{half open}\\ \text{gate valve}}} = 2.1\text{E0}$$

$$K_{\substack{90^\circ\ \text{threaded}\\ \text{elbow}}} = 1.5\text{E0}$$

Re-arrange Eqn. (1) to solve for $H_{S,12}$ and substitute the values given above.

$$H_{S,12} = \frac{p_2 - p_1}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} - \alpha_1 \frac{\bar{V}_1^2}{2g} + z_2 - z_1 + H_{L,12}$$

$$= \frac{p_2}{\rho g} + \frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2} + z_2 - z_1 + H_{L,12}$$

$$= \frac{(2.0\text{E5 Pa})}{(1.0\text{E3 kg/m}^3)(9.8\text{E0 m/s}^2)} + \frac{Q^2}{2(9.8\text{E0 m/s}^2)\left[\frac{\pi}{4}(1.5\text{E-1 m})^2\right]^2} + (1.0\text{E1 m}) - (5.0\text{E0 m})$$

$$+\left[8.6\text{E0} + 8.0\text{E-1} + 2.1\text{E0} + 2(1.5\text{E0})\right]\frac{Q^2}{2(9.8\text{E0 m/s}^2)\left[\frac{\pi}{4}(1.5\text{E-1 m})^2\right]^2}$$

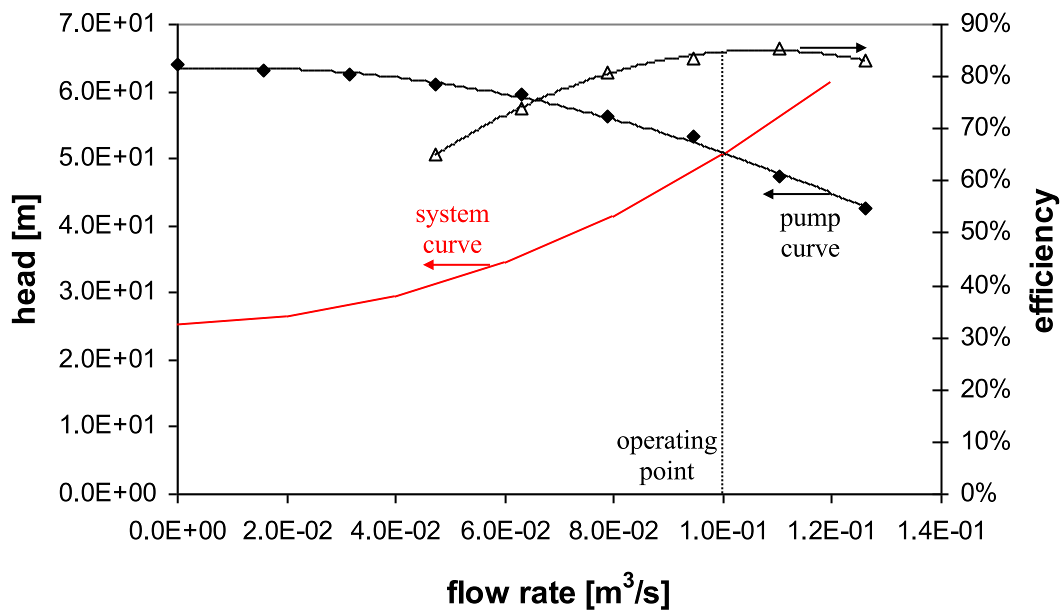
$$H_{S,12} = (2.5\text{E1 m}) + (2.5\text{E3 s}^2/\text{m}^5)Q^2$$

Equate the system head curve with the given pump head curve to determine the operating point.

$$\underbrace{(2.5\text{E1 m}) + (2.5\text{E3 s}^2/\text{m}^5)Q^2}_{\text{system curve}} = \underbrace{(6.3\text{E1 m}) + (2.8\text{E1 s/m}^2)Q + (\text{-}1.5\text{E3 s}^2/\text{m}^5)Q^2}_{\text{pump curve}}$$

$$(4.0\text{E3 s}^2/\text{m}^5)Q^2 + (\text{-}2.8\text{E1 s/m}^2)Q + (\text{-}3.8\text{E1 m}) = 0$$

$$\boxed{Q = 1.0\text{E-1 m}^3/\text{s}}$$



The velocity corresponding to this flow rate is:

$$\bar{V}_2 = \frac{Q}{\frac{\pi}{4}D^2} = 5.7\text{E}0 \text{ m/s}$$

and the corresponding Reynolds number is:

$$\text{Re} = \frac{\bar{V}_2 D}{\nu} = 6.5\text{E}5$$

Hence, the assumption of fully rough turbulent flow is ok.

The pump efficiency at this flow rate is found using the given pump efficiency curve.

$$\eta_P = \left(\text{-}5.6\text{E}1 \text{ s}^2/\text{m}^6\right)Q^2 + \left(1.2\text{E}1 \text{ s/m}^3\right)Q + \left(2.1\text{E-}1\right)$$

$$\eta_P = 85\%$$

Since this efficiency is very close to the best efficiency point, this is an efficient pump for this application.

To determine if cavitation will occur in the pump, we would need to compare the NPSH available at the pump inlet to the NPSH required by the pump (need NPSHA > NPSHR to avoid cavitation). The NPSHA can be determined by apply the Extended Bernoulli Equation from point 1 to a point at the inlet of the pump and using the definition of NPSH.

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_{\text{inlet}} = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_{L,1-\text{inlet}} + H_{S,1-\text{inlet}}$$

$$NPSH \equiv \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g}\right)_{\text{inlet}} - \frac{p_v}{\rho g}$$

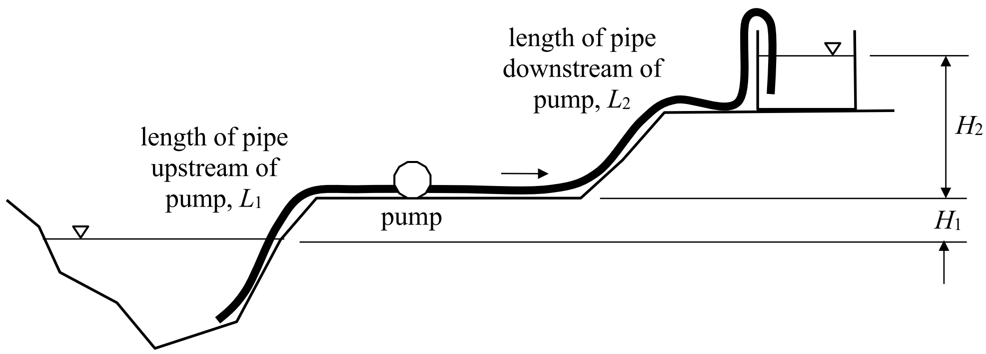
where

$$\alpha_{\text{inlet}} \approx 1 \qquad p_1 = p_{\text{atm}} \qquad H_{S,1-\text{inlet}} = 0$$

$$NPSHA = \frac{p_{\text{atm}} - p_v}{\rho g} + z_1 - z_2 - H_{L,1-\text{inlet}}$$

Since $p_{\text{atm}} > p_v$, $z_1 > z_2$, and $H_{L,1\text{-inlet}}$ will be relatively small since there are few loss mechanisms occurring upstream of the pump, cavitation in the pump will most likely not be an issue.

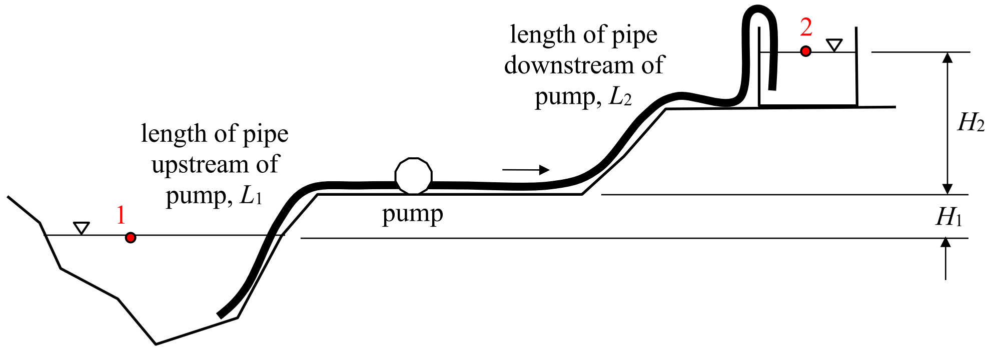
Consider the pipe system containing a pump shown in the figure below. The fluid being pumped from the lake to the tank is water (density = 1000 kg/m^3, kinematic viscosity = 1.0*10^{-6} m^2/s).



diameter of both lengths of pipe, $D_1 = D_2 = 10$ cm
length of pipe upstream of pump, $L_1 = 5$ m
length of pipe downstream of pump, $L_2 = 15$ m
roughness of both lengths of pipe, $\varepsilon_1 = \varepsilon_2 = 1.5*10^{-4}$ m
total minor loss upstream of pump, $K_{\text{minor},1} = 1.0$
total minor loss downstream of pump, $K_{\text{minor},2} = 2.0$
$H_1 = 3$ m
$H_2 = 10$ m
pump head rise curve: H [m] = $(-1.5*10^3\ \text{s}^2/\text{m}^5)Q^2 + (2.8*10^1\ \text{s/m}^2)Q + (6.3*10^1\ \text{m})$
pump efficiency curve: $\eta = (-5.6*10^1\ \text{s}^2/\text{m}^6)Q^2 + (1.2*10^1\ \text{s/m}^3)Q + (2.1*10^{-1})$

a. Determine the operating flow rate for the system.
b. What power must be supplied to the pump by the motor to operate at the flow rate found in part (a)?

SOLUTION:
To determine the operating flow rate, first determine the system head curve by applying the extended Bernoulli equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \ , \tag{1}$$

where

$p_1 = p_{atm}$ $\qquad$ $p_2 = p_{atm}$

$\bar{V}_1 \approx 0$ $\qquad$ $\bar{V}_2 \approx 0$

$z_1 = -H_1$ $\qquad$ $z_2 = H_2$

$$\begin{aligned} H_L &= \sum_i K_i \frac{\bar{V}_i^2}{2g} \\ &= \left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor,1}}+K_{\text{minor,2}}\right]\frac{\bar{V}_p^2}{2g}=\left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor,1}}+K_{\text{minor,2}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2} \ . \end{aligned} \tag{2}$$

The friction factor may be found from the Moody diagram. Since the flow rate is unknown, try assuming that the flow is in the fully turbulent region of the Moody diagram (this assumption will need to be verified). In this region, the friction factor is only a function of the pipe's relative roughness,

$$\frac{\varepsilon}{D}=\frac{1.5*10^{-4}\text{ m}}{0.10\text{ m}}=1.5*10^{-3} \tag{3}$$

From the Moody diagram, $f = 0.022$.

Combining these relations gives the system head curve,

$$H_{S,\text{system}}=(z_2-z_1)+\left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor,1}}+K_{\text{minor,2}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}=s_0+s_2Q^2 \ , \tag{4}$$

where,

$$s_0 = z_2 - z_1 \ \text{, and} \tag{5}$$

$$s_2=\frac{1}{2g\left(\frac{\pi}{4}D^2\right)^2}\left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor,1}}+K_{\text{minor,2}}\right] \ . \tag{6}$$

Determine the operating point by equating the system head curve to the pump head curve,

$$s_0+s_2Q^2=p_0+p_1Q+p_2Q^2 \ , \tag{7}$$

$$(p_2-s_2)Q^2+p_1Q+(p_0-s_0)=0 \ , \tag{8}$$

$$Q=\frac{-p_1\pm\sqrt{p_1^2-4(p_2-s_2)(p_0-s_0)}}{2(p_2-s_2)} \ . \tag{9}$$

Using the given data,

s_0 = 13 m
s_2 = $6.08*10^3$ s^2/m^5
p_0 = $6.30*10^1$ m
p_1 = $2.80*10^1$ s/m^2
p_2 = $-1.50*10^3$ s^2/m^5

$\boxed{Q = 8.3*10^{-2}\ \text{m}^3/\text{s}}$

Check the Reynolds number assumption of fully turbulent flow,

$$\bar{V} = \frac{Q}{\frac{\pi}{4}D^2} \Rightarrow \bar{V} = 10.6\ \text{m/s}\ , \tag{10}$$

$$\text{Re} = \frac{\bar{V}D}{\nu} \Rightarrow \text{Re} = 1.1*10^6 \tag{11}$$

Checking the Moody diagram shows that the flow is in the fully rough zone for this Reynolds number and relative roughness. Thus, our assumption of fully rough flow was a good one.

The power input to the fluid by the pump at these conditions is,

$$\dot{W}_{\substack{\text{into}\\ \text{fluid}}} = \rho QgH\ , \tag{12}$$

where H = 55.0 m at the operating flow rate of $8.3*10^{-2}$ m^3/s (found using either the system or pump head curves). Hence,

$$\Rightarrow \dot{W}_{\substack{\text{into}\\ \text{fluid}}} = 44.8\ \text{kW}\ . \tag{13}$$

Since the pump isn't 100% efficient, the power that must be supplied to the pump is,

$$\dot{W}_{\substack{\text{into}\\ \text{pump}}} = \frac{\dot{W}_{\substack{\text{into}\\ \text{fluid}}}}{\eta} \Rightarrow \boxed{\dot{W}_{\substack{\text{into}\\ \text{pump}}} = 54.6\ \text{kW}}. \tag{14}$$

where the pump efficiency at the operating point is η = 82% (using the given efficiency curve for the pump, $\eta = (-5.6*10^1\ \text{s}^2/\text{m}^6)Q^2 + (1.2*10^1\ \text{s/m}^3)Q + (2.1*10^{-1})$).

12.9. Review Questions

(1) What class of pumps generates large Δps? What class of pumps generates large Q?
(2) What is the fundamental principle for the operation of positive displacement pumps?
(3) Describe the fundamental principle behind the operation of dynamic pumps.
(4) What information is presented on a pump performance plot?
(5) Why do some pump performance plots have different impeller diameter curves?
(6) What is meant by the "Best Efficiency Point"?
(7) Can a pump operate at a flow rate different from the BEP?
(8) Assuming the same diameter inlet and outlet pipe, how does the average flow speed change across a pump? How does the pressure change?
(9) Describe the various components in a centrifugal pump.
(10) What is meant by the "shut-off head"?
(11) How does one determine the operating flow rate for a pump in a pipe system?
(12) What is the definition of NPSH and how is it used?
(13) What is the difference between NPSHA and NPSHR?