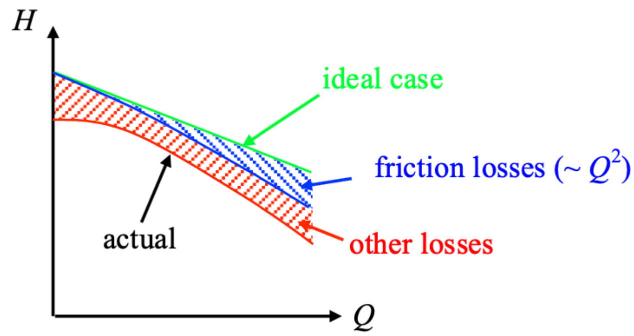


Figure 12.27. Effects of losses on the theoretical head rise across an idealized centrifugal pump.

(5) Pump efficiency is defined as,

$$\eta_P \coloneqq \frac{\dot{m}gH}{\omega T}, \tag{12.17}$$

where $\dot{m}gH$ is the known as the water or hydraulic horsepower, i.e., the power that makes it into the fluid, and ωT is known as the brake horsepower, i.e., the power put into the pump.

(a) Typical pump efficiencies are between $\eta_P = 60\% - 85\%$.

(b) As pump size decreases, the ratio of surface area to volume increases $\Longrightarrow$ frictional losses increase and the pump efficiency decreases.

(6) The head rise, brake horsepower, and efficiency for a pump are provided in a pump performance plot, as shown in Figure 12.28. The top figure shows the Best Efficiency Point (BEP) for the pump, which is the flow rate at which the efficiency is a maximum. Ideally, the pump would operate at this flow rate since it has the highest efficiency of converting the shaft work into an increase in fluid pressure, but the pump can operate at other flow rates.

The top figure also shows the shut-off head (aka dead-head), which is the head rise across the pump when there is no flow through the pump. One would have this situation if a valve placed downstream of the pump was closed, but the pump was still operating. The pump would continue to do work on the fluid, but that work would go mainly into increasing the fluid pressure (and the temperature) rather than increasing kinetic energy.

The bottom chart in Figure 12.28 shows a different, and more common, pump performance plot style. Here, there are three different head curves: one for three different impeller diameters. In practice, it's common to make available several pump impellers that can fit within the same pump housing to expand the range of operation of a pump. There are three different head curves for this particular pump housing corresponding to the three different impellers. The solid lines with numbers along the top are the efficiency curves and the dashed lines with numbers along the bottom right are the brake horsepower curves. The dashed line at the very bottom with the label "NPSHR" is discussed in the next section.

Data measured during tests of a centrifugal pump at 3500 rpm are given in the table below:

Parameter	**Inlet Section**	**Outlet Section**
gage pressure, p [kPa]	95.2	412
elevation above datum, z [m]	1.25	2.75
avg speed of flow, V [m/s]	2.35	3.62

The working fluid is water. The flow rate is 11.5 m^3/hr and the torque applied to the pump shaft is 3.68 N·m. Evaluate the head rise across the pump, the hydraulic power input to the fluid, and the pump efficiency. If the electric motor efficiency is 85%, calculate the electric power requirement.

SOLUTION:

First determine the total heads at the inlet and outlet to the pump. The total head is given by,

$$H = \frac{p}{\rho g} + \alpha \frac{\bar{V}^2}{2g} + z \,. \tag{1}$$

Using the given data (and noting that $D = [Q/(^{\pi}/_{4}V)]^{1/2}$ and $\text{Re} = VD/\nu$ so that $\text{Re}_{\text{inlet}} = 9.78\text{e}4$ and $\text{Re}_{\text{outlet}} = 1.21\text{e}5 \Rightarrow \alpha_{\text{inlet}} \approx \alpha_{\text{outlet}} \approx 1$) and using absolute pressures when calculating the head:

$H_{\text{inlet}} = 21.6$ m
$H_{\text{outlet}} = 55.7$ m
$\boxed{\Delta H = 34.1 \text{ m}}$

The hydraulic power input to the fluid is given by,

$$\dot{W}_{\text{fluid}} = \dot{m}g\left(H_{\text{outlet}} - H_{\text{inlet}}\right), \tag{2}$$

$\boxed{\dot{W}_{\text{fluid}} = 1.07 \text{ kW}}$

The power required to drive the pump is,

$$\dot{W}_{\text{shaft}} = T\omega \,, \tag{3}$$

$\dot{W}_{\text{shaft}} = 1.35$ kW

The efficiency of the pump is given by,

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{fluid}}}{\dot{W}_{\text{shaft}}}, \tag{4}$$

$\boxed{\eta_{\text{pump}} = 79.4\%}$

The electric power required is,

$$\dot{W}_{\substack{\text{required}\\ \text{for motor}}} = \dot{W}_{\text{shaft}} / \eta_{\text{motor}} \,, \tag{5}$$

$\boxed{\dot{W}_{\substack{\text{required}\\ \text{for motor}}} = 1.59 \text{ kW}}$

Data measured during tests of a centrifugal pump at 3500 rpm are given in the table below.

Parameter	Inlet Section	Outlet Section
gage pressure, p [kPa]	85.2	412
elevation above datum, z [m]	1.25	2.75
avg speed of flow, V [m/s]	2.35	3.62

The flow rate is 11.5 m^3/hr and the torque applied to the pump shaft is 3.68 N·m. Evaluate the total heads at the pump inlet and outlet, the hydraulic power input to the fluid, and the pump efficiency. Specify the electric motor size needed to drive the pump. If the electric motor efficiency is 85%, calculate the electric power requirement.

SOLUTION:
At the outlet,

$$H_{\text{outlet}} = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_{\text{outlet}}, \tag{1}$$

where,

p_{outlet} = 85.2 kPa (gage),
ρ = 1000 kg/m^3,
g = 9.81 m/s^2,
$\alpha_{\text{outlet}} \approx 1$ (assuming turbulent flow),
$\bar{V}_{\text{outlet}}$ = 2.35 m/s,
z_{outlet} = 1.25 m,
=> $\boxed{H_{\text{outlet}} = 10.2\text{ m}}$.

At the inlet,

$$H_{\text{inlet}} = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_{\text{inlet}}, \tag{2}$$

where,

p_{inlet} = 412 kPa (gage),
ρ = 1000 kg/m^3,
g = 9.81 m/s^2,
$\alpha_{\text{inlet}} \approx 1$ (assuming turbulent flow),
$\bar{V}_{\text{inlet}}$ = 3.62 m/s,
z_{inlet} = 2.75 m,
=> $\boxed{H_{\text{inlet}} = 45.4\text{ m}}$.

The rate at which energy is put into the fluid is,

$$\dot{W}_{\text{on fluid}} = \rho g Q H, \tag{3}$$

where,

Q = 11.5 m^3/hr = 3.19*10^{-3} m^3/s,
$H = H_{\text{outlet}} - H_{\text{inlet}}$ = 35.2 m (head rise across the pump),
=> $\boxed{\dot{W}_{\text{on fluid}} = 1100\text{ W}}$.

The rate at which energy is put into the pump is,

$$\dot{W}_{\text{on pump}} = \omega T, \tag{4}$$

where,

ω = 3500 rpm = 366.5 rad/s,
T = 3.68 N.m,
=> $\dot{W}_{\text{on pump}}$ = 1349 W.

The pump efficiency is,

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{on fluid}}}{\dot{W}_{\text{on pump}}}, \tag{5}$$

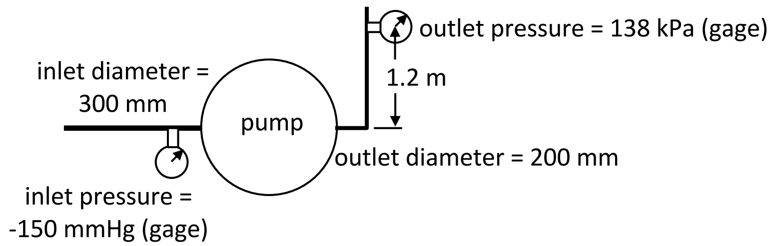
Using the previously calculated values, $\boxed{\eta_{\text{pump}} = 0.82 = 82\%.}$

The electric motor size required for operation is $\boxed{1350\text{ W} = 1.8\text{ hp}}$.

If the <u>motor</u> is 85% efficient, then we need to supply the motor with,

$$\dot{W}_{\text{on motor}} = \frac{\dot{W}_{\text{on pump}}}{\eta_{\text{motor}}} \implies \boxed{\dot{W}_{\text{on motor}} = 1590\text{ W} = 2.1\text{ hp}}. \tag{6}$$

Brine, with a specific gravity of 1.2, passes through an 85% efficient pump at a flow rate of 125 L/s. The centerlines of the pump's 300 mm diameter inlet and 200 mm diameter outlet are at the same elevation. The inlet suction gage pressure is 150 mm of mercury (specific gravity of 13.6) below atmospheric pressure. The discharge pressure is measured 1.2 m above the centerline of the pump's outlet and indicates 138 kPa (gage). Neglecting losses in the pipes, what is the input power to the pump?



SOLUTION:

The power into the pump may be found from the head rise across the pump, the flow rate through the pump, the brine properties, and the pump efficiency,

$$\eta = \frac{\dot{W}_{\text{into fluid}}}{\dot{W}_{\text{into pump}}} \Rightarrow \dot{W}_{\text{into pump}} = \frac{\dot{W}_{\text{into fluid}}}{\eta}, \tag{1}$$

where,

$$\dot{W}_{\text{into fluid}} = \rho Q g H, \tag{2}$$

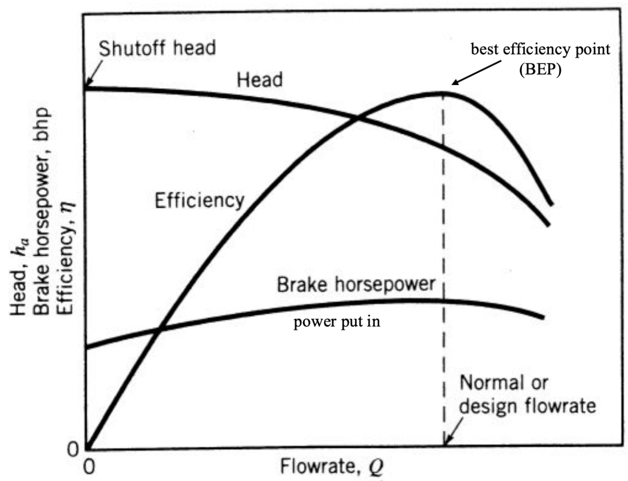
and,

$$H = \left(\frac{p_{\text{outlet}} - p_{\text{inlet}}}{\rho g}\right) + \left(\frac{V_{\text{outlet}}^2 - V_{\text{inlet}}^2}{2g}\right) + \left(z_{\text{outlet}} - z_{\text{inlet}}\right). \tag{3}$$

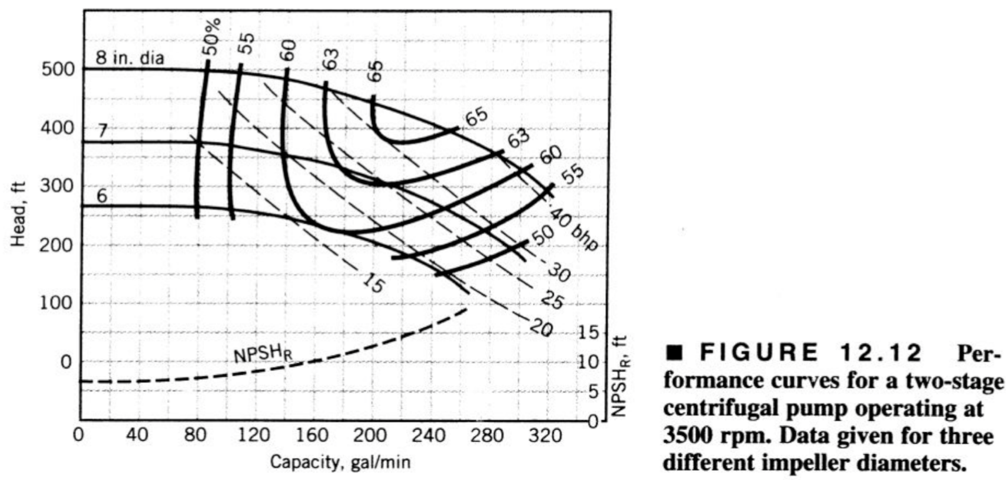
Using the given data:

η = 85%
ρ = (1.2)(1000 kg/m^3) = 1200 kg/m^3 (4)
g = 9.81 m/s^2
$z_{\text{outlet}} - z_{\text{inlet}}$ = 1.2 m
p_{outlet} = 138*10^3 Pa (gage)
p_{inlet} = $\rho_{\text{Hg}} g h$ = -(13.6)(1000 kg/m^3)(9.81 m/s^2)(0.150 m) = -20000 Pa (gage) (5)
Q = 125 L/s = 0.125 m^3/s
D_{outlet} = 0.200 m
D_{inlet} = 0.300 m
V_{outlet} = $Q/\left(\frac{\pi}{4} D_{\text{outlet}}^2\right)$ = 3.98 m/s (6)
V_{outlet} = $Q/\left(\frac{\pi}{4} D_{\text{inlet}}^2\right)$ = 1.77 m/s (7)

$\Rightarrow H$ = 15.3 m
$\Rightarrow \dot{W}_{\text{into fluid}} = 22.5\text{ kW}$
$\therefore \boxed{\dot{W}_{\text{into pump}} = 26.4\text{ kW}}$



(A) This figure is from Munson, B.R., Young, D.F., and Okiishi, T.H., *Fundamentals of Fluid Mechanics*, 3rd ed., Wiley.



(B) This figure is from Munson, B.R., Young, D.F., and Okiishi, T.H., *Fundamentals of Fluid Mechanics*, 3rd ed., Wiley.

Figure 12.28. Two examples of pump performance plots.

12.5. Net Positive Suction Head (NPSH)

Along the suction side of the impeller blade near the pump inlet are regions of low pressure (Figure 12.29).

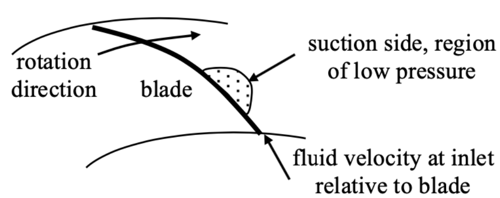


Figure 12.29. A sketch showing the region of low pressure on the suction side of an impeller blade.