solution procedure is consistent. If not, then the laminar flow assumption was incorrect and a turbulent flow assumption should be made and the problem re-solved.

11.7. Pipe Systems

Pipe flow systems can be classified as being of one of three types:

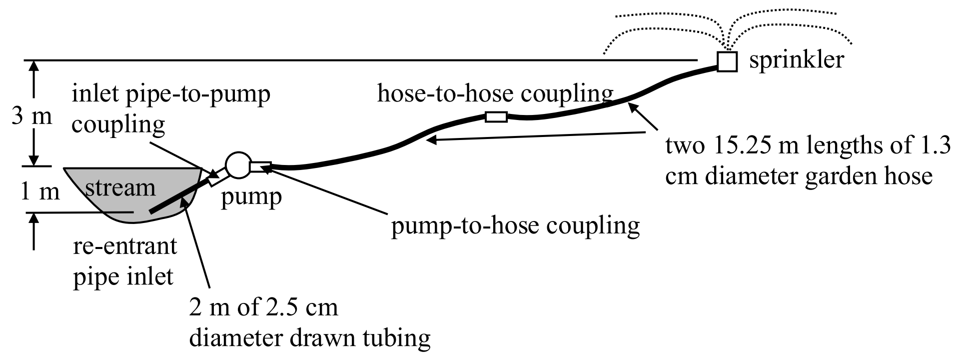
- Type I: The desired flow rate is specified and the required pressure drop must be determined.
- Type II: The desired pressure drop is specified and the required flow rate must be determined.
- Type III: The desired flow rate and pressure drop are specified and the required pipe diameter must be determined.

Type I pipe systems are the easiest to solve. Since the flow velocity and diameter are known, calculation of the major loss coefficient, and the friction factor in particular, is straightforward. Type II and Type III problems are more challenging to solve since the friction factor is unknown. These types of pipe systems usually require iteration to solve.

Notes:

(1) There is no unique iterative scheme that must be used to solve Type II and Type III pipe flow problems. Different people may propose different algorithms. In addition, there is no guarantee that a particular iterative scheme will converge to a solution.
(2) When using an iterative scheme, choose an initial flow rate or diameter that is reasonable. Don't start with an exceedingly small or large value. For example, for a Type II pipe system, choose a starting flow rate that corresponds to the fully turbulent zone region.
(3) It's often worthwhile to first assume that a Type II and Type III flow system is operating in the fully rough zone of the Moody plot. Using this assumption will often avoid the need for iteration. However, one must verify at the end of the solution that the assumption of fully rough flow was correct. If not, then an iterative solution should be considered with the fully rough zone conditions used as a starting point for the iterations.

A homeowner plans to pump water from a stream in their backyard to water their lawn. A schematic of the pipe system is shown in the figure.



Details of the system are given in the following table. The design flow rate for the system is $2.5*10^{-4}$ m^3/s.

Item	**Value**
water density	1000 kg/m^3
water dynamic viscosity	$1*10^{-3}$ kg/(m.s)
inlet pipe length	2 m
inlet pipe diameter	2.5 cm
inlet pipe material	drawn tubing
hose length	two 15.25 m lengths
hose diameter	1.3 cm
hose roughness	smooth
pipe inlet loss coefficient	0.8
inlet pipe-to-pump coupling loss coefficient	0.1
pump-to-hose coupling loss coefficient	0.2
hose-to-hose coupling loss coefficient	0.5
pressure drop across the sprinkler	210 kPa at $2.5*10^{-4}$ m^3/s flow rate
sprinkler nozzle exit diameter	4 mm

For the following questions, circle the answer that is most correct.

1. What is the loss coefficient for the sprinkler at design conditions? Base the sprinkler loss coefficient on the velocity just upstream of the sprinkler.
2. What is the friction factor for the hose?
3. What is the velocity head, including the kinetic energy correction factor, at the sprinkler exit?

For the next two questions, assume that the loss coefficient for the sprinkler is 100 and the friction factor for the hose is 0.01.

4. What is the total *minor* head loss in the system?
5. What is the total *major* head loss in the system?

For the next question, assume that the velocity head at the sprinkler exit, including the kinetic energy correction factor, is 10 m and the total head loss is 10 m.

6. What is the shaft head required to operate the system at the design flow rate?

For the next question, assume that the shaft head required to operate the system at the design flow rate is 100 m.

7. What power must be supplied to the pump if the pump is 65% efficient?

SOLUTION:

1. The loss coefficient for the sprinkler may be found using the definition of a loss coefficient.

$$K_{\text{sprinkler}} = \frac{\Delta p_{\text{sprinkler}}}{\frac{1}{2}\rho \bar{V}_{\text{hose}}^2} \tag{1}$$

where

$$\bar{V}_{\text{hose}} = \frac{Q}{\frac{\pi}{4}D_{\text{hose}}^2} \tag{2}$$

Using the given parameters,

$$\bar{V}_{\text{hose}} = 1.88 \text{ m/s} \quad \text{and} \quad \boxed{K_{\text{sprinkler}} = 118} \tag{3}$$

2. The friction factor for the hose may be found using the Moody plot. The Reynolds number for the flow in the hose is:

$$\text{Re}_D = \frac{\rho \bar{V}_{\text{hose}} D_{\text{hose}}}{\mu} \tag{4}$$

Using the given parameters, the hose Reynolds number is $\text{Re}_D = 24500$. From the Moody chart and using the smooth curve (we're told to consider the hose to be "smooth"), the friction factor for the hose is $\boxed{f_{\text{hose}} = 0.0244}$.

3. The velocity head, including the kinetic energy correction factor, at the sprinkler's exit is:

$$\left.\alpha \frac{\bar{V}^2}{2g}\right|_{\substack{\text{sprinkler}\\\text{exit}}} \tag{5}$$

where

$$\bar{V}_{\substack{\text{sprinkler}\\\text{exit}}} = \frac{Q}{\frac{\pi}{4}D_{\substack{\text{sprinkler}\\\text{exit}}}^2} \Rightarrow \bar{V}_{\substack{\text{sprinkler}\\\text{exit}}} = 19.9 \text{ m/s} \tag{6}$$

and

$$\alpha = 1 \tag{7}$$

since

$$\text{Re}_D = \frac{\rho \bar{V}_{\substack{\text{sprinkler}\\\text{exit}}} D_{\substack{\text{sprinkler}\\\text{exit}}}}{\mu} \Rightarrow \text{Re}_D = 79600 > 2300 \Rightarrow \text{Turbulent flow at the exit!} \tag{8}$$

Thus,

$$\boxed{\left.\alpha \frac{\bar{V}^2}{2g}\right|_{\substack{\text{sprinkler}\\\text{exit}}} = 20.2 \text{ m}} \tag{9}$$

4. The total minor head loss for the system includes minor losses at the pipe inlet, the pump couplings, the hose coupling, and the sprinkler.

$$H_{L,\text{minor}} = \left[K_{\text{inlet}} + K_{\substack{\text{inlet pipe-pump}\\\text{coupling}}}\right]\frac{\bar{V}_{\text{pipe}}^2}{2g} + \left[K_{\substack{\text{pump-hose}\\\text{coupling}}} + K_{\substack{\text{hose-hose}\\\text{coupling}}} + K_{\text{sprinkler}}\right]\frac{\bar{V}_{\text{hose}}^2}{2g} \tag{10}$$

Using the given parameters,

$$\boxed{H_{L,\text{minor}} = 21.5 \text{ m}}$$

5. The total major head loss for the system includes the major losses in the inlet pipe and the hoses.

$$H_{L,\text{major}} = \left. f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}\right|_{\text{pipe}} + \left. f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}\right|_{\text{hoses}} \tag{11}$$

$$\boxed{H_{L,\text{major}} = 10.4 \text{ m}}$$

6. The required shaft head may be found by applying the Extended Bernoulli Equation from the free surface of the stream to the outlet of the sprinkler.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \tag{12}$$

$$H_S=\left(\frac{p_2-p_1}{\rho g}\right)+\left(\alpha\frac{\bar{V}_2^1}{2g}-\alpha\frac{\bar{V}_1^1}{2g}\right)+\left(z_2-z_1\right)+H_L \tag{13}$$

where

$$p_1=p_2=p_{\text{atm}} \tag{14}$$

$$\bar{V}_1\approx 0 \tag{15}$$

$$\bar{V}_2=\bar{V}_{\substack{\text{sprinkler}\\\text{exit}}} \tag{16}$$

$$H_L=H_{L,\text{minor}}+H_{L,\text{major}} \tag{17}$$

Using the given parameters, $\boxed{H_S = 55.1\text{ m}}$.

7. The power that must be supplied to the pump with the given efficiency is:

$$\dot{W}_{\substack{\text{into}\\\text{pump}}}=\frac{\dot{W}_{\substack{\text{into}\\\text{water}}}}{\eta}=\frac{\dot{m}gH_S}{\eta}=\frac{\rho QgH_S}{\eta} \tag{18}$$

where, using the given parameters,

$$\boxed{\dot{W}_{\substack{\text{into}\\\text{pump}}}=210\text{ W}}$$

It rains during the construction of a building and water fills a recently excavated pit to a depth 0.5 m. In order to continue construction, the water must first be pumped out of the pit. A hose with a length of 50 m, a diameter of $2.5*10^{-2}$m, and a surface roughness of $5.0*10^{-5}$m is attached to a pump. Note that the kinematic viscosity of the water is $1.005*10^{-6}$ m^2/s and the density is 1000 kg/m^3.

a. If the pump is placed at the pit's surface (figure A), what is the maximum depth of the pit, H, for which water can be pumped out at a velocity of 1 m/s without causing cavitation in the pipe? The vapor pressure of water for the current temperature is 2.337 kPa (abs) and atmospheric pressure is 101 kPa (abs).

b. If the pump is placed at the bottom of the pit (figure B), what is the maximum depth of the pit, H, for which water can be pumped out at a velocity of 1 m/s? Assume that the pump supplies a power of 200 W to the fluid.

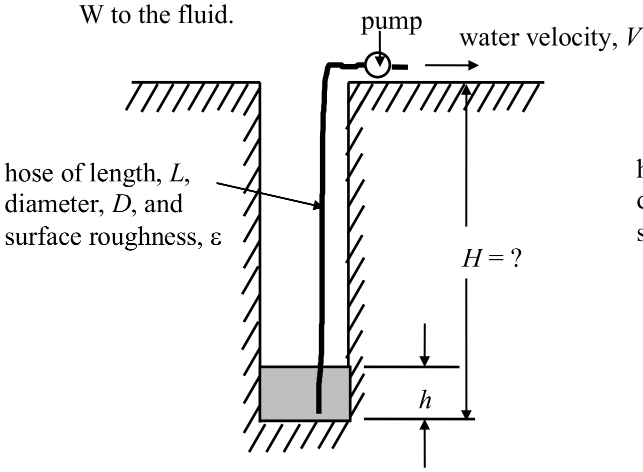


Figure A

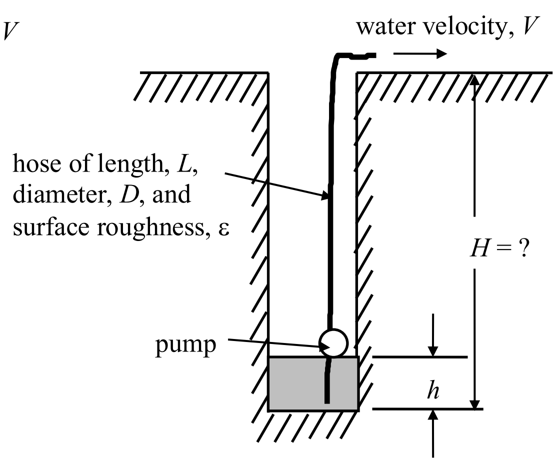
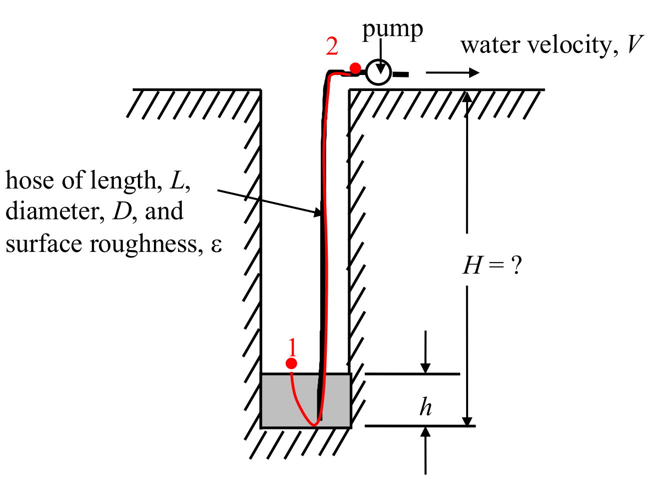


Figure B

SOLUTION:



Apply the Extended Bernoulli Equation from point 1 to point 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\overline{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\overline{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2} \quad (1)$$

where,

$p_1 = p_{atm}$,

$p_2 = p_v$ (point 2 is located just before the pump; the lowest pressure that can be reached is vapor pressure),

$z_1 = h$,

$z_2 = H = ?$,

$\bar{V}_1 \approx 0$,

$\bar{V}_2 = V$,

$\alpha_2 \approx 1$ (assuming turbulent flow),

$H_{S,1\to2} = 0$ (there's no pump between points 1 and 2),

$$H_{L,12} = \sum_i K_i \frac{\bar{V}_i^2}{2g} = \left(K_{\text{minor}} + K_{\text{major}}\right)\frac{\bar{V}^2}{2g}. \tag{2}$$

Now determine the flow Reynolds number,

$$\text{Re}_D = \frac{\bar{V}_2 D}{\nu} = \frac{(1\ \text{m/s})(2.5\text{e-}2\ \text{m})}{(1.005\text{e-}6\ \text{m}^2/\text{s})} = 25,000 \ \Rightarrow \text{ turbulent flow assumption is justified!} \tag{2}$$

Also determine the relative roughness of the pipe,

$$\frac{\varepsilon}{D} = \frac{5.0\text{e-}5\ \text{m}}{2.5\text{e-}2\ \text{m}} = 0.002 \tag{3}$$

Use the Moody chart to determine the pipe's friction factor,

$$\Rightarrow \underline{f = 0.0289} \tag{4}$$

The major loss coefficient, $K_{\text{major}} = f(L/D)$ is,

$K_{\text{major}} = 57.7$.

Since the major loss coefficient is so large, it's reasonable to neglect the minor loss coefficients.

Substituting into the EBE and simplifying,

$$\frac{p_v}{\rho g} + \frac{V^2}{2g} + H = \frac{p_{atm}}{\rho g} + h - K_{\text{major}}\frac{V^2}{2g}, \tag{6}$$

$$H = \frac{p_{atm} - p_v}{\rho g} + h - \left[1 + K_{\text{major}}\right]\frac{V^2}{2g}. \tag{7}$$

Using the given parameters,

$p_{atm} = 101*10^3$ Pa (abs),

$p_v = 2.337*10^3$ Pa (abs),

$\rho = 1000\ \text{kg/m}^3$,

$g = 9.81\ \text{m/s}^2$,

$h = 0.5$ m,

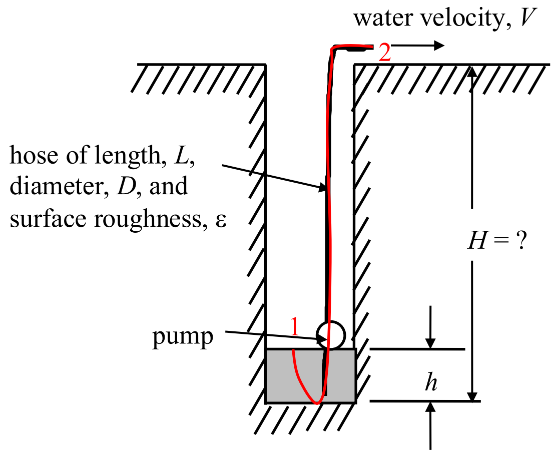
$K_{\text{major}} = 57.7$,

$V = 1$ m/s,

=> $\boxed{H = 7.57\ \text{m}}$.

The height is short because we're relying on atmospheric pressure to push the water up through the hose.

Now consider the case when the pump is in the pit.



Apply the Extended Bernoulli Equation from point 1 to point 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to2}+H_{S,1\to2} \quad (8)$$

where,

$p_1 = p_{atm}$,
$p_2 = p_{atm}$ (flow exits to the atmosphere),
$z_1 = h$,
$z_2 = H = ?$,
$\bar{V}_1 \approx 0$,
$\bar{V}_2 = V$,
$\alpha_2 \approx 1$ (turbulent flow, as before),
$H_{S,12} = \frac{P}{\dot{m}g}$ (there's a pump between points 1 and 2),
$H_{L,12} = \sum_i K_i \frac{\bar{V}_i^2}{2g} = K_{\text{major}} \frac{\bar{V}^2}{2g}$ (neglecting minor losses, as before) (9)

Substitute and solve for the height,

$$\frac{p_{atm}}{\rho g}+\frac{V^2}{2g}+H=\frac{p_{atm}}{\rho g}+h-K_{\text{major}}\frac{V^2}{2g}+\frac{P}{\dot{m}g}, \quad (10)$$

$$H=h-\left(1+K_{\text{major}}\right)\frac{V^2}{2g}+\frac{P}{\dot{m}g}. \quad (11)$$

Using the given values,

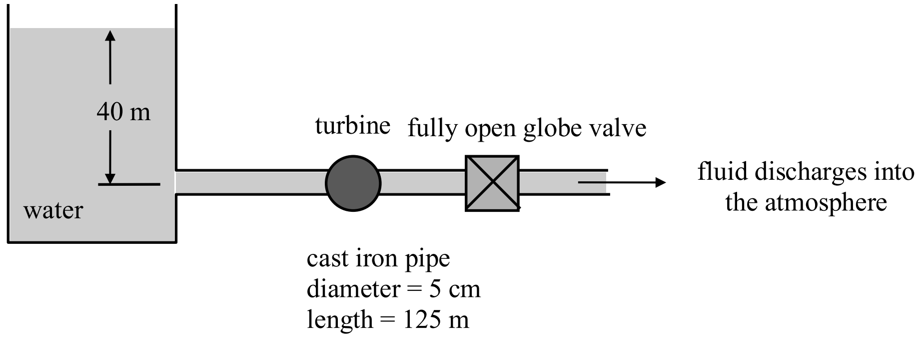
h = 0.5 m,
K_{major} = 57.7,
V = 1 m/s,
g = 9.81 m/s^2,
P = 200 W,
=> $\dot{m} = \rho V \frac{\pi D^2}{4} = 0.491$ kg/s,
=> $\boxed{H = 39.0 \text{ m}}$.

The height in part (b) is much higher than the height in part (a) because in part (b) the pump is used to increase the pressure beyond atmospheric pressure, which pushes the fluid up the hose.

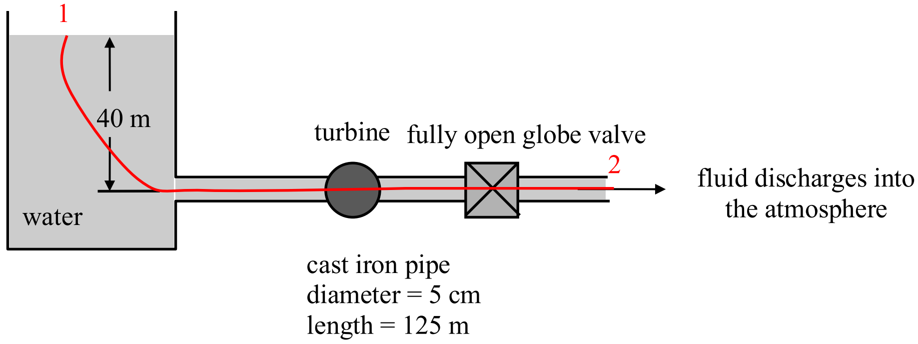
Following is python code used to perform the calculations.

```
# pipe_03.py

import numpy as np

def f_Haaland(Re, e_D):  # Calculate the friction factor using the Haaland formula
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

def f_Colebrook(Re, e_D):  # Calculate the friction factor using the Colebrook formula
    fprime = f_Haaland(Re, e_D)
    freldiff = 1
    tol = 0.001
    while (freldiff > tol):
        f = fprime
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2
        freldiff = np.absolute((fprime-f)/f)
    return f

# Initialize variable values.
g = 9.81 # m/s^2; gravitational acceleration
rho = 1000 # kg/m^3; water density
nu = 1.005e-6  # m^2/s; water kinematic viscosity
h = 0.5  # m; water free surface height
V = 1 # m/s; average flow speed
L = 50 # m; hose length
D = 2.5e-2  # m; hose diameter
e = 5.0e-5  # m; hose roughness
patm = 101e3  # Pa; atmospheric pressure
pv = 2.337e3  # Pa; vapor pressure
P = 200 # W; pump power

# Calculate the Reynolds number.
Re = V*D/nu
print("Re = %.3e" % Re)

# Calculate the relative roughness.
e_D = e/D
print("e/D = %.3e" % e_D)

# Determine the friction factor.
f = f_Colebrook(Re, e_D)
print("f = %.3e" % f)

# Calculate the viscous loss coefficient.
K_major = f*(L/D)
print("K_major = %.3e" % K_major)

# Calculate the height in part (A).
H = (patm-pv)/rho/g + h - (1 + K_major)*(V**2)/2/g
print("H = %.3e m" % H)

# Calculate the mass flow rate.
mdot = rho*V*np.pi*D*D/4
print("mdot = %.3e kg/s" % mdot)

# Now calculate the height in part (B).
H = h - (1 + K_major)*(V**2)/2/g + P/mdot/g
print("H = %.3e m" % H)
```

Determine the power, in kW, extracted by the turbine in the system shown below. The pipe entrance is sharp-edged and the volumetric flow rate is 0.004 m^3/s. The density of water is 998 kg/m^3 and the kinematic viscosity is $1.005*10^{-6}$ m^2/s.



SOLUTION:



Apply the Extended Bernoulli Equation from point 1 to point 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \tag{1}$$

where,

$p_1 = p_2 = p_{atm}$,

$\bar{V}_1 \approx 0$,

$$\bar{V}_2=\frac{Q}{\pi D^2/4}, \tag{2}$$

$$z_2 - z_1 = -H, \tag{3}$$

$$H_{L,12}=\sum_i K_i\frac{\bar{V}_i^2}{2g}=\left(K_{\text{square inlet}}+K_{\text{valve}}+K_{\text{major}}\right)\frac{\bar{V}_2^2}{2g}, \tag{4}$$

$$H_{S,12}=\frac{P}{\dot{m}g}. \tag{5}$$

Substitute and solve for the power,

$$\alpha_2\frac{\bar{V}_2{}^2}{2g}+H=-\left(K_{\text{square inlet}}+K_{\text{valve}}+K_{\text{major}}\right)\frac{\bar{V}_2^2}{2g}+\frac{P}{\dot{m}g}, \tag{6}$$

$$P=\dot{m}g\left[H+\left(\alpha_2+K_{\text{square inlet}}+K_{\text{valve}}+K_{\text{major}}\right)\frac{\bar{V}_2^2}{2g}\right]. \tag{7}$$

Using the given data,

$\rho = 998$ kg/m^3 and $\nu = 1.005*10^{-6}$ m^2/s,

$D = 0.05$ m,

$L = 125$ m,

$Q = 0.004$ m^3/s => $\dot{m}=\rho Q$=3.992 kg/s and $\bar{V}_2 = 2.037$ m/s, (8)

$g = 9.81$ m/s^2,

$H = 40$ m,

$K_{\text{square inlet}} = 0.5$ (from minor loss table),

$K_{\text{valve}} = 10$ (from minor loss table),

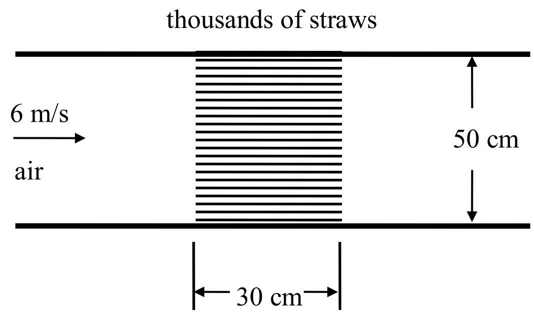
$\epsilon = 0.26*10^{-3}$ m (cast iron pipe) => $\epsilon/D = 0.0052$,

$\mathrm{Re}_D = \frac{\bar{V}_2 D}{\nu} = 101{,}400$ (turbulent flow => $\alpha_2 \approx 1$), (9)
$f = 0.0316$ (from Moody plot or Colebrook Formula),
$K_{\text{major}} = f(L/D) = 79.0$ (The inlet loss coefficient is much smaller than the valve and major loss coefficients and, thus, could be reasonably neglected.)
=> $\boxed{P = 2.32\text{ kW}}$. This is the power extracted by the turbine.

Following is a python code used to perform the calculations.

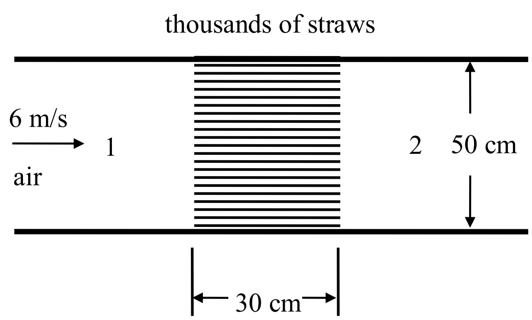
```
# pipe_04.py

import numpy as np

def f_Haaland(Re, e_D):  # Calculate the friction factor using the Haaland formula
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

def f_Colebrook(Re, e_D):  # Calculate the friction factor using the Colebrook formula
    fprime = f_Haaland(Re, e_D)
    freldiff = 1
    tol = 0.001
    while (freldiff > tol):
        f = fprime
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2
        freldiff = np.absolute((fprime-f)/f)
    return f

# Initialize variable values.
g = 9.81 # m/s^2; gravitational acceleration
rho = 998 # kg/m^3; water density
nu = 1.005e-6  # m^2/s; water kinematic viscosity
H = 40  # m; water free surface height
Q = 0.004 # m^3/s; volumetric flow rate
L = 125 # m; pipe length
D = 0.05  # m; pipe diameter
e = 0.26e-3  # m; pipe roughness (cast iron)
K_inlet = 0.5  # square-edged pipe inlet
K_valve = 10  # fully open globe valve
alpha2 = 1  # kinetic energy correction factor

# Calculate the mass flow rate.
mdot = rho*Q
print("mdot = %.3e kg/s" % mdot)

# Calculate the average flow speed.
V2 = Q/(np.pi/4*D*D)
print("V2 = %.3e m/s" % V2)

# Calculate the Reynolds number.
Re = V2*D/nu
print("Re = %.3e" % Re)

# Calculate the relative roughness.
e_D = e/D
print("e/D = %.3e" % e_D)

# Determine the friction factor.
f = f_Colebrook(Re, e_D)
print("f = %.3e" % f)

# Calculate the viscous loss coefficient.
K_major = f*(L/D)
print("K_major = %.3e" % K_major)

# Calculate the power extracted from the turbine.
P = mdot*g*(H + (alpha2 + K_inlet + K_valve + K_major)*(V2**2)/2/g)
print("P = %.3e kW" % (P/1000))
```

For straightening and smoothing an air flow in a 50 cm diameter duct, the duct is packed with a "honeycomb" of 30 cm long, 4 mm diameter thin straws. The inlet flow is air moving at an average velocity of 6 m/s. Estimate the pressure drop across the honeycomb. The density of the air is 1.2 kg/m^3 and the kinematic viscosity is 1.5e-5 m^2/s. You may neglect inlet and outlet minor losses.



SOLUTION:

Apply the Extended Bernoulli's Equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2} \tag{1}$$

where

$$\frac{p_2-p_1}{\rho g}=\frac{\Delta p}{\rho g}=?\ \text{(This is what we're trying to find.)} \tag{2}$$

$$\left(\alpha\frac{\bar{V}^2}{2g}\right)_2=\left(\alpha\frac{\bar{V}^2}{2g}\right)_1 \tag{3}$$

$$z_2=z_1 \tag{4}$$

$$H_{S,1\to 2}=0\ \text{(There is no shaft work between points 1 and 2.)} \tag{5}$$

$$H_{L,1\to 2}=\left(f\frac{L_S}{D_S}\right)\frac{\bar{V}_S^2}{2g}\ \text{(where the subscript "}S\text{" refers to the conditions in the straw)} \tag{6}$$

Note that minor losses have been neglected.

Substitute and re-arrange.

$$\frac{\Delta p}{\rho g}=-\left(f\frac{L_S}{D_S}\right)\frac{\bar{V}_S^2}{2g} \tag{7}$$

Now determine the average flow velocity in the straw, $\bar{V}_S$. Note that the flow rate through the pipe must be the same as the flow rate through all of the straws.

$$\bar{V}_P \frac{\pi}{4} D_P^2 = N_S \bar{V}_S \frac{\pi}{4} D_S^2 \tag{8}$$

where the number of straws, N_S, is:

$$\frac{\pi}{4} D_P^2 = N_S \frac{\pi}{4} D_S^2 \Rightarrow N_S = \left(\frac{D_P}{D_S}\right)^2 \tag{9}$$

and thus:

$$\bar{V}_S = \bar{V}_P \tag{10}$$

Now determine the friction factor for the flow through the straw. First calculate the straw's Reynolds number.

$$\mathrm{Re}_S = \frac{\bar{V}_S D_S}{\nu} = \frac{\left(6 \text{ m/s}\right)\left(4e-3 \text{ m}\right)}{\left(1.5e-5 \text{ m}^2\text{/s}\right)} = 1600 \Rightarrow \text{laminar flow in the straws} \tag{11}$$

$$\Rightarrow f = \frac{64}{\mathrm{Re}_S} \tag{12}$$

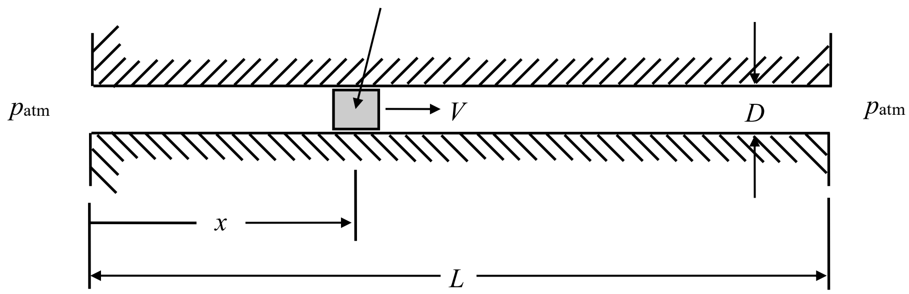
Substitute Eqn. (12) into Eqn. (7) and solve for the pressure drop. using the given data.

$$\boxed{\frac{\Delta p}{\rho g} = -\left(\frac{64}{\mathrm{Re}_S}\frac{L_S}{D_S}\right)\frac{\bar{V}_S^2}{2g}} \quad (L_S = 0.30 \text{ m}; \rho = 1.2 \text{ kg/m}^3) \tag{13}$$

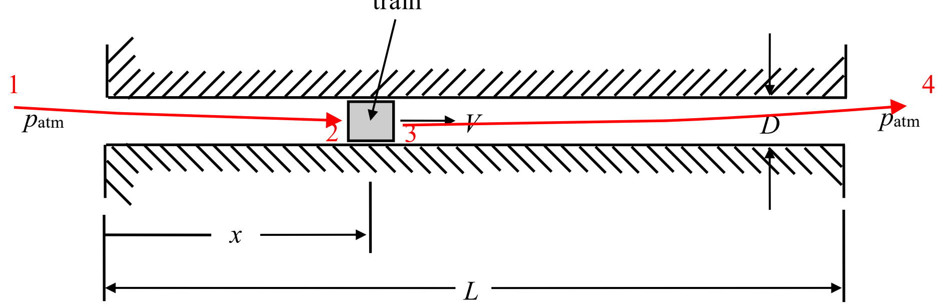
$\boxed{\therefore \Delta p = -64.8 \text{ Pa}}$

A train travels through a tunnel as shown in the figure. The train and tunnel are both circular in cross section. The tunnel has a diameter of D = 3 m, a total length of L = 2000 m, and walls comprised of concrete. The clearance between the train and the tunnel wall is small so that it may be assumed that the air in front of the train is pushed through the tunnel with the same speed as the train, V = 20 m/s.

1. Determine the pressure difference between the front and rear of the train when the train is a distance, x, from the tunnel entrance.
2. Determine the power, P, required to produce the air flow in the tunnel when the train is a distance, x, from the tunnel entrance.



SOLUTION:



Apply the Extended Bernoulli Equation from 1 to 2 and from 3 to 4 to obtain the pressures on the front and back faces of the train,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \quad (1)$$

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_4=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_3-H_{L,34}+H_{S,34}, \quad (2)$$

where,

$$p_1 = p_3 = p_{atm}, \quad (3)$$

$$\bar{V}_1=\bar{V}_4=0, \quad (4)$$

$$\bar{V}_2=\bar{V}_3=V \quad \text{(same speed as the train)}, \quad (5)$$

$$\alpha_2\approx\alpha_3\approx 1 \quad \text{(turbulent flow, } Re_D = VD/\nu = (20\text{ m/s})(3\text{ m})/(1.5*10^{-5}\text{ m}^2/\text{s}) = 4.00*10^6\text{)}, \quad (6)$$

$$z_1\approx z_2\approx z_3\approx z_4 \quad \text{(the train is moving through air so the elevation differences are negligible)}, \quad (7)$$

$$H_{L,12}=f_{D,12}\left(\frac{x}{D}\right)\frac{\bar{V}_2^2}{2g} \quad \text{(neglecting minor losses since the tunnel is long)}, \quad (8)$$

$$H_{L,34}=f_{D,34}\left(\frac{L-x}{D}\right)\frac{\bar{V}_3^2}{2g} \quad \text{(neglecting minor losses since the tunnel is long)}, \quad (9)$$

$$H_{S,12}=H_{S,34}=0 \quad \text{(no fluid machinery)}, \quad (10)$$

Substitute and solve for the pressures on the front and back faces of the train.

$$\frac{p_2}{\rho g}+\frac{V^2}{2g}=\frac{p_{atm}}{\rho g}-f_{D,12}\left(\frac{x}{D}\right)\frac{V^2}{2g} \Rightarrow \frac{p_2-p_{atm}}{\rho g}=-\left[1+f_D\left(\frac{x}{D}\right)\right]\frac{V^2}{2g}, \quad (11)$$

$$\frac{p_{atm}}{\rho g}=\frac{p_3}{\rho g}+\frac{V^2}{2g}-f_{D,34}\left(\frac{L-x}{D}\right)\frac{V^2}{2g} \Rightarrow \frac{p_3-p_{atm}}{\rho g}=-\left[1-f_D\left(\frac{L-x}{D}\right)\right]\frac{V^2}{2g}, \quad (12)$$

Note that the friction factor will be the same along both paths. Subtract Eq. (11) from Eq. (12),

$$\frac{p_3-p_2}{\rho g} = f_D\left(\frac{L}{D}\right)\frac{V^2}{2g} \Rightarrow p_3 - p_2 = f_D\left(\frac{L}{D}\right)\frac{1}{2}\rho V^2. \quad (13)$$

Find the friction factor from the Moody plot or the Colebrook Formula. The Reynolds number was calculated previously in Eq. (6). The relative roughness is,

$\epsilon/D = (1.7*10^{-3}$ m)/(3 m) $= 5.67*10^{-4}$ (the wall material is concrete), (14)

=> $f_D = 0.0173$.

Using the given and previously calculated values,

$\rho = 1.23$ kg/m^3,
$L = 2000$ m,
$D = 3$ m,
$V = 20$ m/s,
=> $\boxed{p_3 - p_2 = 2.84 \text{ kPa}}$.

Thus, there is a net pressure force acting on the train to resist its motion. The force is,

$$F = (p_3 - p_2)\frac{\pi D^2}{4}. \quad (15)$$

The power required to overcome this force at the given speed is,

$$P = FV = (p_3 - p_2)\frac{\pi D^2}{4}V. \quad (16)$$

Using the given and previously calculated values,

$F = 20.1$ kN,
$\boxed{P = 402 \text{ kW}}$.

The following python code was used to perform the calculations.

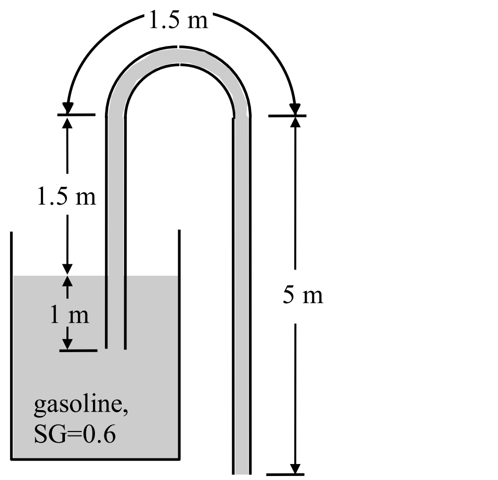
```
# pipe_08.py

import numpy as np

def f_Haaland(Re, e_D):  # Calculate the friction factor using the Haaland formula
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

def f_Colebrook(Re, e_D):  # Calculate the friction factor using the Colebrook formula
    fprime = f_Haaland(Re, e_D)
    freldiff = 1
    tol = 0.001
    while (freldiff > tol):
        f = fprime
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2
        freldiff = np.absolute((fprime-f)/f)
    return f

# Initialize variable values.
g = 9.81 # m/s^2; gravitational acceleration
rho = 1.23 # kg/m^3; air density
nu = 1.5e-5  # m^2/s; air kinematic viscosity
V = 20 # m/s; train speed
L = 2000 # m; tunnel length
D = 3  # m; tunnel diameter
e = 1.7e-3  # m; roughness of concrete

# Calculate the Reynolds number.
Re = V*D/nu
print("Re = %.3e" % Re)

# Calculate the relative roughness.
e_D = e/D
print("e/D = %.3e" % e_D)

# Determine the friction factor.
f = f_Colebrook(Re, e_D)
print("f = %.3e" % f)

# Determine the pressure difference.
Delta_p = f*(L/D)*rho*(V**2)/2
```

```
print("Delta p = %.3e kPa" % (Delta_p/1000))

# Determine the pressure force on the train.
F = Delta_p*np.pi/4*D**2
print("F = %.3e kN" % (F/1000))

# Determine the power required to overcome the pressure force at this speed.
P = F*V
print("P = %.3e kW" % (P/1000))
```

Gasoline at 20 °C is being siphoned from a tank through a rubber hose having an inside diameter of 25 mm. The roughness for the hose is 0.01 mm.

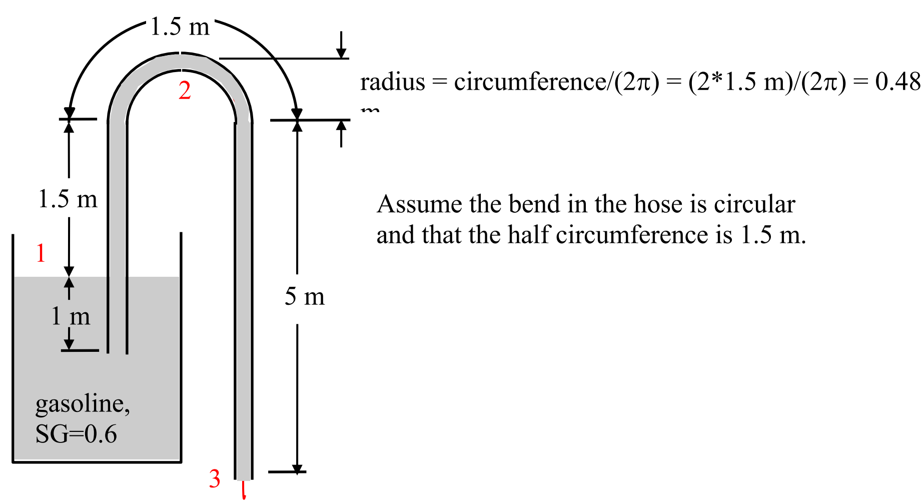
1. What is the volumetric flow rate of the gasoline through the hose?
2. What is the minimum pressure in the hose and where does it occur?

You may neglect minor losses. The kinematic viscosity of gasoline is 4.294e-7 m^2/s.



SOLUTION:

Apply the Extended Bernoulli Equation from points 1 to 3.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_3=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \quad (1)$$

where

$p_1 = p_3 = p_{atm}$ (2)

$\bar{V}_1 \approx 0$ (surface of a large tank) (3)

$\bar{V}_3 = ?$ ($\alpha_3 \approx 1$, assuming turbulent flow in the hose) (4)

$z_1 - z_3 = 3.5$ m (5)

$H_S = 0$ (6)

$$H_L = f\left(\frac{L}{D}\right)\frac{\bar{V}_3^2}{2g} \quad \text{(Neglecting minor losses.)} \tag{7}$$

where f may be found from the Moody diagram. The relative roughness is

$$\frac{\varepsilon}{D} = \frac{0.01\text{ mm}}{25\text{ mm}} = 4*10^{-4} \tag{8}$$

Assuming fully turbulent flow so that f is not a function of the Reynolds number, the Moody diagram gives, $f = 0.016$.

Substitute into Eq. (1) and solve for $\bar{V}_3$.

$$\frac{\bar{V}_3^2}{2g}\left[1+f\left(\frac{L}{D}\right)\right] = (z_1 - z_3) \tag{9}$$

$$\bar{V}_3 = \sqrt{\frac{2g(z_1 - z_3)}{1+f\left(\frac{L}{D}\right)}} \tag{10}$$

Using the given data,

$g = 9.81\text{ m/s}^2$
$z_1 - z_3 = 3.5\text{ m}$
$f = 0.016$
$L = 1\text{ m} + 1.5\text{ m} + 1.5\text{ m} + 5\text{ m} = 9\text{ m}$
$D = 0.025\text{ m}$
$\Rightarrow \bar{V}_3 = 3.2\text{ m/s}$

Check the Reynolds number to verify the fully turbulent assumption.

$$\text{Re}_D = \frac{\bar{V}_3 D}{\nu} = \frac{(3.2\text{ m/s})(0.025\text{ m})}{\left(4.294*10^{-7}\text{ m}^2/\text{s}\right)} = 1.9*10^5 \tag{11}$$

The given relative roughness and this Reynolds number puts the flow in the fully turbulent range so the assumptions made in the problem are consistent.

The flow rate is given by,

$$Q = \bar{V}_3\frac{\pi D^2}{4} = (3.2\text{ m/s})\frac{\pi(0.025\text{ m})^2}{4} \Rightarrow \boxed{Q = 1.57*10^{-3}\text{ m}^3\text{/s}} \tag{12}$$

The minimum pressure occurs near point 2 in the figure shown previously. Apply the Extended Bernoulli Equation from points 2 to 3.

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_3 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 - H_L + H_S \tag{13}$$

where

$p_2 = ?$
$p_3 = p_{\text{atm}}$ (14)
$\bar{V}_2 = \bar{V}_3$ ($\alpha_2 \approx \alpha_3 \approx 1$ (15)
$z_2 - z_3 = 0.48\text{ m} + 5\text{ m} = 5.48\text{ m}$ (16)
$H_S = 0$ (17)

$$H_L = f\left(\frac{L}{D}\right)\frac{\bar{V}_3^2}{2g} \tag{18}$$

where $f = 0.016$ was found previously.

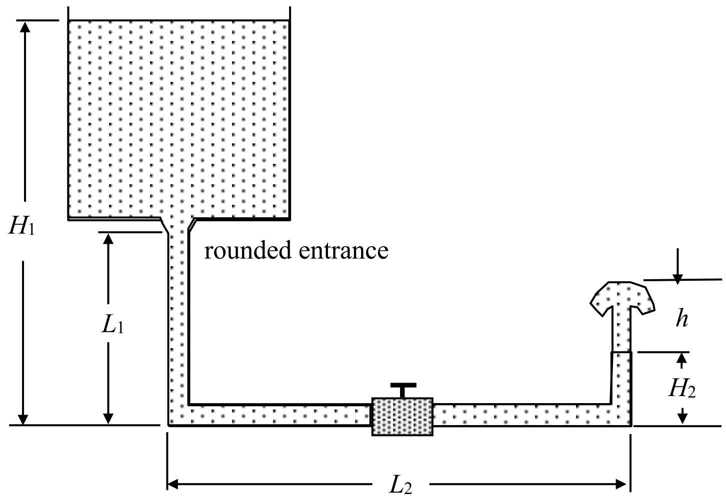
Substitute into Eq. (13) and solve for p_2.

$$p_2 = p_3 + \rho g\left(z_3 - z_2\right) + f\left(\frac{L}{D}\right)\frac{1}{2}\rho \bar{V}_3^2 \tag{19}$$

Using the given data,

p_3 = 101 kPa (abs)
ρ = 0.6(1000 kg/m^3) = 600 kg/m^3
g = 9.81 m/s^2
$z_3 - z_2$ = -5.48 m
f = 0.016
L = (1.5 m)/2 + 5 m = 5.75 m
D = 0.025 m
$\bar{V}_3 = 3.2$ m/s
$\Rightarrow$ $\boxed{p_2 = 80.0 \text{ kPa (abs)}}$

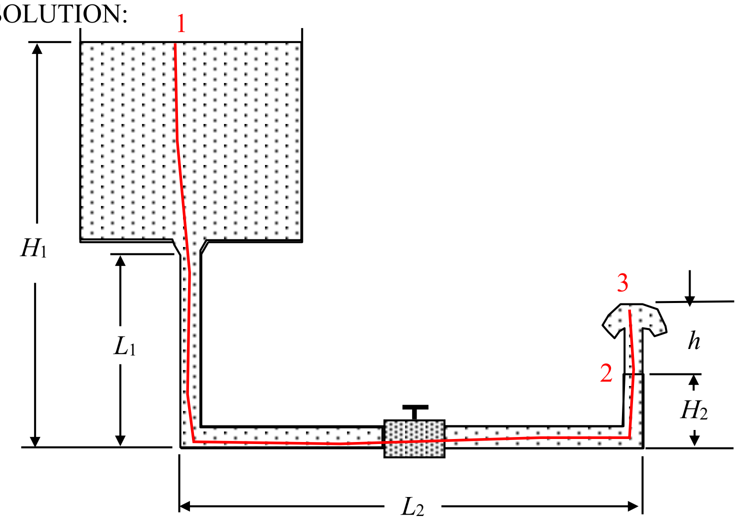
Water flows from a container as shown in the figure. Determine the loss coefficient needed in the valve if the water is to "bubble up" a distance h above the outlet pipe.



H_1 = 45 in
L_1 = 18 in
L_2 = 32 in
H_2 = 2 in
h = 3 in

The pipe is ½ in diameter galvanized iron pipe with threaded fittings.

SOLUTION:



Apply the Extended Bernoulli Equation from point 1 to point 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \quad (1)$$

where,

$$p_1 = p_2 = p_{atm}, \quad (2)$$
$$\bar{V}_1 \approx 0, \quad (3)$$
$$\bar{V}_2 = ?, \quad (4)$$
$$\alpha_2 \approx 1 \text{ (assume turbulent flow)}, \quad (5)$$
$$z_1 = H_1 \text{ and } z_2 = H_2 \quad (6)$$
$$H_{L,12}=\sum_i K_i\frac{\bar{V}_i^2}{2g}=\left(K_{\text{inlet}}+2K_{\text{elbow}}+K_{\text{valve}}+K_{\text{major}}\right)\frac{\bar{V}_2^2}{2g}, \quad (7)$$
$$H_{S,12}=0. \quad (8)$$

Substitute and solve for the valve loss coefficient,

$$\frac{\bar{V}_2^2}{2g}+H_2 = H_1-\left(K_{\text{inlet}}+2K_{\text{elbow}}+K_{\text{valve}}+K_{\text{major}}\right)\frac{\bar{V}_2^2}{2g}, \quad (9)$$

$$K_{\text{valve}} = \frac{2g(H_1 - H_2)}{\bar{V}_2^2} - 1 - K_{\text{inlet}} - 2K_{\text{elbow}} - K_{\text{major}}. \tag{10}$$

The major loss coefficient is,

$$K_{\text{major}} = f\left(\frac{L_1 + L_2 + H_2}{D}\right), \tag{11}$$

where the friction factor is found using the Moody plot or the Colebrook formula using,

$$\text{Re}_D = \frac{\bar{V}_2 D}{\nu}, \tag{12}$$

and the relative roughness ϵ/D.

The velocity in the pipe (and at location 2) can be found by applying Bernoulli's equation from point 2 to point 3,

$$\left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_3 = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_2, \tag{13}$$

where,

$p_3 = p_{\text{atm}}$,
$\bar{V}_3 \approx 0$,
$z_3 = H_2 + h$.

Substitute and solve for the average velocity at 2,

$$\bar{V}_2 = \sqrt{2gh}. \tag{14}$$

Note that the "ordinary" Bernoulli's equation was used from 2 to 3 since these points are outside the pipe.

Using the given parameters,

$g = 32.2$ ft/s^2,
$\nu = 1.21*10^{-5}$ ft^2/s,
$H_1 = 45$ in. = 3.75 ft,
$H_2 = 2$ in. = 0.167 ft,
$h = 3$ in. = 0.25 ft,
$L_1 = 18$ in. = 1.5 ft,
$L_2 = 32$ in. = 2.67 ft,
$D = 0.5$ in. = 0.0417 ft,
$\epsilon = 0.0005$ ft (galvanized iron pipe) => $\epsilon/D = 0.0120$,
$K_{\text{inlet}} = 0.05$ (rounded inlet; from minor loss table),
$K_{\text{elbow}} = 1.5$ (90 deg threaded elbow; from minor loss table).
=> $\bar{V}_2 = 4.012$ ft/s,
=> $\text{Re}_D = 13{,}820$ (turbulent flow assumption is ok!)
=> $f = 0.04387$,
=> $K_{\text{major}} = 4.562$,
=> $\boxed{K_{\text{valve}} = 5.72}$.

The following python code was used to perform the computations.

```
# pipe_12.py

import numpy as np

def f_Haaland(Re, e_D):  # Calculate the friction factor using the Haaland formula
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

def f_Colebrook(Re, e_D):  # Calculate the friction factor using the Colebrook formula
    fprime = f_Haaland(Re, e_D)
    freldiff = 1
    tol = 0.001
    while (freldiff > tol):
        f = fprime
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2
        freldiff = np.absolute((fprime-f)/f)
    return f

# Initialize variable values.
g = 32.2 # ft/s^2; gravitational acceleration
```

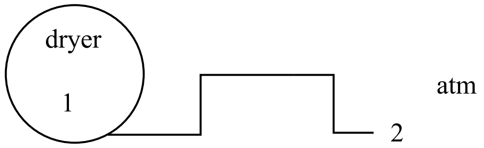
```
nu = 1.21e-5  # ft^2/s; water kinematic viscosity
H1 = 45/12  # ft; tank free surface height
H2 = 2/12  # ft; exit pipe height
h = 3/12  # ft; fountain height
L1 = 18/12 # ft; vertical pipe length
L2 = 32/12 # ft; horizontal pipe length
D = 0.5/12  # ft; pipe diameter
e = 0.0005  # ft; pipe roughness (galvanized iron)
K_inlet = 0.05  # rounded pipe inlet
K_elbow = 1.5  # 90 deg threaded elbow

# Print the lengths in feet.
print("H1 = %.3e ft, H2 = %.3e ft, h = %.3e ft, L1 = %.3e ft, L2 = %.3e ft, D = %.3e
ft" % (H1, H2, h, L1, L2, D))

# Calculate the average velocity in the pipe.
V2 = np.sqrt(2*g*h)
print("V2 = %.3e ft/s" % V2)

# Calculate the Reynolds number.
Re = V2*D/nu
print("Re = %.3e" % Re)

# Calculate the relative roughness.
e_D = e/D
print("e/D = %.3e" % e_D)

# Determine the friction factor.
f = f_Colebrook(Re, e_D)
print("f = %.3e" % f)

# Calculate the viscous loss coefficient.
K_major = f*((L1+L2+H2)/D)
print("K_major = %.3e" % K_major)

# Calculate the valve loss coefficient.
K_valve = 2*g*(H1-H2)/V2/V2 - 1 - K_inlet - 2*K_elbow - K_major
print("K_valve = %.3e" % K_valve)
```

According to an appliance manufacturer, the 4 in diameter galvanized iron vent on a clothes dryer is not to contain more than 20 ft of pipe and four 90° elbows. Under these conditions, determine the air flow rate if the gage pressure within the dryer is 1.04 psf. You may assume the following:

kinematic viscosity of air: 1.79e-4 ft^2/s
density of air: 2.20e-3 slugs/ft^3
$K_{90° \text{ bend}} = 1.5$
$K_{\text{entrance}} = 0.5$

SOLUTION:

Apply the Extended Bernoulli Equation from point 1 to point 2 as shown in the figure below.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \tag{1}$$

where

$p_2 = 0$ (gage) and $p_1 = 1.04$ psfg
$\bar{V}_2 = ?$ (Assume turbulent flow so that $\alpha_2 \approx 1$.)
$\bar{V}_1 << \bar{V}_2$ (The air in the dryer is relatively stagnant compared to the outflowing air.)
$z_2 - z_1 \approx 0$
$H_S = 0$

$$H_L = f\left(\frac{L}{D}\right)\frac{\bar{V}_P^2}{2g}+K_{\text{entrance}}\frac{\bar{V}_P^2}{2g}+4K_{\text{bends}}\frac{\bar{V}_P^2}{2g} \tag{2}$$

Note that $\bar{V}_P = \bar{V}_2$ since the pipe and exit diameters are the same. Also, there is no exit loss since point 2 is located just at the exit of the pipe. The air has not undergone any exit losses at this point.

Substitute and simplify.

$$\frac{\bar{V}_2^2}{2}\left[f\left(\frac{L}{D}\right)+K_{\text{entrance}}+4K_{\text{bends}}+1\right]=\frac{p_{1,g}}{\rho} \tag{3}$$

It's given that

L = 20 ft
D = 4 in. = 0.33 ft
K_{elbow} = 1.5
K_{entrance} = 0.5
$p_{1,g}$ = 1.04 lb$_f$/ft^2
ρ = 2.20*10^{-3} slug/ft^3

Using these parameters, Eq. (3) becomes,

$$\bar{V}_2^2\left[60f+7.5\right]=945.5\ \text{ft}^2/\text{s}^2 \tag{4}$$

Note that f is dependent on the Reynolds number and relative roughness,

$$\mathrm{Re}_D = \frac{\bar{V}_P D}{\nu} = (1862\ \mathrm{s/ft})\bar{V}_2 \tag{5}$$

where $\nu_{air} = 1.79*10^{-4}$ ft^2/s and $\bar{V}_P = \bar{V}_2$. The roughness of galvanized iron pipe is $e = 0.0005$ ft so that the relative roughness is,

$$\frac{e}{D} = 0.0015 \tag{6}$$

To solve for $\bar{V}_2$, we must iterate to a solution since f is also a (complex) function of $\bar{V}_2$ because of the Reynolds number dependence. One iterative procedure that can be used is given below.

1. Choose a value for f.
2. Calculate $\bar{V}_2$ using Eq. (4).
3. Calculate Re_D using Eq. (5).
4. Use the Moody diagram with the Re_D calculated from Step 3 and the relative roughness given in Eq. (6) to find f'.
5. Is $f' = f$? If so, then the iterations are complete and $\bar{V}_2$ is the value found in Step 2. Otherwise, use f' as the new value for f and go to Step 2.

Using this iterative algorithm and an initial guess of $f = 0.025$,

1. $f = 0.025$
   a. $\bar{V}_2 = 10.25$ ft/s
   b. $\mathrm{Re}_D = 19{,}000$
   c. $f' = 0.029$ (This value is different than our original guess, must continue iterations.)
2. $f = 0.029$
   a. $\bar{V}_2 = 10.11$ ft/s
   b. $\mathrm{Re}_D = 18{,}800$
   c. $f' = 0.029$ (This value matches our initial guess! Iterations complete!)

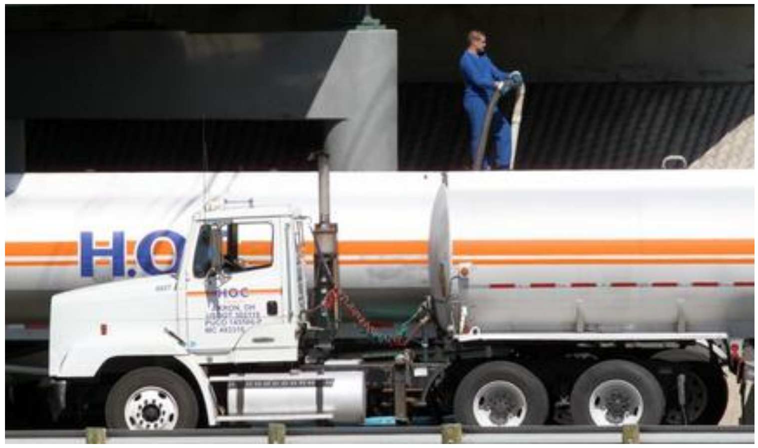
Note that the flow is turbulent, which is consistent with the assumption that $\alpha_2 \approx 1$.

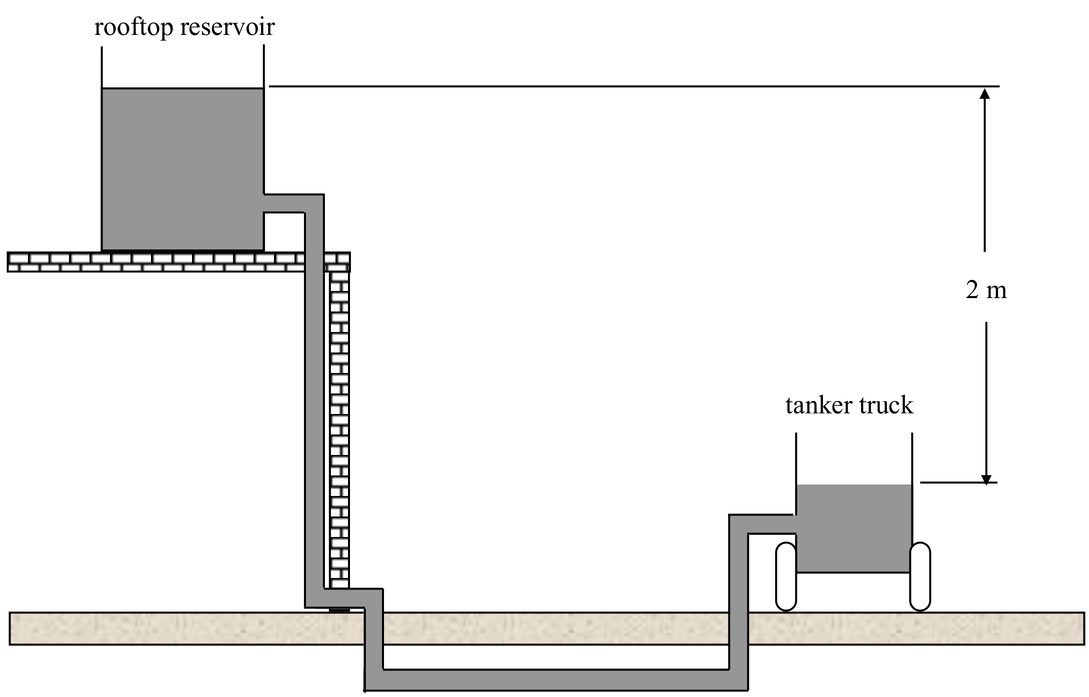
The volumetric flow rate may be found using,

$$Q = \bar{V}_2 \frac{\pi D^2}{4} \Rightarrow \boxed{Q = 0.882\ \mathrm{ft^3/s}} \tag{7}$$

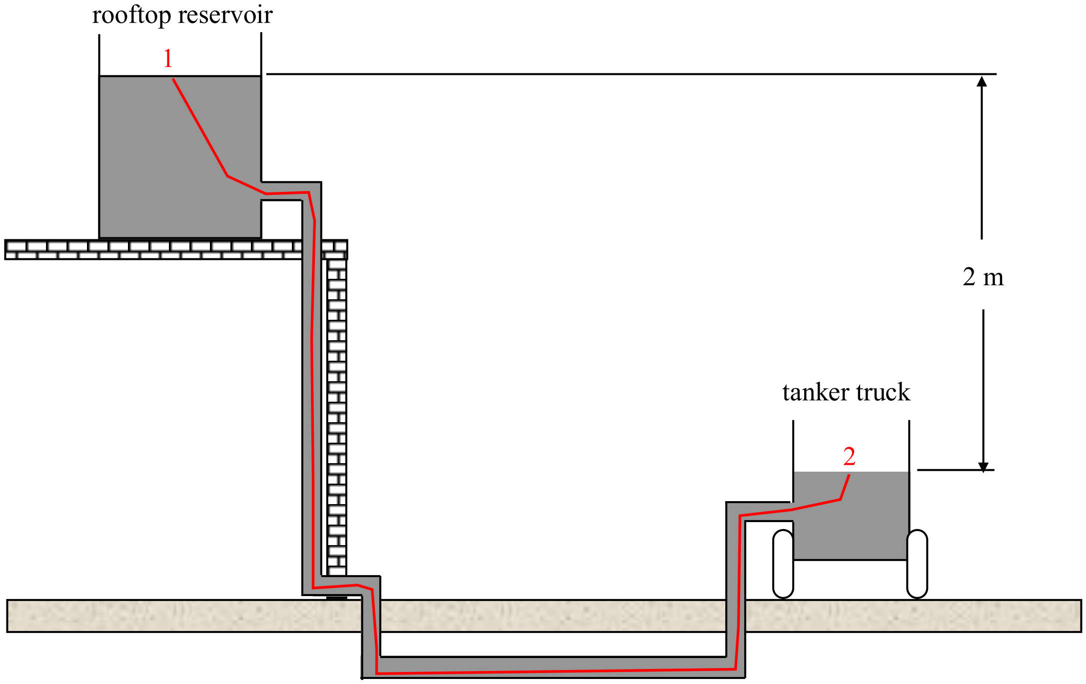
where $\bar{V}_2 = 10.11$ ft/s and $D = 0.33$ ft.

Water at 10 °C (kinematic viscosity of $1.307*10^{-6}$ m^2/s) is to flow from a roof-top reservoir to a tanker truck through a cast iron pipe (roughness of 0.26 mm) of length 20 m at a flow rate of 0.0020 m^3/s. The roof-top tank water level is located 2 m above the tanker truck fluid level. The system contains a sharp-edged entrance, six threaded 90° elbows, and a sharp-edged exit. Determine the required pipe diameter for the given flow conditions.





SOLUTION:



Apply the Extended Bernoulli Equation from point 1 to point 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \quad (1)$$

where,

$p_1 = p_2 = p_{atm}$,

$\bar{V}_1 \approx \bar{V}_2 \approx 0$,

$z_2 - z_1 = -H$, (2)

$H_{L,12} = \sum_i K_i \frac{\bar{V}_i^2}{2g} = \left(K_{\text{square inlet}} + 4K_{\text{elbow}} + K_{\text{exit}} + K_{\text{major}}\right)\frac{\bar{V}^2}{2g}$, (3)

$H_{S,12} = 0$, (4)

$\bar{V} = \frac{Q}{\pi D^2/4}$ (relating the average flow speed in the pipe to the volumetric flow rate). (5)

Substitute and simplify,

$$H = \left(K_{\text{square inlet}} + 4K_{\text{elbow}} + K_{\text{exit}} + K_{\text{major}}\right)\frac{\bar{V}^2}{2g}, \quad (6)$$

$$H = \left(K_{\text{square inlet}} + 4K_{\text{elbow}} + K_{\text{exit}} + K_{\text{major}}\right)\frac{8Q^2}{\pi^2 D^4 g}, \quad (7)$$

$$D^4 = \left(K_{\text{square inlet}} + 4K_{\text{elbow}} + K_{\text{exit}} + K_{\text{major}}\right)\frac{8Q^2}{\pi^2 gH}. \quad (8)$$

The major loss coefficient is,

$$K_{\text{major}} = f\left(\frac{L}{D}\right), \quad (9)$$

where the friction factor is a function of the relative roughness and the Reynolds number,

$$f = f\left(\frac{\epsilon}{D}, \text{Re}_D\right), \quad (10)$$

where,

$$\text{Re}_D = \frac{\bar{V}D}{\nu}. \quad (11)$$

Since the velocity, Reynolds number, and relative roughness are all functions of the pipe diameter, Eq. (5) cannot be solved explicitly for the pipe diameter. Instead, an iterative solution must be used. The following algorithm is implemented in the python code at the end of this solution.

1. Choose a value for the diameter, D.
2. Calculate the average pipe speed using Eq. (5).
3. Calculate the Reynolds number using Eq. (11).
4. Calculate the relative roughness.
5. Determine the friction factor using the Colebrook Formula (or use the Moody plot).
6. Calculate the major loss coefficient using Eq. (9).
7. Use the EBE (Eq. (8)) to solve for the pipe diameter, D'.
8. Is D' equal to D? If so, then the iterations are complete. If not, let $D = D'$ and return to step 2.

The given parameters are,

$\nu = 1.307*10^{-6}$ m^2/s,
$g = 9.81$ m/s^2,
$H = 2.0$ m,
$L = 20$ m,
$Q = 0.0020$ m^3/s,
$\epsilon = 0.26$ mm (cast iron pipe),
$K_{\text{inlet}} = 0.5$ (sharp-edge entrance; from a minor loss table),
$K_{\text{elbow}} = 1.5$ (90 deg, threaded elbow; from a minor loss table),
$K_{\text{exit}} = 1$.

Running the python code generates the following output,

```
Iterating:  D= 4.000e-02 m, Re= 4.871e+04, e/D= 6.500e-03, f= 3.451e-02, K_major= 1.725e+01, D' = 4.497e-02 m
Iterating:  D= 4.497e-02 m, Re= 4.332e+04, e/D= 5.781e-03, f= 3.364e-02, K_major= 1.496e+01, D' = 4.389e-02 m
Iterating:  D= 4.389e-02 m, Re= 4.439e+04, e/D= 5.924e-03, f= 3.381e-02, K_major= 1.541e+01, D' = 4.411e-02 m
Iterating:  D= 4.411e-02 m, Re= 4.417e+04, e/D= 5.895e-03, f= 3.378e-02, K_major= 1.531e+01, D' = 4.406e-02 m
Final:  D = 4.411e-02 m, Re = 4.417e+04, e/D = 5.895e-03, f = 3.378e-02, D' = 4.406e-02 m
```

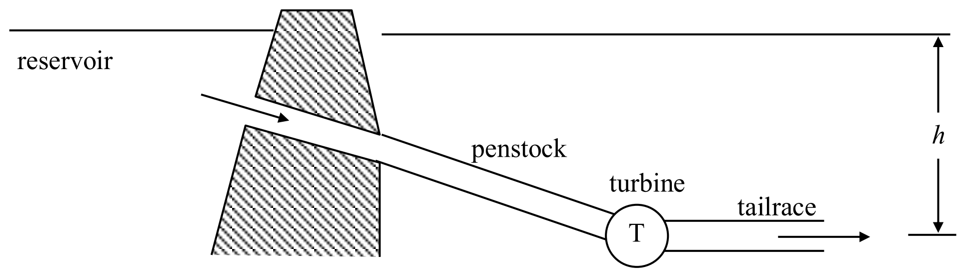
Thus, the pipe diameter should be $\boxed{D = 4.41 \text{ cm}}$.

Following is the python code used for the computations.

```
# pipe_11.py

import numpy as np

# Initialize variable values.
g = 9.81  # m/s^2, gravitational acceleration
H = 2.0  # m, height difference
L = 20  # m, pipe length
Q = 0.0020  # m^3/s, volumetric flow rate
K_inlet = 0.5  # sharp-edge entrance loss coefficient
K_elbow = 1.5  # 90 deg threaded elbow loss coefficient
K_exit = 1.0  # exit loss coefficient
nu = 1.307e-6  # m^2/s, kinematic viscosity
e = 0.26e-3  # m, roughness (cast iron pipe)

def Re_fcn(D):  # Calculate the Reynolds number given the diameter
    return 4*Q/np.pi/nu/D

def f_Haaland(Re, e_D):  # Calculate the friction factor using the Haaland formula
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

def f_Colebrook(Re, e_D):  # Calculate the friction factor using the Colebrook formula
    fprime = f_Haaland(Re, e_D)
    freldiff = 1
    tol = 0.001
    while (freldiff > tol):
        f = fprime
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2
        freldiff = np.absolute((fprime-f)/f)
    return f

def D_fcn(f, K_major):  # Calculate the diameter from the EBE
    D = ((K_inlet + 4*K_elbow + K_exit + K_major)*8*Q*Q/np.pi/np.pi/g/H)**(1/4)
    return D
```

```
# Initial guess for D and set the iteration tolerance.
Dprime = 0.04  # m, first guess at the diameter
Dreldiff = 1  # a large number to start with
tol = 0.001  # tolerance
countmax = 1000  # maximum number of iterations before giving up

# Iterate until D and Dprime are nearly equal or we reach the maximum
# number of iterations.
count = 0
while ((Dreldiff > tol) and (count < countmax)):
    count = count + 1
    D = Dprime
    Re = Re_fcn(D)
    e_D = e/D
    f = f_Colebrook(Re, e_D)
    K_major = f*(L/D)
    Dprime = D_fcn(f, K_major)
    Dreldiff = np.absolute((D-Dprime)/D)  # find the relative difference
    print("Iterating:  D= %.3e m," % D, "Re= %.3e," % Re, "e/D= %.3e," %
e_D,"f= %.3e," % f, "K_major= %.3e," % K_major, "D\' = %.3e m" % Dprime)


if (count == countmax):
    print("Didn't converge to a solution after %d iterations." % countmax)
else:
    print("Final:  D = %.3e m," % D, "Re = %.3e," % Re, "e/D = %.3e," % e_D,"f
= %.3e," % f, "D\' = %.3e m" % Dprime)
```

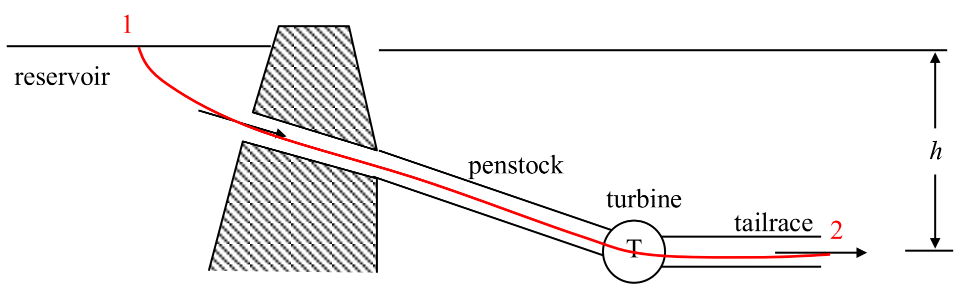
The tailrace (discharge pipe) of a hydro-electric turbine installation is at an elevation, h, below the water level in the reservoir:



The losses in the penstock (the pipe leading to the turbine) and the tailrace are represented by the loss coefficient, K, based on the mean velocity, U, in those pipes, which have the same cross-sectional area, A. The flow discharges to atmospheric pressure at the exit from the tailrace. The water density is denoted by ρ and the acceleration due to gravity by g. Assume turbulent flow conditions.

a. What is the drop in total head across the turbine?
b. What is the power developed by the turbine assuming that it has an efficiency η?
c. What is the optimum velocity, U_{opt}, that will produce the maximum power output from the turbine assuming that h, K, A, ρ, and g are constant?

SOLUTION:



Apply the Extended Bernoulli Equation from 1 to 2,

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_{L,12} + H_{S,12}, \quad (1)$$

where,

$$p_1 = p_2 = p_{atm}, \quad (2)$$

$$\bar{V}_1 \approx 0, \quad (3)$$

$$\bar{V}_2 = U \quad \text{(assume } \alpha_2 \approx 1\text{)}, \quad (4)$$

$$z_2 - z_1 = -h, \quad (5)$$

$$H_{L,12} = \sum_i K_i \frac{\bar{V}_i^2}{2g} = K\frac{U^2}{2g}, \quad (6)$$

$$H_{S,12} = \frac{P}{\rho Q g} = \frac{P}{\rho U A g}. \quad (7)$$

Substitute and simplify,

$$\frac{U^2}{2g} - h = -K\frac{U^2}{2g} + \frac{P}{\rho g U A}, \quad (8)$$

$$\boxed{\frac{P}{\rho g U A} = (1+K)\frac{U^2}{2g} - h}. \quad (9)$$

Note that since it's a turbine, we expect the right-hand side of Eq. (9) to be negative. Thus, the total drop in head across the turbine is equal to the absolute value of the right-hand side of Eq. (9).

The power developed by the turbine, assuming an efficiency of η is,

$$\boxed{P = \eta \rho g U A \left[(1+K)\frac{U^2}{2g} - h \right]}. \tag{10}$$

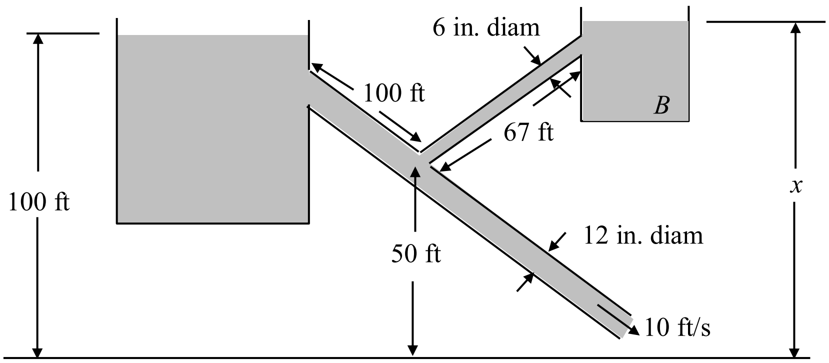
To optimize the power output, take the derivative of Eq. (10) and set it equal to zero,

$$\frac{dP}{dU} = 0 = \eta \rho g A \frac{d}{dU}\left[(1+K)\frac{U^3}{2g} - hU \right], \tag{11}$$

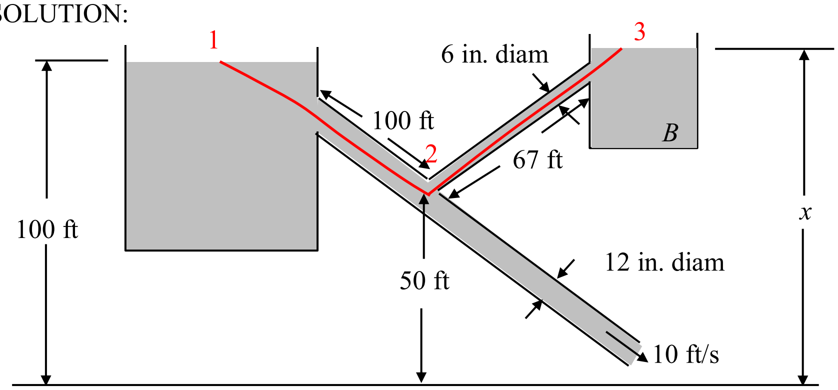
$$0 = 3(1+K)\frac{U^2}{2g} - h, \tag{12}$$

$$\boxed{U_{opt} = \sqrt{\frac{2gh}{3(1+K)}}}. \tag{13}$$

In the water flow system shown, reservoir *B* has variable elevation, *x*. Determine the water level in reservoir *B* so that no water flows into or out of that reservoir. The speed in the 12 in. diameter pipe is 10 ft/s. Assume the pipes are constructed of cast iron and that the entrances are sharp-edged.



SOLUTION:



Apply the Extended Bernoulli Equation from point 1 to point 2,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \tag{1}$$

where,

$$p_1 = p_{\text{atm}}, \tag{2}$$

$$\bar{V}_1 \approx 0, \tag{3}$$

$$\bar{V}_2 \text{ is given,} \tag{4}$$

$$z_2 - z_1 \text{ is given,} \tag{5}$$

$$H_{L,12} = \textstyle\sum_i K_i \frac{\bar{V}_i^2}{2g} = \left(K_{\text{inlet}} + K_{\text{major}}\right)\frac{\bar{V}_2^2}{2g}, \tag{6}$$

$$H_{S,12} = 0. \tag{7}$$

The pressure at point 2 can be found using hydrostatics since there is no flow from point 3 to point 2,

$$p_2 = p_{atm} + \rho g(x - z_2). \tag{8}$$

Substitute and solve for *x*,

$$\frac{p_{atm}}{\rho g} + (x - z_2) + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2 = \frac{p_{atm}}{\rho g} + z_1 - \left(K_{\text{inlet}} + K_{\text{major}}\right)\frac{\bar{V}_2^2}{2g}, \tag{9}$$

$$x = z_1 - \left(K_{\text{inlet}} + K_{\text{major}} + \alpha_2\right)\frac{\bar{V}_2^2}{2g}, \tag{10}$$

Using the given parameters,

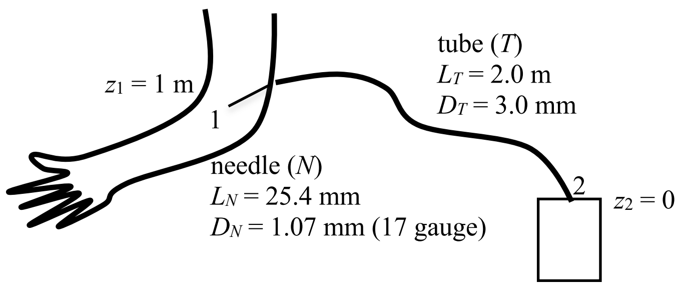
$z_1 = 100$ ft,
$K_{\text{inlet}} = 0.5$ (sharp-edge inlet; from a minor loss table),
$\bar{V}_2 = 10$ ft/s,
$\nu = 1.08 * 10^{-5}$ ft^2/s,
$g = 32.2$ ft/s^2,
$D = 12$ in. $= 1$ ft,
$L_{12} = 100$ ft,
$\epsilon = 0.00085$ ft (cast iron) => $\epsilon/D = 0.00085$,
$\text{Re}_D = \bar{V}_2 D/\nu = 925{,}900$ (turbulent flow => $\alpha_2 \approx 1$),
=> $f = 0.01924$ (from the Moody plot or the Colebrook formula),
=> $K_{\text{major}} = f(L_{12}/D) = 1.924$,
=> $\boxed{x = 94.7 \text{ ft}}$.

The following python code was used for the computations.

```
# pipe_15.py

import numpy as np

def f_Haaland(Re, e_D):  # Calculate the friction factor using the Haaland formula
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

def f_Colebrook(Re, e_D):  # Calculate the friction factor using the Colebrook formula
    fprime = f_Haaland(Re, e_D)
    freldiff = 1
    tol = 0.001
    while (freldiff > tol):
        f = fprime
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2
        freldiff = np.absolute((fprime-f)/f)
    return f

# Initialize variable values.
g = 32.2 # ft/s^2; gravitational acceleration
nu = 1.08e-5  # ft^2/s; kinematic viscosity
z1 = 100  # ft; free surface height
V2 = 10 # ft/s; flow rate
L12 = 100 # ft; pipe length
D = 12/12  # ft; pipe diameter
e = 0.00085  # ft; pipe roughness (cast iron)
K_inlet = 0.5  # square-edged pipe inlet

# Calculate the Reynolds number.
Re = V2*D/nu
print("Re = %.3e" % Re)

# Calculate the relative roughness.
e_D = e/D
print("e/D = %.3e" % e_D)

# Determine the friction factor.
f = f_Colebrook(Re, e_D)
print("f = %.3e" % f)

# Calculate the viscous loss coefficient.
K_major = f*(L12/D)
print("K_major = %.3e" % K_major)

# Calculate the elevation x.
x = z1 - (K_inlet + K_major + 1)*(V2**2)/2/g
print("x = %.3e ft" % x)
```

Consider the process of donating blood. Blood flows from a vein in which the pressure is greater than atmospheric, through a long small-diameter tube, and into a plastic bag that is essentially at atmospheric pressure. Based on fluid mechanics principles, estimate the amount of time it takes to donate a pint of blood. List all assumptions and show calculations.

SOLUTION:



First, a few assumptions:

1. Treat blood as a Newtonian fluid. Blood is actually slightly non-Newtonian with shear-thinning behavior, but we'll model it as Newtonian here for simplicity. Assume the density of blood is ρ = 1060 kg/m^3 and its dynamic viscosity is μ = 3.5 cP = 3.5*10^{-3} kg/(m.s). Hence, the kinematic viscosity is $\nu = \mu/\rho$ = 3.30*10^{-6} m^2/s.
2. Steady flow through the needle and tube. In fact, the real flow will be pulsatile due to the fluctuating pressure in the vein.
3. Constant pressure in the arm and collection bag. Again, the real flow will have periodic pressure variations in the arm.
4. The mean arterial pressure in the arm is at 93.3 mm Hg (gage). In practice, there are two values given for blood pressure: a systolic pressure and a diastolic pressure. The systolic pressure is the pressure when the heart is contracted while the diastolic pressure is when the heart is relaxed. A value of (systolic/diastolic) 120/80 mm Hg (gage) is within the normal range of blood pressures. The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and can be approximated as: MAP = $p_{\text{diastolic}} + (1/3)*(p_{\text{systolic}} - p_{\text{diastolic}})$.
5. The pipe system consists of a needle, plastic tubing, and a plastic collection bag. A 17 gauge (1.07 mm inner diameter) needle diameter is often used for collecting blood. These needles are approximately 2.54 cm (1 in.) in length. The plastic tubing is assumed to be approximately 2.0 m in length with an inner diameter of 3.0 mm.
6. The flow in the needle and tube is laminar since the diameters are small.
7. The collection bag is located below the person's arm. We'll assume an elevation difference of 0.5 m.

Apply the Extended Bernoulli Equation from point 1 (in the vein) to point 2 (just upstream of the tube exit leading into the bag),

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2}\,, \tag{1}$$

where

p_1 = p_{arm} = 93.3 mm Hg (gage) (use the mean arterial pressure in the arm)
p_2 = 0 (gage) (discharging into a bag that is at atmospheric pressure)

$\bar{V}_1 \approx 0$ (blood speed in the vein is small compared to the speed in the needle and tube)
$\bar{V}_2 = \bar{V}_T$ (blood speed just before the exit of the tube)
$\alpha_2 = \alpha_T = 2$ (because the tube diameter is small, assume the flow is laminar at the tube exit)
$z_1 = 0.5$ m (assume the person's arm is 0.5 m above the bag)
$z_2 = 0$
$H_{S,1\to 2} = 0$ (no fluid machinery in the process)

$$H_{L,1\to 2} = \frac{\bar{V}_N^2}{2g}\left[f_N\left(\frac{L_N}{D_N}\right) + K_{\text{entrance}} + K_{\text{expansion}}\right] + \frac{\bar{V}_T^2}{2g}\left[f_T\left(\frac{L_T}{D_T}\right)\right] \tag{2}$$

(major losses in the needle and tube and a minor loss at the inlet and at the transition from the need to the tube)

Note that,

$$\bar{V}_N = \frac{Q}{\frac{\pi}{4}D_N^2} = \frac{4Q}{\pi D_N^2} \quad \text{and} \quad \bar{V}_T = \frac{Q}{\frac{\pi}{4}D_T^2} = \frac{4Q}{\pi D_T^2} \tag{3}$$

$$f_N = \frac{64}{\mathrm{Re}_{D_N}} = \frac{64\nu}{\bar{V}_N D_N} = \frac{16\pi\nu D_N}{Q} \quad \text{and} \quad f_T = \frac{64}{\mathrm{Re}_{D_T}} = \frac{64\nu}{\bar{V}_T D_T} = \frac{16\pi\nu D_T}{Q} \quad \text{(since the flow is laminar)} \tag{4}$$

Substitute and simplify,

$$\alpha_T\frac{\bar{V}_T^2}{2g} = \left(\frac{p_{g,\text{arm}}}{\rho g} + z_1\right) - \frac{\bar{V}_N^2}{2g}\left[f_N\left(\frac{L_N}{D_N}\right) + K_{\text{entrance}} + K_{\text{expansion}}\right] - \frac{\bar{V}_T^2}{2g}\left[f_T\left(\frac{L_T}{D_T}\right)\right], \tag{5}$$

$$\left(\frac{p_{g,\text{arm}}}{\rho g} + z_1\right) = \frac{8Q^2}{\pi^2 g D_N^4}\left(\frac{16\pi\nu L_N}{Q} + K_{\text{entrance}} + K_{\text{expansion}}\right) + \frac{8Q^2}{\pi^2 g D_T^4}\left(\frac{16\pi\nu L_T}{Q} + \alpha_T\right), \tag{6}$$

$$\left(\frac{p_{g,\text{arm}}}{\rho g} + z_1\right) = \frac{8Q^2}{\pi^2 g D_N^4}\frac{16\pi\nu L_N}{Q} + \frac{8Q^2}{\pi^2 g D_T^4}\frac{16\pi\nu L_T}{Q} + \frac{8Q^2}{\pi^2 g D_N^4}\left(K_{\text{entrance}} + K_{\text{expansion}}\right) + \frac{8Q^2}{\pi^2 g D_T^4}\alpha_T, \tag{7}$$

$$\left(\frac{p_{g,\text{arm}}}{\rho g} + z_1\right) = \frac{128\nu Q}{\pi g}\left(\frac{L_N}{D_N^4} + \frac{L_T}{D_T^4}\right) + \frac{8Q^2}{\pi^2 g}\left(\frac{K_{\text{entrance}} + K_{\text{expansion}}}{D_N^4} + \frac{\alpha_T}{D_T^4}\right), \tag{8}$$

$$\underbrace{\frac{8}{\pi^2 g}\left(\frac{K_{\text{entrance}} + K_{\text{expansion}}}{D_N^4} + \frac{\alpha_T}{D_T^4}\right)}_{=A}Q^2 + \underbrace{\frac{128\nu}{\pi g}\left(\frac{L_N}{D_N^4} + \frac{L_T}{D_T^4}\right)}_{=B}Q + \underbrace{-\left(\frac{p_{g,\text{arm}}}{\rho g} + z_1\right)}_{=C} = 0\,. \tag{9}$$

Using the given data,

g = 9.81 m/s^2
K_{entrance} = 0.78 (re-entrant inlet)
$K_{\text{expansion}}$ = 0.8 (area ratio = $(D_N/D_T)^2$ = 0.126)
D_N = 1.07*10^{-3} m
α_T = 2 (laminar flow at tube exit)
D_T = 3.0*10^{-3} m
ν = 3.30*10^{-6} m^2/s
L_N = 2.54*10^{-2} m
L_T = 2.0 m
$p_{g,\text{arm}}$ = $\rho g H$ = (13.6*1000 kg/m^3)(9.81 m/s^2)(93.3*10^{-3} m Hg) = 12.5*10^3 Pa
ρ = 1060 kg/m^3
z_1 = 0.5 m
$\Rightarrow$ A = 1.03*10^{11} s^2/m^5, B = 6.07*10^5 s/m^2, C = -1.70 m
$\Rightarrow$ $\underline{Q = 2.07\text{*}10^{-6}\ \text{m}^3/\text{s}}$ $\Rightarrow$ $\bar{V}_N$ = 2.31 m/s, $\bar{V}_T$ = 0.29 m/s $\Rightarrow$ Re_{DN} = 748, Re_{DT} = 266 $\Rightarrow$ The laminar flow assumptions are good ones!

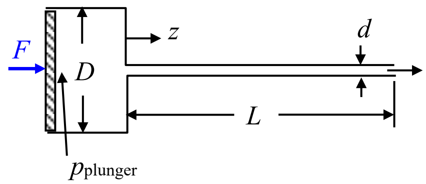
One pint is equivalent to V_{collect} = 4.73*10^{-4} m^3, thus the expected time required to collect one pint of blood is,

$$T = \frac{V_{\text{collect}}}{Q} \Rightarrow \boxed{T = 229\ \text{s} = 3.8\ \text{min.}}$$

In practice, the time required to donate a pint of blood is approximately 8 – 10 minutes, so this prediction, although in the right ballpark, is too small when compared to reality. Two assumptions likely factor into this error. First, we've assumed fully developed flow in the needle, which is most likely not the case. The pressure drop in the needle will be larger than what we've predicted from our fully developed flow model and thus the flow rate will decrease and the predicted donation time will increase. Secondly, there will be additional losses due to the bends in the tubing and, especially, due to the clamp located on the tubing to make it easier to stop the flow, if needed. These additional minor losses aren't negligible and will contribute to make the flow rate smaller.

A hypodermic needle, with an inside diameter of 0.1 mm and a length of 25 mm is used to inject saline solution with a dynamic viscosity five times that of water. The plunger diameter is 10 mm and the maximum force that can be exerted by a thumb on the plunger is 45 N. Estimate the volume flow rate of saline that can be produced.

SOLUTION:



For a viscous, laminar, fully developed flow in a circular pipe (Poiseuille flow), the average velocity is

$$\bar{u} = \frac{d^2}{32\mu}\left(-\frac{dp}{dz}\right) \tag{1}$$

and the volumetric flow rate is:

$$Q = \bar{u}\frac{\pi d^2}{4} = \frac{\pi d^4}{128\mu}\left(-\frac{dp}{dz}\right) \tag{2}$$

The pressure gradient, assuming fully developed flow in the needle, is:

$$\frac{dp}{dz} = \frac{\Delta p}{L} = \frac{p_{\text{atm}} - p_{\text{plunger}}}{L} = \frac{-p_{\text{plunger,gage}}}{L} \tag{3}$$

where $p_{\text{plunger,gage}}$ is:

$$p_{\text{plunger,gage}} = \frac{F}{\left(\pi D^2 / 4\right)} \tag{4}$$

Using the given data:

d	= 0.1e-3 m
D	= 10e-3 m
L	= 25e-3 m
F	= 45 N
μ	= 5e-3 N·m/s
$\Rightarrow$	$p_{\text{plunger,gage}}$ = 5.73e5 Pa
$\Rightarrow$	dp/dz = -2.29e7 Pa/m
$\Rightarrow$	$\bar{u}$ = 1.43 m/s
$\Rightarrow$	$\boxed{Q = 1.13\text{e-8 m}^3\text{/s} = 11.3\text{ mm}^3\text{/s}}$

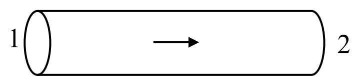
Check the Reynolds number to verify that the laminar flow assumption is ok.

$$\text{Re} = \frac{\rho \bar{u} d}{\mu} \quad \text{(Use } \rho \approx 1000\text{ kg/m}^3\text{.)} \tag{5}$$

$\Rightarrow$ Re = 28.8 < 2300 $\Rightarrow$ The laminar flow assumption is justified!

The average flow speed in a constant-diameter section of the Alaskan pipeline is 8.27 ft/s. At the inlet, the pressure is 1200 psig and the elevation is 150 ft; at the outlet, the pressure is 50 psig and the elevation is 375 ft. Calculate the head loss in this section of pipeline.

SOLUTION:



Apply the Extended Bernoulli's Equation from 1 to 2.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2} \tag{1}$$

where

$$\frac{p_2-p_1}{\rho g}=\frac{(50-1200)\ \cancel{\text{lb}_\text{f}}/\cancel{\text{in}^2}\cdot 144\ \cancel{\text{in}^2}/\cancel{\text{ft}^2}}{\underbrace{(0.9)}_{=\text{SG}_\text{crude oil}}\left(1.94\ \cancel{\text{slug}}/\cancel{\text{ft}^3}\right)\left(32.2\ \cancel{\text{ft}}/\cancel{\text{s}^2}\right)}=-2940\text{ ft} \tag{2}$$

$$\left(\alpha\frac{\bar{V}^2}{2g}\right)_2=\left(\alpha\frac{\bar{V}^2}{2g}\right)_1 \quad \text{(since the flow is fully developed and mass is conserved)} \tag{3}$$

$$z_2-z_1=(375-150)\text{ ft}=225\text{ ft} \tag{4}$$

$$H_{L,1\to 2}=?\quad \text{(This is what we're trying to find.)} \tag{5}$$

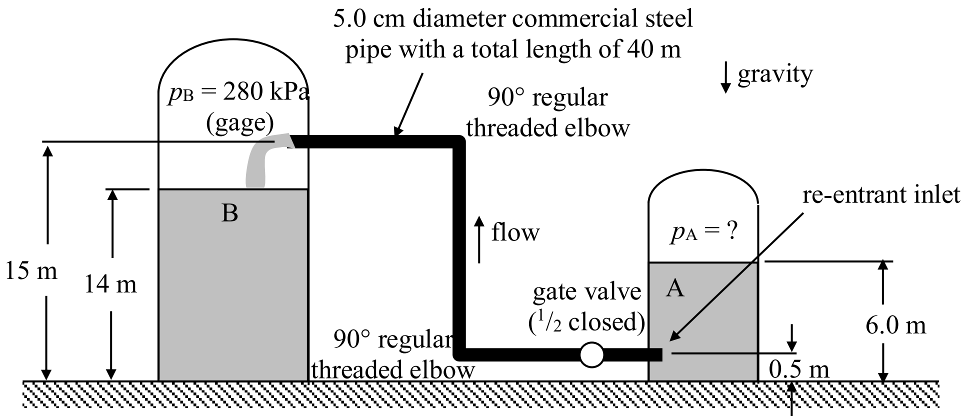
$$H_{S,1\to 2}=0\quad \text{(There is no shaft work between points 1 and 2.)} \tag{6}$$

Substitute and solve for H_L.

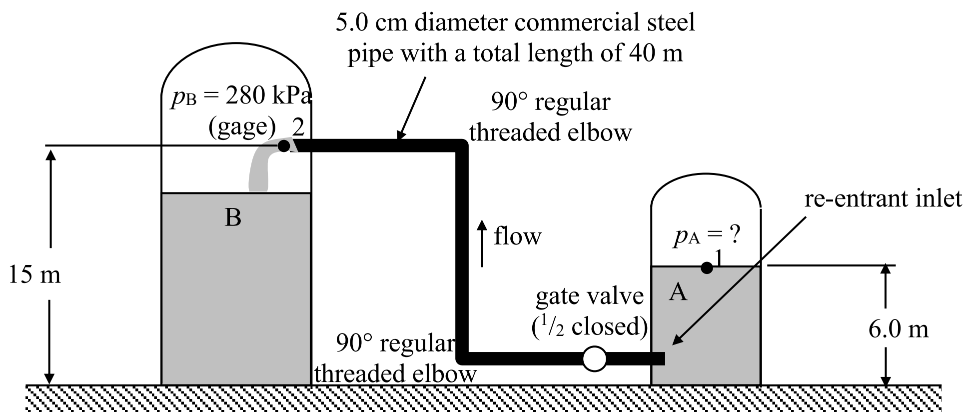
$$\boxed{H_{L,1\to 2}=-\left(\frac{p_2-p_1}{\rho g}\right)-(z_2-z_1)} \tag{7}$$

$$\boxed{\therefore H_{L,1\to 2}=2940-225\text{ ft}=2720\text{ ft}} \tag{8}$$

Consider the pipe system shown below in which water (with a density of 1.0E3 kg/m^3 and a dynamic viscosity of 1.3E-3 Pa·s) flows from the tank A to tank B. If the required flow rate is 1.0E-2 m^3/s, what is the required gage pressure in tank A?



SOLUTION:



Apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the end of the pipe leading into tank B (point 2).

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12} \quad (1)$$

where

$p_1 = p_A = ?$ and $p_2 = p_B = 2.8\text{E}5$ Pa (gage) (given)

$\bar{V}_1 \approx 0$ (large tank) $\quad \bar{V}_2 = \bar{V} \quad \alpha_2 \approx 1$ (Assume turbulent flow.)

$z_1 = 6.0\text{E}0$ m and $z_2 = 1.5\text{E}1$ m (given)

$H_{S,12} = 0$ (no fluid machinery between points 1 and 2)

$$H_{L,12} = f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}+K_{\substack{\text{re-entrant}\\ \text{inlet}}}\frac{\bar{V}^2}{2g}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}\frac{\bar{V}^2}{2g}+2K_{\substack{90^\circ\ \text{threaded}\\ \text{elbow}}}\frac{\bar{V}^2}{2g}$$

$$H_{L,12} = \left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\ \text{inlet}}}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}+2K_{\substack{90^\circ\ \text{threaded}\\ \text{elbow}}}\right]\frac{\bar{V}^2}{2g} \quad (2)$$

(Note that there are no exit losses at point 2 since no mixing occurs there.)

The mean velocity in the pipe is determined from the volumetric flow rate and the pipe area.

$$\bar{V} = \frac{Q}{\frac{\pi}{4}D^2}$$

Using the given data:

$$\bar{V} = \frac{1.0\text{E-2 m}^3\text{/s}}{\frac{\pi}{4}\left(5.0\text{E-2 m}\right)^2} = 5.1\text{E0 m/s}$$

The friction factor, f, is determined from the Moody chart using the Reynolds number in the pipe, Re, and the relative roughness, ε/D.

$$\text{Re} = \frac{\rho\bar{V}D}{\mu} = \frac{\left(1.0\text{E3 kg/m}^3\right)\left(5.1\text{E0 m/s}\right)\left(5.0\text{E-2 m}\right)}{\left(1.3\text{E-3 Pa}\cdot\text{s}\right)} = 2.0\text{E5} \quad \text{(Turbulent flow assumption ok.)}$$

$$\frac{\varepsilon}{D} = \frac{\left(4.5\text{E-5 m}\right)}{\left(5.0\text{E-2 m}\right)} = 9.0\text{E-4}$$

$$f = 2.1\text{E-2}$$

Hence, the major loss coefficient for the system is:

$$K_{\text{major}} = f\left(\frac{L}{D}\right) = \left(2.1E-2\right)\left(\frac{4.0\text{E1 m}}{5.0\text{E-2 m}}\right) = 1.7\text{E1}$$

The minor loss coefficients are found from minor loss tables to be:

$$K_{\substack{\text{re-entrant}\\ \text{inlet}}} = 8.0\text{E-1}$$

$$K_{\substack{\text{half open}\\ \text{gate valve}}} = 2.1\text{E0}$$

$$K_{\substack{90^\circ\text{ threaded}\\ \text{elbow}}} = 1.5\text{E0}$$

Using the given data, the total head loss (from Eqn. (2)) is:

$$H_{L,12} = \left[\left(2.1E-2\right)\left(\frac{4.0\text{E1 m}}{5.0\text{E-2 m}}\right) + 8.0\text{E-1} + 2.1\text{E0} + 2\left(1.5\text{E0}\right)\right]\frac{\left(5.1\text{E0 m/s}\right)^2}{2\left(9.8\text{E0 m/s}^2\right)}$$

$$\therefore H_{L,12} = 3.0\text{E1 m}$$

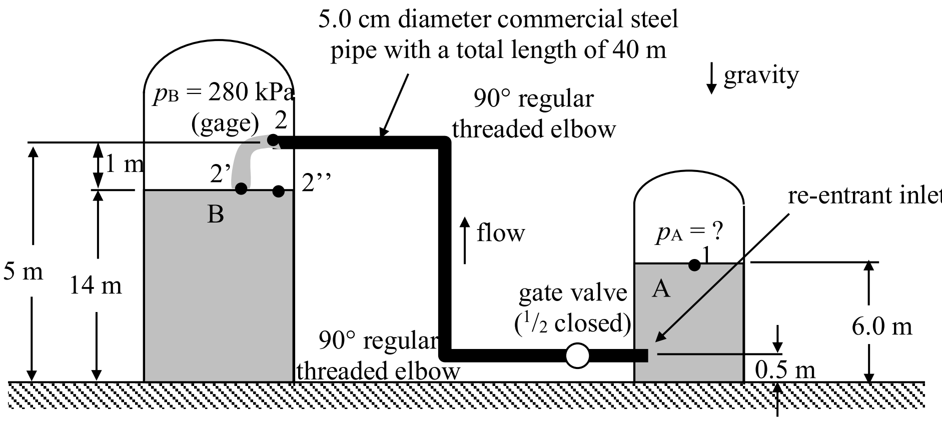
Re-arranging Eqn. (1) to solve for p_1 gives:

$$p_A = p_B + \tfrac{1}{2}\rho\bar{V}^2 + \rho g\left(z_2 - z_1 + H_{L,12}\right)$$

$$= \left(2.8\text{E5 Pa}\right) + \tfrac{1}{2}\left(1.0\text{E3 kg/m}^3\right)\left(9.5\text{E0 m/s}\right)^2 + \left(1.0\text{E3 kg/m}^3\right)\left(9.8\text{E0 m/s}^2\right)\left(1.5\text{E1 m} - 6.0\text{E0 m} + 3.0\text{E1 m}\right)$$

$$\boxed{p_A = 6.8\text{E5 Pa}}$$

Now let's solve the problem using points 2' and 2'' as shown in the figure below.



Apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the end of the stream at the surface of the free jet (point 2').

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{2'}=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{1}-H_{L,12'}+H_{S,12'} \tag{3}$$

where

$p_1 = p_A = ?$ and $p_{2'} = p_B = 2.8\text{E}5$ Pa (gage) (given)

$\bar{V}_1 \approx 0$ (large tank)

$\bar{V}_{2'}^2 = \bar{V}_2^2 + 2g(1\text{ m})$ (using Bernoulli's Eqn applied from the end of the pipe to the surface of tank B)

$\bar{V}_{2'} = 6.7\text{E}0$ m/s

$\alpha_{2'} \approx 1$ (Assume turbulent flow.)

$z_1 = 6.0\text{E}0$ m and $z_{2'} = 1.4\text{E}1$ m (given)

$H_{S,12'} = 0$ (no fluid machinery between points 1 and 2)

$$H_{L,12'} = f\left(\frac{L}{D}\right)\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{re-entrant}\\\text{inlet}}}\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}\frac{\bar{V}_2^2}{2g}+2K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}\frac{\bar{V}_2^2}{2g}$$

$$H_{L,12'} = \left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\\text{inlet}}}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}+2K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}\right]\frac{\bar{V}_2^2}{2g} \tag{4}$$

(Note that there are no exit losses from point 2 to point 2' since the kinetic energy in the stream hasn't been dissipated.)

Using the same data as in the previous solution, except for the velocity at 2' and the elevation at 2',

$$p_A = p_B + \tfrac{1}{2}\rho\bar{V}_{2'}^2 + \rho g\left(z_{2'} - z_1 + H_{L,12'}\right)$$

$$= \left(2.8\text{E}5\text{ Pa}\right)+\tfrac{1}{2}\left(1.0\text{E}3\text{ kg/m}^3\right)\left(6.7\text{E}0\text{ m/s}\right)^2+\left(1.0\text{E}3\text{ kg/m}^3\right)\left(9.8\text{E}0\text{ m/s}^2\right)\left(1.4\text{E}1\text{ m}-6.0\text{E}0\text{ m}+3.0\text{E}1\text{ m}\right)$$

$\boxed{p_A = 6.8\text{E}5\text{ Pa}}$ (Same answer as found previously!)

Now apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the surface of the tank (point 2'').

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{2''}=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{1}-H_{L,12''}+H_{S,12''} \tag{5}$$

where

$p_1 = p_A = ?$ and $p_{2'} = p_B = 2.8\text{E}5 \text{ Pa (gage)}$ (given)

$\bar{V}_1 \approx 0$ (large tank)

$\bar{V}_{2''} \approx 0$ (surface of large tank)

$z_1 = 6.0\text{E}0 \text{ m}$ and $z_{2''} = 1.4\text{E}1 \text{ m}$ (given)

$H_{S,12''} = 0$ (no fluid machinery between points 1 and 2)

$$H_{L,12''} = f\left(\frac{L}{D}\right)\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{re-entrant}\\ \text{inlet}}}\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}\frac{\bar{V}_2^2}{2g}+2K_{\substack{90^\circ\text{ threaded}\\ \text{elbow}}}\frac{\bar{V}_2^2}{2g}+K_{\text{exit}}\frac{\bar{V}_{2'}^2}{2g}$$

$$H_{L,12''} = \left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\ \text{inlet}}}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}+2K_{\substack{90^\circ\text{ threaded}\\ \text{elbow}}}\right]\frac{\bar{V}_2^2}{2g}+K_{\text{exit}}\frac{\bar{V}_{2'}^2}{2g} \tag{6}$$

(Note that the kinetic energy in the stream is dissipated when going from point 2' to point 2''. Thus, the correct velocity to use in the velocity head term is the velocity at 2'.)

Using the same data as in the previous solution, except for the velocity at 2' and the elevation at 2',

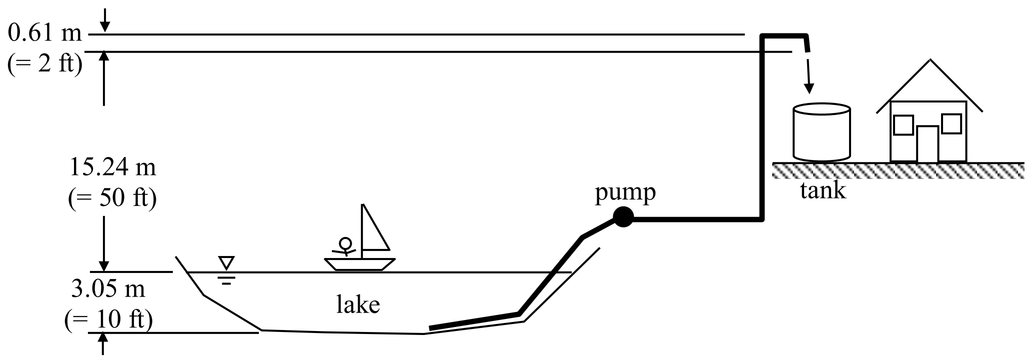
$$p_A = p_B + \rho g\left(z_{2''} - z_1 + H_{L,12''}\right)$$
$$= \left(2.8\text{E}5 \text{ Pa}\right)+\left(1.0\text{E}3 \text{ kg/m}^3\right)\left(9.8\text{E}0 \text{ m/s}^2\right)\left(1.4\text{E}1 \text{ m}-6.0\text{E}0 \text{ m}+3.2\text{E}1 \text{ m}\right)$$

$\boxed{p_A = 6.8\text{E}5 \text{ Pa}}$ (Same answer as found previously!)

You purchase a cottage at a lake and need to install a pump to feed water to the house. You plan to pump water at night to fill a storage tank you've installed next to the cottage. The pipes and fittings you have chosen to use for the installation are listed in the table below.

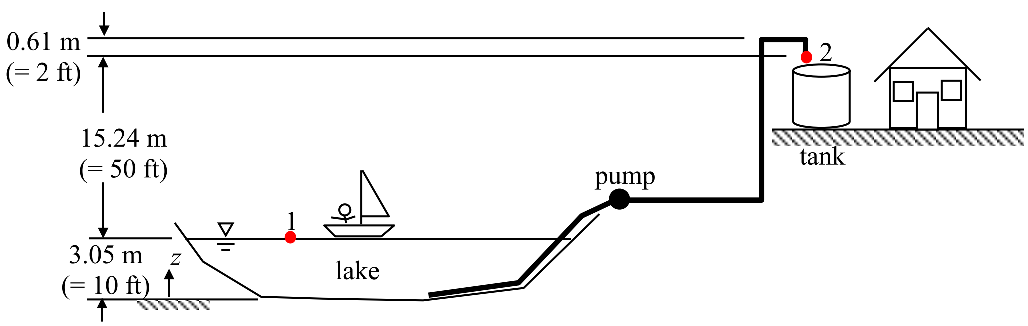
a. What is the minimum head rise across a pump that is capable of providing a flow rate of 18.93 liters per minute (= 5 gpm) of water to the tank?

b. What power should be supplied to the pump assuming the pump efficiency is 65%.



System Component	Amount
straight, smooth plastic pipe, 5.08 cm (= 2 in.) diameter	28.96 m (=95 ft)
re-entrant inlet	1
regular 90º flanged elbow	10
union	8
fully open globe valve	1
fully open gate valve	4

SOLUTION:



Apply the Extended Bernoulli Equation from point 1 to point 2.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2} \quad (1)$$

where

$$p_2=p_1=p_{\text{atm}} \quad (2)$$

$$\bar{V}_1\approx 0 \quad (3)$$

$$\bar{V}_2=\bar{V}_P=\frac{Q}{\frac{\pi D^2}{4}} \quad \text{and} \quad \alpha_2\approx 1 \text{ (assuming turbulent flow)} \quad (4)$$

$$z_2-z_1=15.24\text{ m} \quad (5)$$

$$H_{L,1\to 2} = \sum_i K_i \frac{\bar{V}_i^2}{2g} = K_{\text{major}}\frac{\bar{V}_P^2}{2g} + K_{\text{inlet}}\frac{\bar{V}_P^2}{2g} + 10K_{\text{elbow}}\frac{\bar{V}_P^2}{2g} + 8K_{\text{union}}\frac{\bar{V}_P^2}{2g} + K_{\substack{\text{globe}\\\text{valve}}}\frac{\bar{V}_P^2}{2g} + 4K_{\substack{\text{gate}\\\text{valve}}}\frac{\bar{V}_P^2}{2g}$$
$$= \left(f\frac{L}{D} + K_{\text{inlet}} + 10K_{\text{elbow}} + 8K_{\text{union}} + K_{\substack{\text{globe}\\\text{valve}}} + 4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{\bar{V}_P^2}{2g} \tag{6}$$

(where $\bar{V}_P = \bar{V}_2 = \frac{Q}{\frac{\pi D^2}{4}}$)

$$H_{S,1\to 2} = ? \tag{7}$$

Substitute into the Extended Bernoulli Equation and re-arrange.

$$\frac{\bar{V}_P^2}{2g} + z_2 = z_1 - \left(f\frac{L}{D} + K_{\text{inlet}} + 10K_{\text{elbow}} + 8K_{\text{union}} + K_{\substack{\text{globe}\\\text{valve}}} + 4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{\bar{V}_P^2}{2g} + H_{S,1\to 2} \tag{8}$$

$$H_{S,1\to 2} = (z_2 - z_1) + \left(1 + f\frac{L}{D} + K_{\text{inlet}} + 10K_{\text{elbow}} + 8K_{\text{union}} + K_{\substack{\text{globe}\\\text{valve}}} + 4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{\bar{V}_P^2}{2g} \tag{9}$$

$$\boxed{H_{S,1\to 2} = (z_2 - z_1) + \left(1 + f\frac{L}{D} + K_{\text{inlet}} + 10K_{\text{elbow}} + 8K_{\text{union}} + K_{\substack{\text{globe}\\\text{valve}}} + 4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{8Q^2}{\pi^2 D^4 g}} \tag{10}$$

Use the given data to determine $H_{S,1\to 2}$.

$z_2 - z_1 = 15.24$ m
$L = 28.96$ m
$D = 5.08$ cm = 5.08e-2 m
$K_{\text{inlet}} = 0.8$
$K_{\text{elbow}} = 0.3$
$K_{\text{threaded union}} = 0.06$
$K_{\text{globe valve}} = 10$
$K_{\text{gate valve}} = 0.15$
Q = 18.93 L/min = 3.154e-4 m^3/s (= 5 gpm)
$g = 9.81$ m/s^2 (= 32.2 ft/s^2)

The friction factor is found using the Moody chart for a smooth pipe and a Reynolds number of:

$$\text{Re} = \frac{\bar{V}_P D}{\nu} = \frac{QD}{\frac{\pi D^2}{4}\nu} = \frac{4Q}{\pi D \nu} = \frac{4\left(3.154\text{e-4 m}^3/\text{s}\right)}{\pi\left(5.08\text{e-2 m}\right)\left(1\text{e-6 m}^2/\text{s}\right)} \approx 7900$$

(The turbulent flow assumption is valid!) (11)

$\Rightarrow f \approx 0.033$ (from the Moody diagram) (12)

$\boxed{\therefore H_{S,1\to 2} = 15.28\text{ m}}$ (= 50.14 ft) (13)

Note that the head loss is much smaller than the elevation head.

The power is related to the shaft head by:

$$H_{S,1\to 2} = \frac{\dot{W}_{S,1\to 2}}{\dot{m}g} = \frac{\dot{W}_{S,1\to 2}}{\rho Q g} \tag{14}$$

$$\therefore \dot{W}_{S,1\to 2} = \rho Q g H_{S,1\to 2} \tag{15}$$

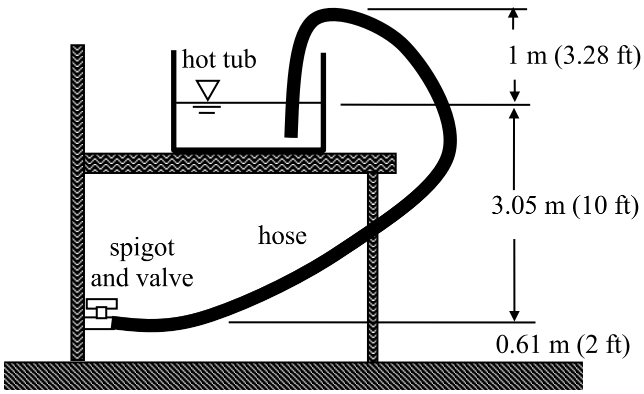
Using the given data (ρ = 1000 kg/m^3):

$\therefore \dot{W}_{S,1\to 2} = 47.3$ W (= 0.06 hp) (16)

Since the efficiency is η = 65%, the power that must be supplied to the pump is:

$$\therefore \dot{W}_{\text{supply}} = \frac{\dot{W}_{S,1\to 2}}{\eta} \Rightarrow \boxed{\therefore \dot{W}_{\text{supply}} = 72.7\text{ W}} \text{ (= 0.1 hp)} \tag{17}$$

A hot tub sits on a deck as shown in the figure below. A homeowner plans to fill the hot tub with water from a 1.91 cm (0.75 in.) diameter, 7.62 m (25 ft) length of old garden hose attached to an outdoor spigot (aka faucet) located underneath the deck. The hose has an internal roughness of 0.5 mm ($1.97*10^{-2}$ in.) and the gage water pressure just upstream of the spigot valve is 379 kPa (55 psig).

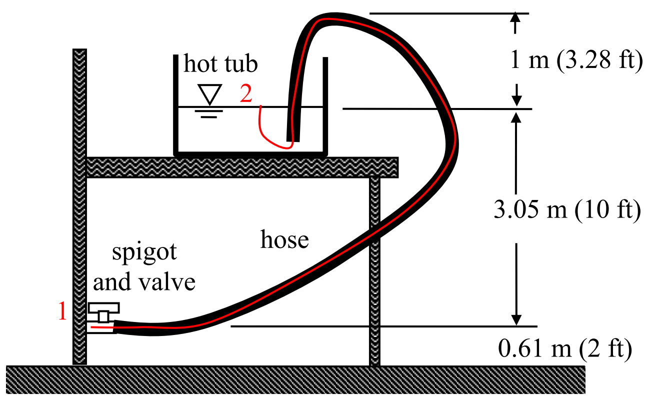


water density = 1000 kg/m^3 (62.4 lb$_m$/ft^3)
water kinematic viscosity = $1.00*10^{-6}$ m^2/s
($1.05*10^{-5}$ ft^2/s)

The minor losses due to the bends in the hose are much smaller than the minor loss due to the valve, which has a loss coefficient of 2.

Determine the volumetric flow rate of water into the hot tub. Clearly state and justify all significant assumptions.

SOLUTION:



Apply the Extended Bernoulli Equation from 1 to 2.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \tag{1}$$

where

$$p_{1,g}=3.79*10^3\text{ Pa} \text{ and } p_{2,g}=0 \tag{2}$$

$$\bar{V}_1=\bar{V}_h \text{ and } \bar{V}_2\approx 0 \tag{3}$$

$$z_2-z_1=\Delta z=3.05\text{ m} \tag{4}$$

$$H_L=\left[f\left(\frac{L}{D}\right)+K_{\text{valve}}+K_{\text{exit}}+K_{\text{bends}}\right]\frac{\bar{V}_h^2}{2g} \tag{5}$$

$$H_S=0 \tag{6}$$

Assume the flow in the hose is fully turbulent so that,

$$\alpha_1 \approx 1, \tag{7}$$

and the friction factor is solely a function of the relative roughness,

$$\frac{e}{D} = \frac{0.5*10^{-3}\ \text{m}}{1.91*10^{-2}\ \text{m}} = 2.62*10^{-2} \tag{8}$$

Using the Moody diagram, the friction factor is,

$$f \approx 0.054 \tag{9}$$

Substitute into Eq. (1) and solve for the average water speed in the hose.

$$\Delta z = \frac{p_{1,g}}{\rho g} + \frac{\bar{V}_h^2}{2g} - \left[f\left(\frac{L}{D}\right) + K_{\text{valve}} + K_{\text{exit}} + K_{\text{bends}} \right] \frac{\bar{V}_h^2}{2g} \tag{10}$$

Note that the major loss coefficient of,

$$K_{\text{major}} = f\left(\frac{L}{D}\right) = 21.65, \tag{11}$$

is larger than the valve and exit losses (recall that from the problem statement, $K_{\text{bends}} << K_{\text{valve}}$, and thus the bend losses may be neglected), with $K_{\text{valve}}/K_{\text{major}} = 0.09$ and $K_{\text{exit}}/K_{\text{major}} = 0.05$. We should retain these minor losses since they are not small enough to be neglected.

Solving Eq. (10) for the hose average velocity gives,

$$\bar{V}_h = \sqrt{\frac{2g\left(\Delta z - \frac{p_{1,g}}{\rho g}\right)}{1 - f\left(\frac{L}{D}\right) - K_{\text{valve}} - K_{\text{exit}}}} \Rightarrow \bar{V}_h = 5.4\ \text{m/s} \tag{12}$$

The volumetric flow rate is,

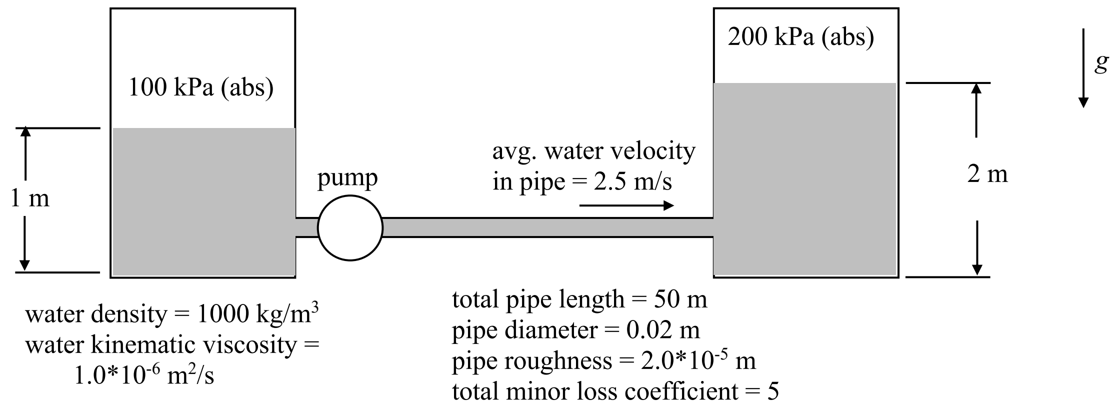
$$Q = \bar{V}_h \tfrac{\pi}{4} D^2 \Rightarrow \boxed{Q = 1.6*10^{-3}\ \text{m}^3/\text{s}} \tag{13}$$

Verify that the flow is indeed in the fully turbulent zone.

$$\text{Re}_D = \frac{\bar{V}_h D}{\nu} = \frac{(5.4\ \text{m/s})(1.91*10^{-2}\ \text{m})}{1.0*10^{-6}\ \text{m}^2/\text{s}} = 1.0*10^5 \tag{14}$$

This value of the Reynolds number is in the fully turbulent zone, so the assumption of fully turbulent flow was a good one.

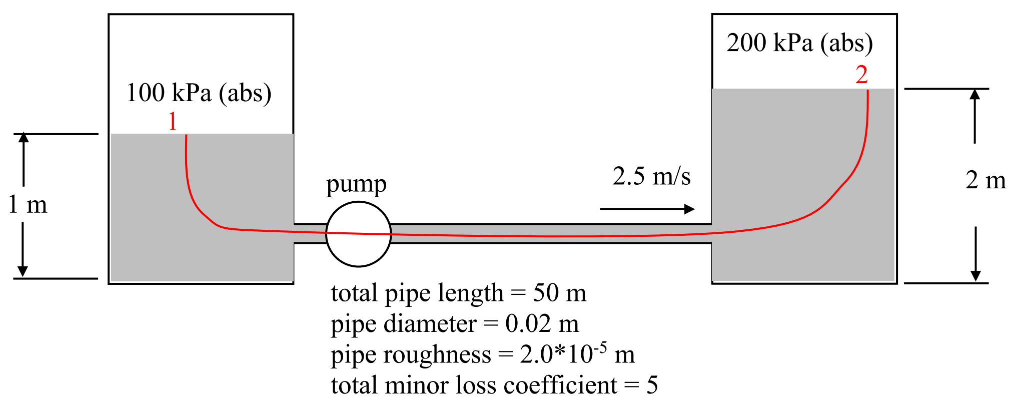
Consider the pipe system shown in the figure below.



Determine the power the pump must provide to the water to maintain the given conditions.

SOLUTION:

Apply the Extended Bernoulli Equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \tag{1}$$

Re-arrange to solve for the shaft head term,

$$H_S=\left(\frac{p_2-p_1}{\rho g}\right)+\left(\alpha_2\frac{\bar{V}_2^2}{2g}-\alpha_1\frac{\bar{V}_1^2}{2g}\right)+\left(z_2-z_1\right)+H_L \tag{2}$$

where

$$g=9.81\ \text{m/s}^2\ \text{ and }\ \rho=1000\ \text{kg/m}^3 \tag{3}$$

$$p_1=100\ \text{kPa (abs)}\ \text{ and }\ p_2=200\ \text{kPa (abs)} \tag{4}$$

$$\bar{V}_1\approx 0\ \text{ and }\ \bar{V}_2\approx 0 \tag{5}$$

$$z_1=1\ \text{m}\ \text{ and }\ z_2=2\ \text{m} \tag{6}$$

$$H_L=\left[f\left(\frac{L}{D}\right)+K_{\text{minor}}\right]\frac{\bar{V}_{\text{pipe}}^2}{2g} \tag{7}$$

and

$$L = 50 \text{ m} \text{ and } D = 0.02 \text{ m} \tag{8}$$

$$K_{\text{minor}} = 5 \tag{9}$$

$$e/D = \left(2.0*10^{-5} \text{ m}\right)/\left(0.02 \text{ m}\right) = 0.001 \tag{10}$$

$$\text{Re} = \frac{\bar{V}_{\text{pipe}} D}{\nu} = \frac{\left(2.5 \text{ m/s}\right)\left(0.02 \text{ m}\right)}{\left(1.0*10^{-6} \text{ m}^2/\text{s}\right)} = 50{,}000 \tag{11}$$

Use the Moody diagram to find the friction factor for this Reynolds number and relative roughness,

$$f = 0.024 \tag{12}$$

Using the given data,

$$H_S = 31.7 \text{ m} \tag{13}$$

The power may be found from the shaft head term using,

$$\dot{W}_S = \rho Q g H_S = \rho V_{\text{pipe}} \tfrac{\pi}{4} D^2 g H_S \tag{14}$$

Thus,

$$\boxed{\dot{W}_S = 244 \text{ W}} \tag{15}$$

11.7.1. Serial Pipe Systems

Serial pipe systems have multiple pipes that have the same inlet conditions and the same outlet conditions (Figure 11.12). For these systems one simply applies the EBE separately for each pipe.

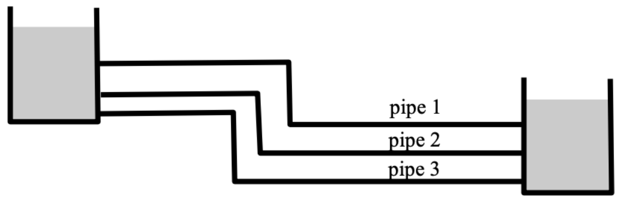


Figure 11.12. An example of a serial pipe system, which has multiple pipes, but the same inlet and outlet conditions.

11.7.2. Parallel Pipe Systems

Parallel pipe systems involve pipes that have intersections, i.e., nodes (Figure 11.13). These pipe systems are more challenging to solve. The EBE can be used between nodes and between nodes and inlets and outlets. Conservation of Mass should be applied at each node. The result will be a system of non-linear equations (due to velocity squared terms that appear in the EBE) that must be solved simultaneously. Often these systems of equations are solved computationally using iterative techniques.

Interestingly, pipe networks have many similarities with electrical networks, with pipe resistances corresponding to electrical resistances, flow rates corresponding to current, and head differences (due to elevation differences or pumps) corresponding to voltage differences. There are other electrical analogies too. For example surge tanks have properties similar to capacitors, heavy paddle wheels have properties similar to inductors, and ball and check valves act as diodes.

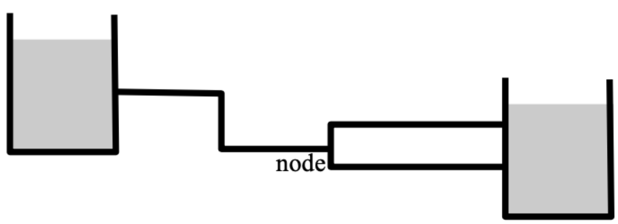
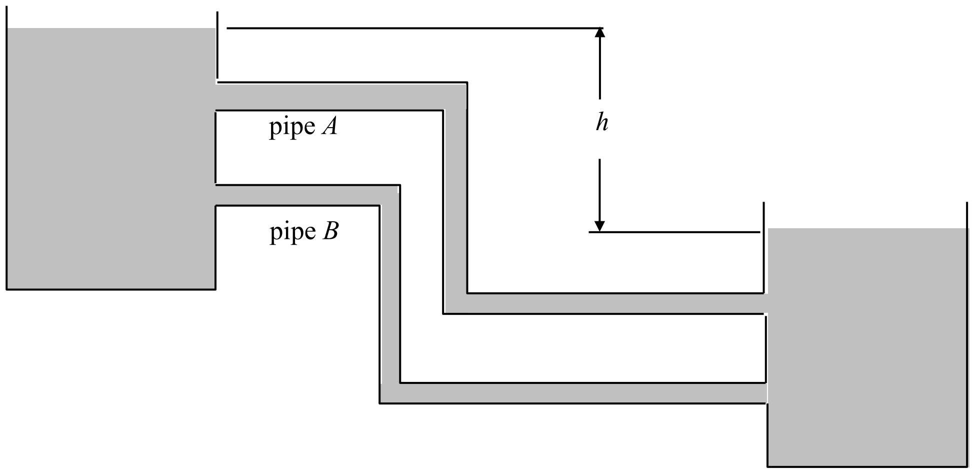


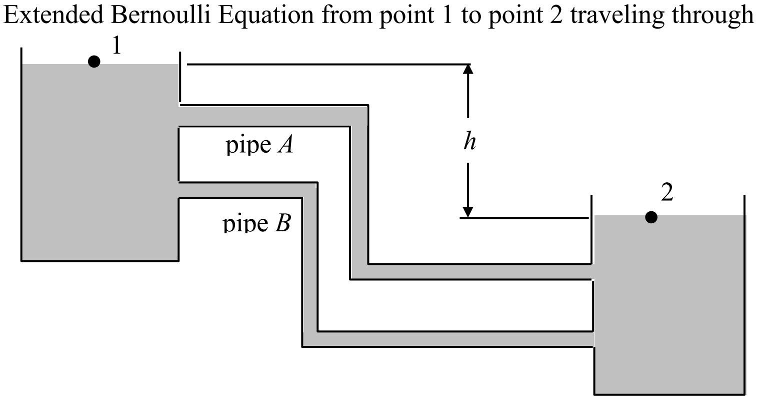
Figure 11.13. An example of a parallel pipe system, which has multiple, interconnecting pipes. The location at which pipes intersect is known as a "node".

Two water reservoirs are connected by galvanized iron pipes. Assume D_A = 75 mm, D_B = 50 mm, and h = 10.5 m. The length of both pipes is 100 m. Compare the head losses in pipes *A* and *B*. Compute the volume flow rate in each pipe.



SOLUTION:

Apply the Extended Bernoulli Equation from point 1 to point 2 traveling through each pipe.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}$$

where

$p_1 = p_2 = p_{atm}$ (free surfaces)

$\bar{V}_1 \approx \bar{V}_2 \approx 0$ (surface of large tanks)

$z_1 - z_2 = h$ (given)

$H_{S,12} = 0$ (no fluid machines between points 1 and 2)

$$H_{L,12,A} = K_{\text{major},A}\frac{\bar{V}_A^2}{2g}+K_{\text{entrance}}\frac{\bar{V}_A^2}{2g}+2K_{\text{elbow}}\frac{\bar{V}_A^2}{2g}+K_{\text{exit}}\frac{\bar{V}_A^2}{2g}$$

$$H_{L,12,B} = K_{\text{major},B}\frac{\bar{V}_B^2}{2g}+K_{\text{entrance}}\frac{\bar{V}_B^2}{2g}+2K_{\text{elbow}}\frac{\bar{V}_B^2}{2g}+K_{\text{exit}}\frac{\bar{V}_B^2}{2g}$$

and

$$K_{\text{major},A} = f_A\left(\frac{L_A}{D_A}\right) \quad \text{and} \quad K_{\text{major},B} = f_B\left(\frac{L_B}{D_B}\right)$$

Substituting into the Extended Bernoulli Equation gives:

$$h = \left[f_A \left(\frac{L_A}{D_A} \right) + K_{\text{entrance}} + 2K_{\text{elbow}} + K_{\text{exit}} \right] \frac{\bar{V}_A^2}{2g} \tag{1}$$

$$h = \left[f_B \left(\frac{L_B}{D_B} \right) + K_{\text{entrance}} + 2K_{\text{elbow}} + K_{\text{exit}} \right] \frac{\bar{V}_B^2}{2g} \tag{2}$$

From Eqns. (1) and (2) we observe that the head loss in each pipe is the same and equal to 10.5 m.

The pipes are made of galvanized iron so the roughness of the pipes is ε = 0.15 mm (found from an average roughness table). Hence, the relative roughness in each pipe is:

$$\left.\frac{\varepsilon}{D}\right|_A = \frac{0.15 \text{ mm}}{75 \text{ mm}} = 0.0020 \quad \text{and} \quad \left.\frac{\varepsilon}{D}\right|_B = \frac{0.15 \text{ mm}}{50 \text{ mm}} = 0.0030$$

Since we don't yet know the velocity in each pipe, assume that the flows are in the wholly turbulent flow region so that the friction factor is independent of the Reynolds number. For this case, the Moody chart (or the Colebrook relation) indicates that the friction factors corresponding to the relative roughnesses determined above are:

$$f_A = 0.0234 \quad \text{and} \quad f_B = 0.0262$$

Substitute the given data into Eqns. (1) and (2).

h	=	10.5 m
K_{entrance}	=	0.5 (assuming a sharp-edged entrance)
K_{elbow}	=	1.5 (assuming 90° threaded elbows)
K_{exit}	=	1.0
g	=	9.81 m/s^2
L_A	=	100 m
L_B	=	100 m
D_A	=	75 mm = 7.5*10^{-2} m
D_B	=	50 mm = 5.0*10^{-2} m
f_A	=	0.0234 (from above)
f_B	=	0.0262 (from above)
ν_{H20}	=	1.01*10^{-6} m^2/s (water at 20 °C)

Note that the sum of the minor loss coefficients (= 4.5 = 0.5 + 2*1.5 + 1) are not insignificant compared to the major loss coefficients so the minor losses cannot be neglected without significant error.

$$K_{\text{major},A} = 31.2 \quad \text{and} \quad K_{\text{major,B}} = 52.4$$

Solving for the average velocities gives:

$$\bar{V}_A = 2.40 \text{ m/s} \quad \text{and} \quad \bar{V}_B = 1.90 \text{ m/s}$$

The corresponding Reynolds numbers are:

$$\text{Re}_A = \frac{\bar{V}_A D_A}{\nu_{\text{H20}}} = 1.78 * 10^5 \quad \text{and} \quad \text{Re}_B = \frac{\bar{V}_B D_B}{\nu_{\text{H20}}} = 9.42 * 10^4 \tag{3}$$

Unfortunately, these Reynolds numbers do not put us in the wholly turbulent zone on the Moody chart (although it's very close) so we must try iterating to a solution instead. For a new choice of friction factors, use the Reynolds number given in Eqn. (3) and consult the Moody chart (or the Colebrook formula).

$$f_A = 0.0244 \quad \text{and} \quad f_B = 0.0275$$

Using these friction factors we find:

$$\bar{V}_A = 2.36 \text{ m/s} \quad \text{and} \quad \bar{V}_B = 1.86 \text{ m/s}$$

and

$$\text{Re}_A = \frac{\bar{V}_A D_A}{\nu_{\text{H20}}} = 1.75*10^5 \quad \text{and} \quad \text{Re}_B = \frac{\bar{V}_B D_B}{\nu_{\text{H20}}} = 9.21*10^4$$

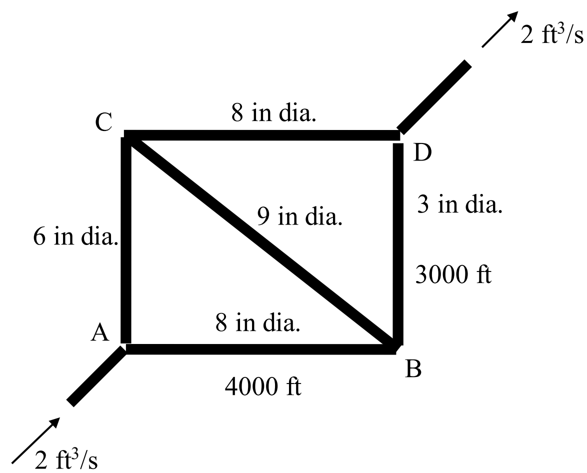
Fortunately these Reynolds numbers give the same friction factors that we started with.

Thus, the volumetric flow rate through each pipe is:

$$\boxed{Q_A = \bar{V}_A \frac{\pi D_A^2}{4} = 1.04*10^{-2} \text{ m}^3/\text{s}}$$

$$\boxed{Q_B = \bar{V}_B \frac{\pi D_B^2}{4} = 3.65*10^{-3} \text{ m}^3/\text{s}}$$

In the five-pipe horizontal network shown in the figure, assume that all pipes have a friction factor f = 0.025. For the given inlet and exit flow rate of 2 ft^3/s of water at 20 °C, determine the flow rate and direction in all pipes. If p_A=120 lb$_f$/in^2 (gage), determine the pressures at points B, C, and D.



SOLUTION:

Apply the Extended Bernoulli Equation around the loops ABCA and DBCD.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_A=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_A-H_{L,ABCA}+H_{S,ABCA} \tag{1}$$

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_D=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_D-H_{L,DBCD}+H_{S,DBCD} \tag{2}$$

Note that the shaft head terms ($H_{S,ABCA}$ and $H_{S,DBCD}$) are zero since there are no fluid machines in the loops. Simplifying Eqns. (1) and (2) gives:

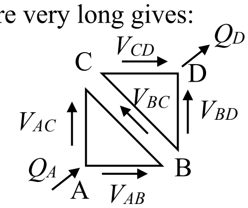
$$H_{L,ABCA}=0 \tag{3}$$

$$H_{L,DBCD}=0 \tag{4}$$

Expanding the head loss term and neglecting minor losses since the pipes are very long gives:

$$H_{L,ABCA}=f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)\frac{\bar{V}_{AB}^2}{2g}+f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\frac{\bar{V}_{BC}^2}{2g}-f_{AC}\left(\frac{L_{AC}}{D_{AC}}\right)\frac{\bar{V}_{AC}^2}{2g}=0 \tag{5}$$

$$H_{L,DBCD}=-f_{BD}\left(\frac{L_{BD}}{D_{BD}}\right)\frac{\bar{V}_{BD}^2}{2g}+f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\frac{\bar{V}_{BC}^2}{2g}+f_{CD}\left(\frac{L_{CD}}{D_{CD}}\right)\frac{\bar{V}_{CD}^2}{2g}=0 \tag{6}$$



Note that particular velocity directions have been assumed in the head loss expressions.

At each pipe node the volumetric flow rate must be conserved (conservation of mass). Hence:

at node A: $Q_A=Q_{AB}+Q_{AC} \quad\Rightarrow\quad Q_A=\bar{V}_{AB}\frac{\pi D_{AB}^2}{4}+\bar{V}_{AC}\frac{\pi D_{AC}^2}{4}$ (7)

at node B: $Q_{AB}=Q_{BC}+Q_{BD} \quad\Rightarrow\quad \bar{V}_{AB}\frac{\pi D_{AB}^2}{4}=\bar{V}_{BC}\frac{\pi D_{BC}^2}{4}+\bar{V}_{BD}\frac{\pi D_{BD}^2}{4}$ (8)

at node C: $Q_{CD}=Q_{AC}+Q_{BC} \quad\Rightarrow\quad \bar{V}_{CD}\frac{\pi D_{CD}^2}{4}=\bar{V}_{AC}\frac{\pi D_{AC}^2}{4}+\bar{V}_{BC}\frac{\pi D_{BC}^2}{4}$ (9)

at node D: $Q_D=Q_{BD}+Q_{CD} \quad\Rightarrow\quad Q_D=\bar{V}_{BD}\frac{\pi D_{BD}^2}{4}+\bar{V}_{CD}\frac{\pi D_{CD}^2}{4}$ (10)

Simplify and summarize Eqns. (5) - (10).

$$f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)\bar{V}_{AB}^2 + f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\bar{V}_{BC}^2 - f_{AC}\left(\frac{L_{AC}}{D_{AC}}\right)\bar{V}_{AC}^2 = 0 \tag{11}$$

$$-f_{BD}\left(\frac{L_{BD}}{D_{BD}}\right)\bar{V}_{BD}^2 + f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\bar{V}_{BC}^2 + f_{CD}\left(\frac{L_{CD}}{D_{CD}}\right)\bar{V}_{CD}^2 = 0 \tag{12}$$

$$\left(\frac{\pi D_{AB}^2}{4}\right)\bar{V}_{AB} + \left(\frac{\pi D_{AC}^2}{4}\right)\bar{V}_{AC} = Q_A \tag{13}$$

$$\left(\frac{\pi D_{BC}^2}{4}\right)\bar{V}_{BC} + \left(\frac{\pi D_{BD}^2}{4}\right)\bar{V}_{BD} - \left(\frac{\pi D_{AB}^2}{4}\right)\bar{V}_{AB} = 0 \tag{14}$$

$$\left(\frac{\pi D_{AC}^2}{4}\right)\bar{V}_{AC} + \left(\frac{\pi D_{BC}^2}{4}\right)\bar{V}_{BC} - \left(\frac{\pi D_{CD}^2}{4}\right)\bar{V}_{CD} = 0 \tag{15}$$

$$\left(\frac{\pi D_{BD}^2}{4}\right)\bar{V}_{BD} + \left(\frac{\pi D_{CD}^2}{4}\right)\bar{V}_{CD} = Q_D \tag{16}$$

Note that Eqn. (16) is not independent since it can be formed by adding Eqns. (13) and (14), subtracting Eqn. (15) and noting that $Q_D = Q_A$. Hence, Eqns. (11) - (15) represent five equations with five unknowns ($\bar{V}_{AB}, \bar{V}_{BC}, \bar{V}_{AC}, \bar{V}_{BD}$, and $\bar{V}_{CD}$). Note that f_{AB}, f_{BC}, f_{AC}, f_{BD}, and f_{CD} are given in the problem statement along with each pipe's length and diameter and the volumetric flowrate Q_A.

Using the given data:

$f_{\text{all pipes}}$	=	0.025
$L_{AB} = L_{CD}$	=	4000 ft
$L_{AC} = L_{BD}$	=	3000 ft
L_{BC}	=	5000 ft (from the Pythagorean theorem)
$D_{AB} = D_{CD}$	=	8/12 ft
D_{AC}	=	6/12 ft
D_{BD}	=	3/12 ft
D_{BC}	=	9/12 ft
Q_A	=	2 ft^3/s

The system of non-linear algebraic equations (Eqns. (11) - (15)) can be solved iteratively. One approach is given below.

1. Assume a value of $\bar{V}_{AB}$.
2. Solve for $\bar{V}_{AC}$ using Eqn. (13).
3. Solve for $\bar{V}_{BC}$ using Eqn. (11).
4. Solve for $\bar{V}_{CD}$ using Eqn. (15).
5. Solve for $\bar{V}_{BD}$ using Eqn. (12).
6. Solve for $\bar{V}_{AB}$ using Eqn. (14).
7. Are the $\bar{V}_{AB}$ s from step 6 and step 1 equal? If so, then the iterations are finished. If not, then choose a new value for $\bar{V}_{AB}$ and go to step 2.

After some iteration.

$\bar{V}_{AB}$	=	$3.40 * 10^0$ ft/s	$\Rightarrow$	Q_{AB}	=	$1.19 * 10^0$ ft^3/s
$\bar{V}_{AC}$	=	$4.14 * 10^0$ ft/s	$\Rightarrow$	Q_{AC}	=	$8.13 * 10^{-1}$ ft^3/s
$\bar{V}_{BC}$	=	$2.24 * 10^0$ ft/s	$\Rightarrow$	Q_{BC}	=	$9.90 * 10^{-1}$ ft^3/s
$\bar{V}_{CD}$	=	$5.17 * 10^0$ ft/s	$\Rightarrow$	Q_{CD}	=	$1.80 * 10^0$ ft^3/s
$\bar{V}_{BD}$	=	$4.02 * 10^0$ ft/s	$\Rightarrow$	Q_{BD}	=	$1.97 * 10^{-1}$ ft^3/s

To find the pressure at the various nodes, apply the Extended Bernoulli Equation between the nodes.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_B=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_A-H_{L,AB}+H_{S,AB}$$

where

$p_A = 120$ psig

$p_B = ?$

$V_A = V_B$ (the velocity just upstream of point B is equal to the velocity just downstream of point A)

$z_A = z_B$

$H_{S,AB} = 0$

$\rho_{H20\ @\ 20\ °C} = 1.94$ slug/ft^3 (Note: 1 lb$_f$ = 1 slug·ft/s^2)

$$H_{L,AB}=f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)\frac{\bar{V}_{AB}^2}{2g}$$

$$\Rightarrow\ p_B=p_A-\tfrac{1}{2}\rho\bar{V}_{AB}^2 f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)$$

$\boxed{p_B = 1.08 * 10^2 \text{ psig}}$

Using a similar approach from A to C (or from B to C):

$\boxed{p_C = 1.03 * 10^2 \text{ psig}}$

Using a similar approach from B to D (or from C to D):

$\boxed{p_D = 7.57 * 10^1 \text{ psig}}$