CHAPTER 11

Pipe Flows

11.1. Entrance Region

The flow in the entrance region is complex (Figure 11.1) and will not be investigated here. Experiments have shown that the dimensionless length of the entrance region depends on whether the entering flow is laminar or turbulent, with,

$$\text{laminar flow:} \quad \frac{L}{D} \approx 0.06\mathrm{Re}_D, \tag{11.1}$$

$$\text{turbulent flow:} \quad \frac{L}{D} \approx 4.4\mathrm{Re}_D^{1/6}, \tag{11.2}$$

where L is the length of the entrance region and D is the pipe diameter. For many engineering flows, $1 \times 10^4 < \mathrm{Re}_D < 1 \times 10^5 \implies 20 < L/D < 30$. The shorter entrance region length for turbulent flows is due to the fact that turbulent mixing rapidly averages the flow speeds across the pipe cross-section.

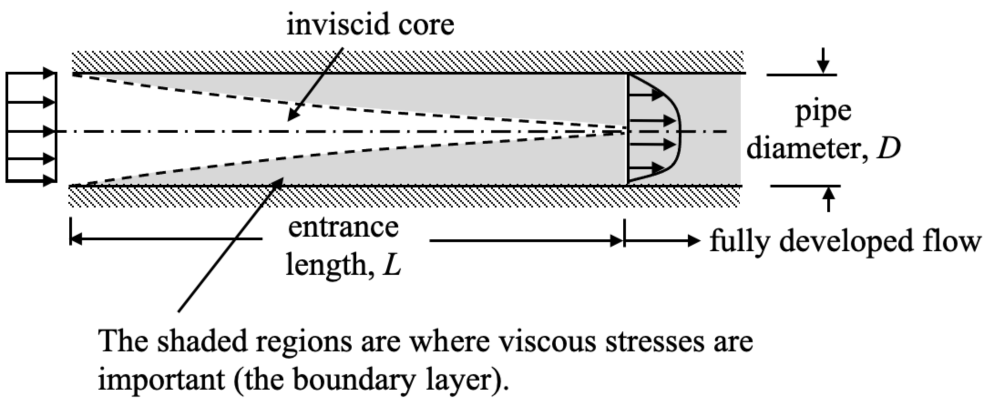


Figure 11.1. The structure of a pipe flow entrance region.

11.2. Fully Developed Laminar Circular Pipe Flow (Poiseuille Flow)

The derivation in this section was previously covered in Chapter 8 and is repeated here, in a slightly condensed form, for convenience. Consider the steady flow of an incompressible, constant viscosity, Newtonian fluid within an infinitely long, circular pipe of radius, R (Figure 11.2).

Make the following assumptions,

(1) The flow is axi-symmetric and there is no "swirl" velocity. $\implies \frac{\partial}{\partial \theta}(\dots) = 0$ and $u_\theta = 0$
(2) The flow is at steady state. $\implies \frac{\partial}{\partial t}(\dots) = 0$
(3) The flow is fully-developed in the z-direction. $\implies \frac{\partial u_r}{\partial z} = \frac{\partial u_z}{\partial z} = 0$
(4) There are no body forces. $\implies f_r = f_\theta = f_z = 0$

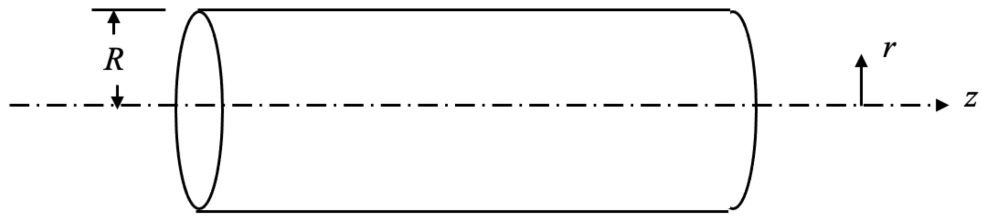


Figure 11.2. A schematic of flow through a circular pipe.

Let's first examine the Continuity Equation,

$$\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0. \tag{11.3}$$

From assumptions #1 and #3,

$$r u_r = \text{constant}. \tag{11.4}$$

Since there is no flow through the walls, the constant must be zero and, thus,

$$u_r = 0 \quad \text{(Call this condition \# 5.)} \tag{11.5}$$

Now let's examine the Navier-Stokes equation in the z-direction,

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho f_z. \tag{11.6}$$

We can simplify this equation using our assumptions,

$$\rho\left(\underbrace{\frac{\partial u_z}{\partial t}}_{=0(\#2)} + \underbrace{u_r}_{=0(\#5)}\frac{\partial u_z}{\partial r} + \underbrace{\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}}_{=0(\#1)} + u_z\underbrace{\frac{\partial u_z}{\partial z}}_{=0(\#3)}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\underbrace{\frac{\partial^2 u_z}{\partial \theta^2}}_{=0(\#1)} + \underbrace{\frac{\partial^2 u_z}{\partial z^2}}_{=0(\#3)}\right] + \rho\underbrace{f_z}_{=0(\#4)}, \tag{11.7}$$

$$\implies \frac{d}{dr}\left(r\frac{du_z}{dr}\right) = \frac{r}{\mu}\frac{dp}{dz}, \tag{11.8}$$

$$\implies r\frac{du_z}{dr} = \frac{r^2}{2\mu}\frac{dp}{dz} + c_1, \tag{11.9}$$

$$\implies u_z = \frac{r^2}{4\mu}\frac{dp}{dz} = c_1 \ln r + c_2. \tag{11.10}$$

Note that in the previous derivation the fact that u_z is a function only of r has been used to change the partial derivatives to ordinary derivatives. Furthermore, examining the Navier-Stokes equations in the r and θ directions demonstrates that the pressure, p, is a function only of z and, thus, ordinary derivatives can be used when differentiating the pressure with respect to z.

Now let's apply boundary conditions to determine the unknown constants c_1 and c_2. First, note that the fluid velocity in a pipe must remain finite as $r \to 0$ so the constant c_1 must be zero (this is a type of kinematic boundary condition). Also, the pipe wall is fixed so we have $u_z(r = R) = 0$ (no-slip condition). After applying boundary conditions we have,

$$\boxed{u_z = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]}. \tag{11.11}$$

This is known as Poiseuille Flow in a Circular Pipe.

Notes:

(1) The velocity profile is a paraboloid with the maximum velocity occurring along the centerline. The average velocity in the pipe is found from,

$$\bar{u} = \frac{1}{\pi R^2}\int_{r=0}^{r=R} u_z(2\pi r dr) = \frac{R^2}{8\mu}\left(-\frac{dp}{dz}\right) = \frac{D^2}{32\mu}\left(-\frac{dp}{dz}\right) = \frac{1}{2}u_{\max}, \tag{11.12}$$

where $u_{\max}$ is the maximum fluid speed and D is the pipe diameter.

(2) The volumetric flow rate through the pipe is,

$$Q = \bar{u}\left(\frac{\pi}{4}D^2\right) = \frac{\pi D^4}{128\mu}\left(-\frac{dp}{dz}\right). \tag{11.13}$$

(3) We can determine stresses using the constitutive relations for a Newtonian fluid. The shear stress that the pipe walls apply to the fluid, τ_w, is,

$$\tau_w = \frac{R}{2}\left(\frac{dp}{dz}\right) = \frac{-4\mu\bar{u}}{R}, \tag{11.14}$$

where $\bar{u}$ is the average speed in the pipe. Note that an alternate method for determining the average wall shear stress, which in this case is equal to the exact wall shear stress, is to balance shear forces and pressure forces on a small slice of the flow as shown in Figure 11.3.

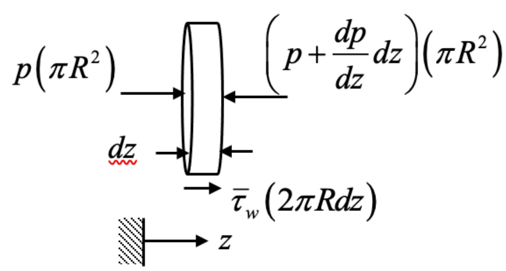


Figure 11.3. A free body diagram showing the forces on a thin disk of fluid in the pipe.

In engineering applications, it is common to express the average shear stress in terms of a dimensionless (Darcy) friction factor, f_D, which is defined as,

$$f_D := \left|\frac{4\overline{\tau_w}}{\frac{1}{2}\rho\bar{u}^2}\right| = 64\left(\frac{\mu}{\rho\bar{u}D}\right) = \frac{64}{\mathrm{Re}_D}, \tag{11.15}$$

where $D = 2R$ is the pipe diameter and Re_D is the Reynolds number based on the pipe diameter. The Darcy friction factor appears in the Moody chart for incompressible, viscous pipe flow. Note again that this solution is only valid only for a laminar flow. The condition for the flow to remain laminar is found experimentally to be,

$$\mathrm{Re}_D = \frac{\rho\bar{u}D}{\mu} < 2300. \tag{11.16}$$

(4) Re-write Eq. (11.12),

$$\bar{u} = \frac{D^2}{32\mu}\left(-\frac{dp}{dz}\right) \implies |\bar{u}| = \frac{D^2}{32\mu}\left(\frac{\Delta p}{L}\right), \tag{11.17}$$

where, in the fully developed region, the pressure gradient remains constant and we may write dp/dz as $\Delta p/L$ where Δp is the pressure drop over a length L of the pipe (Figure 11.4). Re-arranging Eq. (11.17) and dropping the absolute value symbol for convenience,

$$\Delta p = \frac{32\mu\bar{u}L}{D^2}. \tag{11.18}$$

Make the previous equation dimensionless by dividing through by the dynamic pressure based on the average flow speed,

$$\frac{\Delta p}{\frac{1}{2}\rho\bar{u}^2} = \frac{64\mu}{\rho\bar{u}D}\left(\frac{L}{D}\right) = \left(\frac{64}{\mathrm{Re}_D}\right)\left(\frac{L}{D}\right). \tag{11.19}$$

The dimensionless pressure drop is also referred to as a loss coefficient, k. Hence, for a laminar flow, the loss coefficient corresponding to the viscous stresses at the pipe walls is,

$$k_{\substack{\text{laminar,}\\ \text{wall stresses}}} = \left(\frac{64}{\mathrm{Re}_D}\right)\left(\frac{L}{D}\right) = f_D\left(\frac{L}{D}\right). \tag{11.20}$$

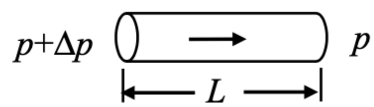


Figure 11.4. A schematic showing the change in pressure over the pipe length.

A liquid with a specific gravity of 0.95 flows steadily at an average velocity of 10 m/s through a horizontal, smooth tube of diameter 5 cm. The fluid pressure is measured at 1 m intervals along the pipe as follows:

x [m]	0	1	2	3	4	5	6
p [kPa]	304	273	255	240	226	213	200

a. Estimate the average wall shear stress, in Pa, in the fully developed region of the pipe.
b. What is the approximate wall shear stress between stations 1 and 2? State any significant assumptions you make.

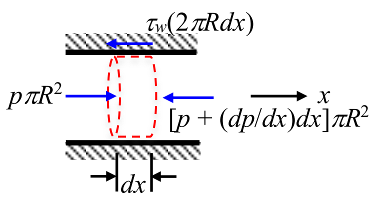
SOLUTION:

First determine the fully developed region by examining the pressure gradient in the pipe. The pressure gradient is constant in the fully developed region.

x [m]	0	1	2	3	4	5	6
p [kPa]	304	273	255	240	226	213	200
dp/dx [kPa/m]		-31	-18	-15	-14	-13	-13

Hence, the fully developed region starts at $x = 4$ m where the pressure drop remains constant at $dp/dx = -13$ kPa/m.

To determine the average wall shear stress in the pipe, apply the linear momentum equation in the x direction to the control volume shown in the figure below.



$$\frac{d}{dt}\int_{CV} u_x \rho \, dV + \int_{CS} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{S,x} + F_{B,x}\,, \tag{1}$$

where,

$$\frac{d}{dt}\int_{CV} u_x \rho \, dV = 0 \quad \text{(steady flow)}, \tag{2}$$

$$F_{B,x} = 0 \quad \text{(no body forces in } x\text{-direction)}, \tag{3}$$

$$F_{S,x} = p\pi R^2 - \left(p + \frac{dp}{dx}dx\right)\pi R^2 - \overline{\tau}_w\left(2\pi R dx\right) = -\frac{dp}{dx}dx\pi R^2 - \overline{\tau}_w\left(2\pi R dx\right), \tag{4}$$

$$\int_{CS} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = 0 \tag{5}$$

(since for a fully-developed flow, the inlet and outlet velocity profiles are identical)

Substitute and simplify,

$$0 = -\frac{dp}{dx}dx\pi R^2 - \overline{\tau}_w\left(2\pi R dx\right), \tag{6}$$

$$\boxed{\overline{\tau}_w = -\frac{R}{2}\frac{dp}{dx}}. \tag{7}$$

Using the given data,

R = (0.05/2) m = 0.025 m

dp/dx = -13 kPa/m

$\boxed{\Rightarrow \bar{\tau}_w = 163 \text{ Pa}}$

For part (b), apply the same linear momentum equation, except that between stations 1 and 2, the velocity profile is not fully developed, hence the momentum flux term in the linear momentum equation (Eq. (5)) won't be zero. However, <u>if the flow is turbulent</u>, as would be expected for such a large velocity and assuming a liquid viscosity similar to that of water, the velocity profile will not change considerably as the flow continues downstream in the entrance region. The reason for this is that a turbulent velocity profile already looks like an average velocity profile due to the radial mixing associated with turbulence. Hence, <u>although the momentum flux term isn't exactly zero, it is expected to be small in comparison to the pressure gradient term</u>. As a result, even in the entrance region the average wall shear stress may be found using,

$$\boxed{\bar{\tau}_w \approx -\frac{R}{2}\frac{dp}{dx}}. \quad (8)$$

Using the given data between stations 1 and 2,

R = (0.05/2) m = 0.025 m

dp/dx = -18 kPa/m

$\boxed{\Rightarrow \bar{\tau}_w = 225 \text{ Pa}}$

Equation (11.29) is implicit in f_D, which means that an iterative approach must be used to solve for f_D as a function of Re_D. A number of approximations to this relation have been proposed that are easier to solve. For example, Blasius, a student of Prandtl's, suggested the following approximation,

$$\boxed{f_D \approx \frac{0.316}{\mathrm{Re}_D^{1/4}}}, \tag{11.30}$$

which is valid for $4000 < \mathrm{Re}_D < 1 \times 10^5$.

11.3.2. Turbulent Flow in a Very Rough Pipe

The roughness of the pipe walls can significantly affect the friction factor for turbulent flows (roughness has a negligible effect on the friction factor for laminar flows). Recall from the Law of the Wall that the time averaged velocity in the laminar sub-layer is,

$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu} \quad \text{for} \quad 0 \leq \frac{yu^*}{\nu} \leq 5. \tag{11.31}$$

Thus, the thickness of the laminar sub-layer, δ_{LSL}, is,

$$\frac{\delta_{\mathrm{LSL}} u^*}{\nu} = 5 \implies \delta_{\mathrm{LSL}} = \frac{5\nu}{u^*}. \tag{11.32}$$

Since,

$$u^* = \sqrt{\frac{\tau_w}{\rho}} = \bar{u}\sqrt{\frac{f_D}{8}} \quad \text{(refer to Eq. (11.24))}, \tag{11.33}$$

we have,

$$\delta_{\mathrm{LSL}} = \frac{5\nu}{\bar{u}}\sqrt{\frac{8}{f_D}} \implies \frac{\delta_{\mathrm{LSL}}}{D} = \frac{5\nu}{\bar{u}D}\sqrt{\frac{8}{f_D}} = \frac{5}{\mathrm{Re}_D}\sqrt{\frac{8}{f_D}}, \tag{11.34}$$

$$\therefore \frac{\delta_{\mathrm{LSL}}}{D} = \frac{14.1}{\mathrm{Re}_D\sqrt{f_D}}. \tag{11.35}$$

Thus, if the wall roughness, ϵ, is much smaller than the laminar sub-layer thickness, then we'll still have a laminar sub-layer and the flow won't be significantly affected by the wall roughness, i.e., we may treat the wall as being smooth (but still frictional). However, if $\epsilon \gg \delta_{\mathrm{LSL}}$, then the laminar sub-layer will be destroyed and the wall roughness becomes the new length scale for use in the Law of the Wall, i.e.,

$$\frac{\bar{u}}{u^*} = f\left(\frac{y}{\epsilon}\right). \tag{11.36}$$

Following the same analysis as that for turbulent flow in a smooth pipe, but using y/ϵ in place of yu^*/ν, we obtain,

$$\boxed{\sqrt{\frac{1}{f_D}} \approx -2.0\log_{10}\left(\frac{\epsilon/D}{3.7}\right)}. \tag{11.37}$$

This is the friction factor for turbulent flow in a very rough pipe. The term ϵ/D is known as the relative roughness. Note that this equation is independent of the Reynolds number.

11.3.3. Turbulent Flow in a Rough Pipe

For the transitional regime where ϵ/D is between "smooth" and "very rough", empirical formulas in which the friction factor is a function of both ϵ/D and Re_D have been developed,

$$\boxed{\sqrt{\frac{1}{f_D}} \approx -2.0\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}_D\sqrt{f_D}}\right) \quad \mathrm{Re}_D > 4000} \quad \text{Colebrook Formula}, \tag{11.38}$$

which is implicit in f_D, or the explicit empirical formula,

$$\boxed{\sqrt{\frac{1}{f_D}} \approx -1.8\log_{10}\left[\frac{6.9}{\text{Re}_D} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right] \quad \text{Re}_D > 4000} \quad \underline{\text{Haaland Formula}}. \tag{11.39}$$

The Haaland formula isn't as accurate as the Colebrook formula, but it's easier to calculate since it's explicit in f_D. To solve the Colebrook formula for f_D, an iterative algorithm must be used. An initial first guess for f_D using the Haaland formula usually results in convergence in the Colebrook formula within one or two iterations.

11.4. The Moody Plot

The previous friction factor relations have been summarized into a single plot known as the Moody Plot, which is shown in Figure 11.6.

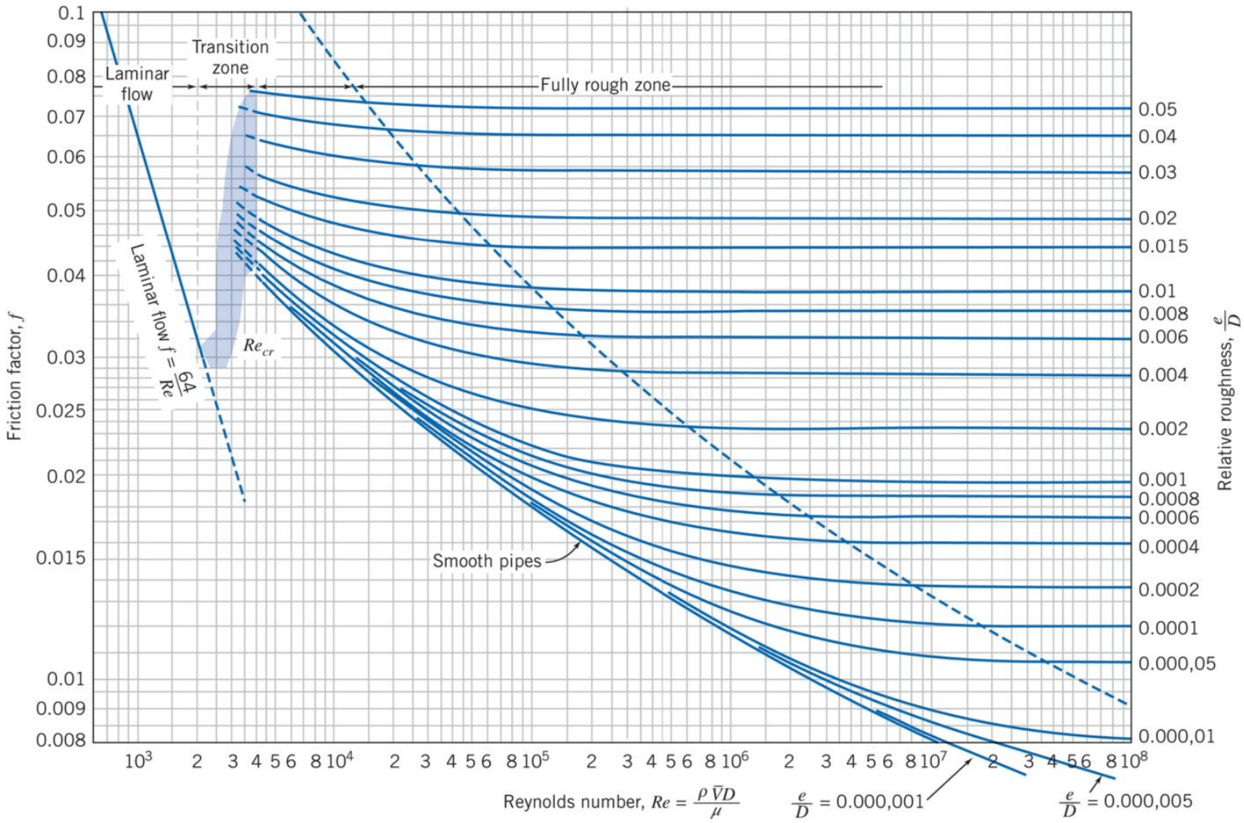


FIGURE 11.6. The Moody plot, which plots the (Darcy) friction factor as a function of Reynolds number for different relative roughnesses. This figure is from Pritchard, P.J. and Mitchell, J.W., *Fox and McDonald's Introduction to Fluid Mechanics*, 9th ed., Wiley.

Notes:

(1) For Reynolds numbers less than 2,300, one may use either the analytical expression for the friction factor,

$$f_D = \frac{64}{\text{Re}_D}, \tag{11.40}$$

or the Moody plot.

(2) Reynolds numbers between approximately 2,300 and 4,000 correspond to the transitional regime between laminar and turbulent flow. The gray region in the Moody plot reflects the fact that the friction factor can vary significantly in this region. At best, bounds can be determined for the friction factor in this region rather than a specific value.
(3) The fully rough zone (aka wholly turbulent zone, fully turbulent zone) in the Moody plot is a region where the friction factor is a weak function of the Reynolds number, but a strong function of the relative roughness. If the Reynolds number of a flow is unknown, but is expected to be large, it is often helpful to assume that the flow is in the fully rough zone as an initial first guess.
(4) The roughnesses of various types of pipe materials have been compiled into tables such as Table 11.1. Note that "smooth" in the table does not mean frictionless.

Table 11.1. A table of pipe material wall roughnesses.

Material (new)	$\boldsymbol{\epsilon}$ (ft)	$\boldsymbol{\epsilon}$ (mm)
riveted steel	0.003 - 0.03	0.9 - 9.0
concrete	0.001 - 0.01	0.3 - 3.0
wood stave	0.0006 - 0.003	0.18 - 0.9
cast iron	0.0085	0.26
galvanized iron	0.0005	0.15
asphalted cast iron	0.0004	0.12
commercial steel or wrought iron	0.00015	0.046
drawn tubing	0.000005	0.0015
glass	smooth	smooth

1. Using the Moody chart, determine the friction factor for a Reynolds number of 10^5 and a relative roughness of 0.001.
2. What is the friction factor for a Reynolds number of 1000?
3. What is the friction factor for a Reynolds number of 10^6 in a smooth pipe?

SOLUTION:

1. The friction factor is $\boxed{f_D \approx 0.0225}$ (Follow the red lines in the following figure.)
2. Since the Reynolds number is less than 2300, we can use the exact laminar flow relation:

$$f_D = \frac{64}{\mathrm{Re}_D} \quad \Rightarrow \quad \boxed{f_D = 0.064}.$$

Alternately, we could use the Moody chart by following the blue lines in the following figure.

3. The friction factor is $\boxed{f_D \approx 0.012}$. (Follow the green lines in the following figure.)

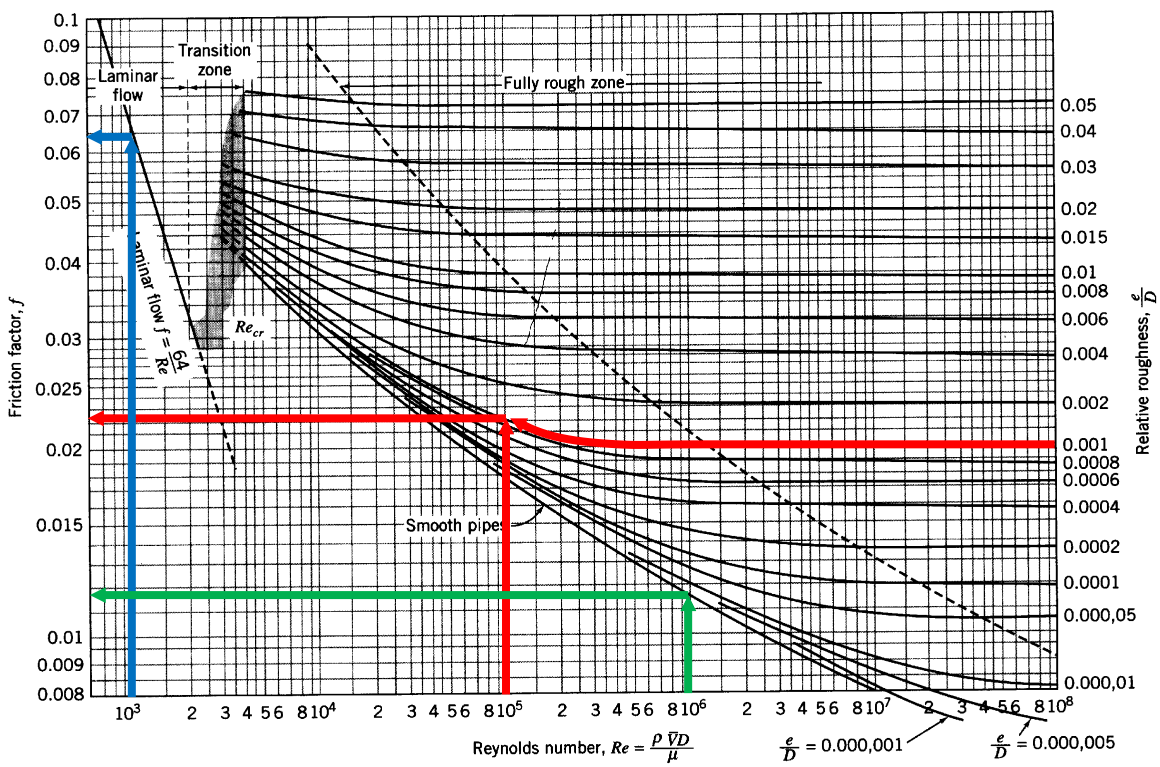


Fig. 8.13 Friction factor for fully developed flow in circular pipes. (Data from [8], used by permission.)

Create a computer program that uses the Colebrook formula to calculate a friction factor. The program inputs should be the Reynolds number and the relative roughness. The output should be the friction factor.

Use your program to create a copy of the Moody plot. Plot the friction factor for Reynolds numbers between 4000 and $1*10^8$ for 10 relative roughness values between $1*10^{-6}$ and $1*10^{-2}$. Use logarithmic horizontal and vertical axes.

SOLUTION:
The Colebrook formula is,

$$\sqrt{\frac{1}{f}} \approx -2.0 \log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}_D\sqrt{f}}\right). \tag{1}$$

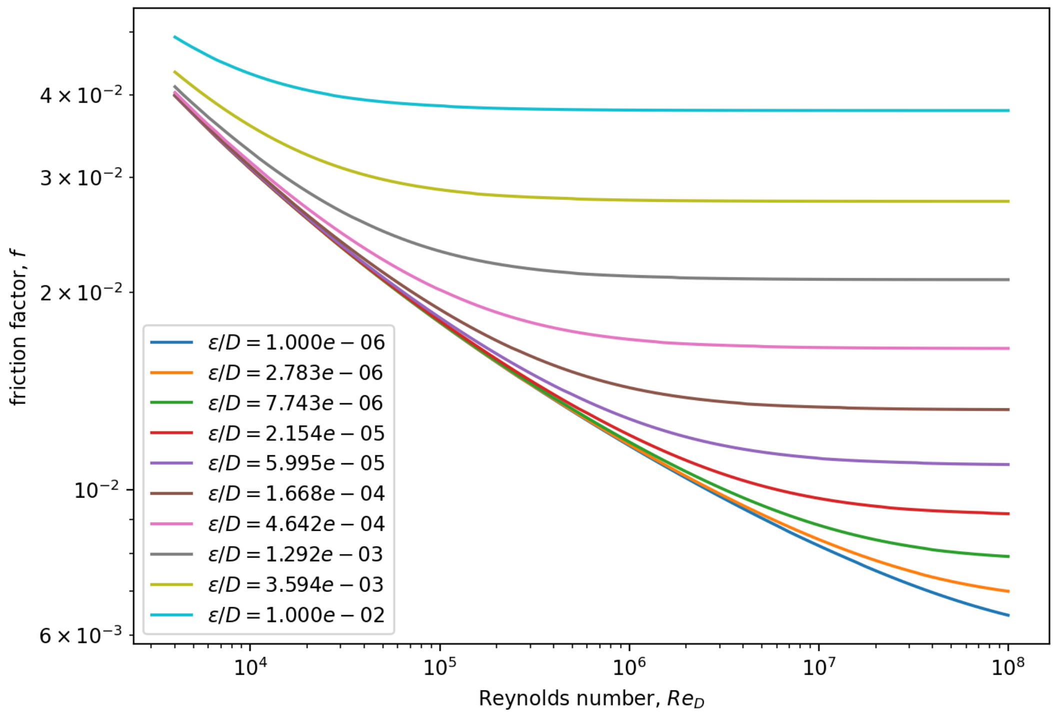
This function is implicit in *f* so an iterative scheme must be used to solve it. There are various algorithms that can be used to solve for *f*. In this solution, the following algorithm is used:

1. Guess a value for the friction factor *f*. Use the Haaland formula for this first guess,
$$\sqrt{\frac{1}{f}} \approx -1.8 \log_{10}\left[\frac{6.9}{\mathrm{Re}_D} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]. \tag{2}$$
2. Solve for the friction factor on the left-hand side of the Colebrook formula (Eq. (1)), call this *f'*, using the guessed value for *f* on the right-hand side.
3. Is the value of *f'* equal to *f* within an acceptable tolerance (*tol*), i.e.,
$$\text{Is } \frac{|f'-f|}{f} < tol? \tag{3}$$
If not, then let *f* = *f'* and repeat step 2. If so, then we now have our value for *f*. A counter is also included in the program to ensure that we don't iterate an unacceptably large number of times. Usually this algorithm solve for *f* within a few iterations, but the counter is a fail-safe measure since the iterative scheme isn't guaranteed to converge.

This algorithm is implemented here using the Python programming language. The plot is shown at the end of this document.

```
# pipe_50.py

# Import some helpful Python libraries.
import numpy as np  # used for numerical routines
import matplotlib.pyplot as plt  # used for making plots

# Create a function for the Haaland formula.
def f_Haaland(Re, e_D):
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

# Create a function for solving the Colebrook formula.
def f_Colebrook(Re, e_D):
    # The first guessed value for the friction factor uses the Haaland formula.
    fprime = f_Haaland(Re, e_D)

    # Set the relative difference between f and fprime large initially
    # to start the loop.
    freldiff = 1

    # Set the acceptable tolerance to be <= 0.1 percent.
    tol = 0.001

    # Only go up to 1000 iterations.
    max_counter = 1000

    # Initialize the counter.
    counter = 0

    # Loop until the relative difference is less than the tolerance or
    # until the maximum number of iterations is reached.
    while ((freldiff > tol) and (counter < max_counter)):
        counter = counter + 1  # Update the counter.
```

```
        f = fprime  # Set f equal to fprime for solving the Colebrook formula.

        # Colebrook formula.
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2

        # Calculate the relative difference
        freldiff = np.absolute((fprime-f)/f)


    # The maximum number of iterations was reached.  Print a warning
    # and exit the program.
    if (counter == max_counter):
        print("The maximum number of iterations was reached.  Did not converge on a value for
f.")
        exit(1)

    # Return the converged value for the friction factor.
    return f


# Make arrays of Reynolds numbers and relative roughnesses values.
Re = np.geomspace(4e3, 1e8, num=100)
e_D = np.geomspace(1e-6, 1e-2, num=10)

# Create an array of friction factor values the same size as the
# Reynolds number array.  Initialize the array with zero values.
f = np.full_like(Re,0)

# Calculate the friction factor for all combinations of the Reynolds
# numbers and relative roughnesses.
for j in range(len(e_D)):
    for i in range(len(Re)):
        f[i] = f_Colebrook(Re[i], e_D[j])
    # Create some text for the plot legend.
    legendtext = '$\epsilon/D = %.3e$' % e_D[j]
    # Plot these friction factors as a function of Reynolds number for
    # this particular relative roughness.
    plt.plot(Re, f, "-", label=legendtext)

# Label the plot, change the axes to be logarithmic, and show the legend.
plt.xlabel(r"Reynolds number, $Re_D$")
plt.ylabel(r"friction factor, $f$")
plt.xscale("log")
plt.yscale("log")
plt.legend()
plt.show()
```


friction factor, f
Reynolds number, Re_D
4 × 10^-2
3 × 10^-2
2 × 10^-2
10^-2
6 × 10^-3
10^4
10^5
10^6
10^7
10^8
ε/D = 1.000e − 06
ε/D = 2.783e − 06
ε/D = 7.743e − 06
ε/D = 2.154e − 05
ε/D = 5.995e − 05
ε/D = 1.668e − 04
ε/D = 4.642e − 04
ε/D = 1.292e − 03
ε/D = 3.594e − 03
ε/D = 1.000e − 02

11.5. Other Losses

The loss due to the viscous resistance caused by the pipe walls is referred to as a major loss. Pressure losses may occur due to viscous dissipation resulting from fluid interactions with other parts of a pipe system such as valves, bends, contractions/expansions, inlets, and connectors. These losses are known as minor losses. The names can be misleading since it's not uncommon in pipe systems to have most of the pressure loss resulting from the minor losses, e.g., a pipe system with a large number of bends and valves, but short sections of straight pipe. What causes these minor losses? The pressure loss results primarily from viscous dissipation in regions with large velocity gradients, such as in a recirculation zone as shown in Figure 11.7.

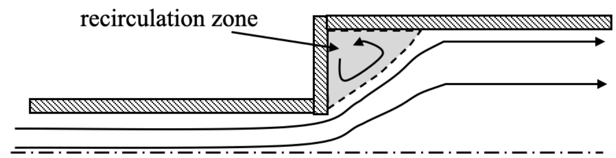


Figure 11.7. A sketch showing where viscous losses occur in a sudden pipe expansion.

A closely related phenomenon known as the vena contracta acts to effectively reduce the diameter at entrances and bends (Figure 11.8). The recirculation zone also results in a pressure loss.

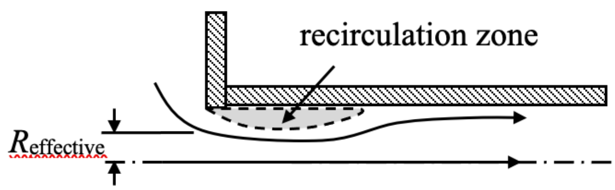


Figure 11.8. A sketch illustrating a vena contracta.

Although minor loss coefficients can be determined analytically for certain situations, most frequently the loss coefficient for a particular device is found experimentally. Essentially, one measures the pressure drop across the device, Δp, and forms the loss coefficient, k, using,

$$k = \frac{\Delta p}{\frac{1}{2}\rho \bar{u}^2}, \tag{11.41}$$

where ρ is the fluid density and $\bar{u}$ is the average speed through the device. Many tables with experimentally determined loss coefficients are available.

Notes:

(1) When using a loss coefficient, it is important to know what velocity has been used to form the coefficient. For example, the loss coefficient for a contraction is typically based on the speed downstream of the contraction, while the loss coefficient for an expansion is based on the speed upstream of the expansion.
(2) Minor losses are sometimes given in terms of equivalent lengths of pipe. An equivalent minor loss of 10 pipe diameters worth of a particular type of pipe means that the major loss caused by a pipe of that type, 10 diameters in length will give the same pressure loss as the minor loss. Thus, a loss coefficient and equivalent pipe length, L_e, can be related by,

$$k = f_D \left(\frac{L_e}{D}\right). \tag{11.42}$$

(3) For non-circular pipes or pipes that are not completely filled, the same methods of determining the friction factor and loss coefficients are used, except that a hydraulic diameter, D_h, is used in place of the diameter. The hydraulic diameter is defined as,

$$D_h := \frac{4A}{P_w}, \tag{11.43}$$

where A and P_w are the cross-sectional *flow area* (not necessarily the pipe cross-sectional area) and wetted perimeter of the pipe, i.e., the part of the pipe that is in contact with the fluid. For example, consider a completely filled square pipe of side length L (Figure 11.9). The hydraulic diameter for such a pipe is,

$$D_h = \frac{4(L^2)}{4L} = L. \tag{11.44}$$

Now consider a completely filled annular pipe with outer diameter D_o and inner diameter D_i

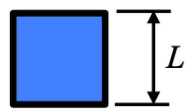


Figure 11.9. A square cross-sectioned pipe filled with fluid.

(Figure 11.10). The hydraulic diameter for this case is,

$$D_h = \frac{4\left(\frac{\pi}{4}D_o^2 - \frac{\pi}{4}D_i^2\right)}{\pi D_o - \pi D_i} = \frac{D_o^2 - D_i^2}{D_o + D_i} = D_o - D_i. \tag{11.45}$$

Now consider a half-filled circular pipe of diameter D (Figure 11.11). The hydraulic diameter for

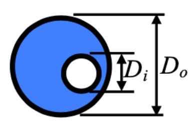


Figure 11.10. An annular cross-sectioned pipe filled with fluid.

this case is,

$$D_h = \frac{4\left(\frac{1}{2}\frac{\pi}{4}D^2\right)}{\left(\frac{1}{2}\pi D\right)} = D. \tag{11.46}$$

Notes:

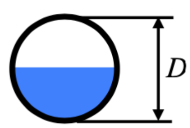


Figure 11.11. A circular cross-sectioned pipe half filled with fluid.

(a) Often a hydraulic radius, R_h, is used instead of a hydraulic diameter for flows in conduits with a free surface. The hydraulic radius is defined as,

$$R_h := \frac{A}{P_w}. \tag{11.47}$$

Using this definition, $D_h \neq 2R_h$, but is instead, $D_h = 4R_h$, which can be confusing. The Manning Formula (not covered in these notes) is frequently used in the analysis of free surface conduit flows.