Pipe Flows – Head Losses

Trans-Alaska Pipeline: 800 miles (1290 km), 48 in. (1.2 m) diameter
Pipe wall thickness ~0.5 in.
Takes oil 11.9 days to travel the length at an average speed of 3.7 mph (6.0 kph)

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$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{out}=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{in}-H_L+H_S$$

where

$$\alpha=\begin{cases}2 & Re_D<2300\text{ (laminar)}\\ 1 & Re_D>2300\text{ (turbulent)}\end{cases}$$

$$H_S=\frac{\dot{W}_{other,on\ CV}}{\dot{m}g}$$

Steady, Laminar, Viscous, Newtonian Fluid Flow Through a Circular Pipe

Assumptions:
1. steady flow
2. fully developed flow in the *z* direction
3. no body forces
4. axi-symmetric flow with no swirl component

Continuity Equation and Navier-Stokes Equation in the z direction:

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{\partial u_z}{\partial z}=0$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial\theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial\theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

Boundary Conditions:
1. No slip at the pipe circumference
2. Flow velocity remains finite

Velocity profile:

$$u_z=\frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1-\frac{r^2}{R^2}\right)$$

$$Re_D=\frac{\bar{u}D}{\nu}<2300$$

Average velocity:

$$\bar{u}=\frac{1}{\pi R^2}\int_{r=0}^{r=R}u_z(r)(2\pi r dr)=\frac{1}{2}u_{max}=\frac{D^2}{32\mu}\left(-\frac{dp}{dz}\right)$$

Wall shear stress:

$$\tau_w=\mu\frac{du}{dr}\bigg|_{r=R}=\frac{R}{2}\left(\frac{dp}{dz}\right)=-\frac{4\mu\bar{u}}{R}$$

$$f_D\equiv\left|\frac{4\tau_w}{\frac{1}{2}\rho\bar{u}^2}\right|=\frac{32\mu}{\rho\bar{u}R}=\frac{64}{Re_D}$$

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Revisiting the average velocity equation:

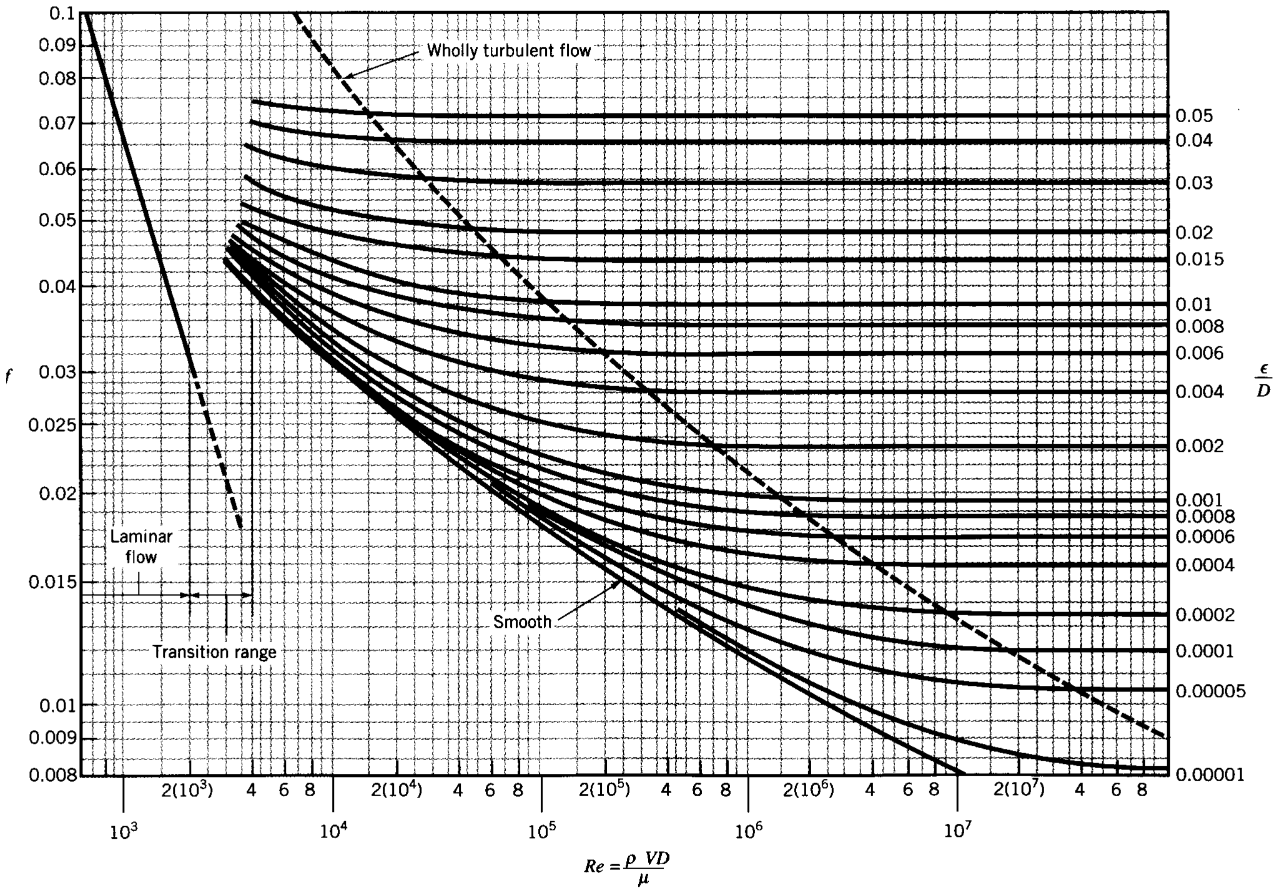
$$\bar{u} = \frac{D^2}{32\mu}\left|\frac{\Delta p}{L}\right| \quad \Longrightarrow \quad \frac{\Delta p}{\frac{1}{2}\rho\bar{u}^2} = \frac{64\mu}{\rho D\bar{u}}\frac{L}{D} = 64\frac{\mu}{\rho\bar{u}D}\frac{L}{D} = \frac{64}{Re_D}\frac{L}{D}$$

$$k \equiv \frac{\Delta p}{\frac{1}{2}\rho\bar{u}^2}$$

$$k_{\text{major}} = f_D\left(\frac{L}{D}\right)$$

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_{out} = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_{in} - H_L + H_S$$

Pipe Flows – Head Losses



Colebrook Formula: $\sqrt{\frac{1}{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}_D\sqrt{f}}\right)$

Haaland Formula $\sqrt{\frac{1}{f}} \approx -1.8\log_{10}\left[\frac{6.9}{\mathrm{Re}_D}+\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$

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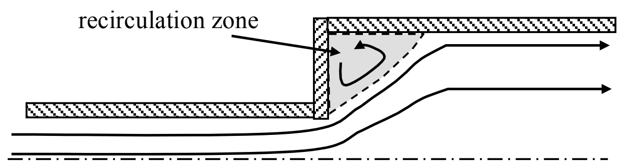

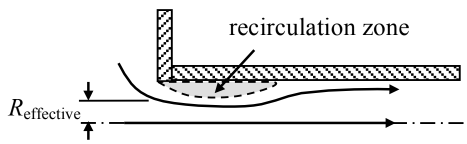

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Average Roughness of Commercial Pipes

Material (new)	ft	mm
Riveted steel	0.003-0.03	0.9-9.0
Concrete	0.001-0.01	0.3-3.0
Wood stave	0.0006-0.003	0.18-0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

Table of Minor Loss Coefficients

Component	K
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bends, flanged	0.2
180° return bends, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.06

Component	K
e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, ¼ closed	0.26
Gate, ½ closed	2.1
Gate, ¾ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, 1/3 closed	5.5
Ball valve, 2/3 closed	210
f. Entrances	
Re-entrant	0.8
Sharp-edged	0.5
Slightly rounded	0.2
Well rounded	0.04
g. Exits	
Re-entrant, sharp-edged, slightly rounded, well-rounded	1

h. Sudden Contraction/Expansion:

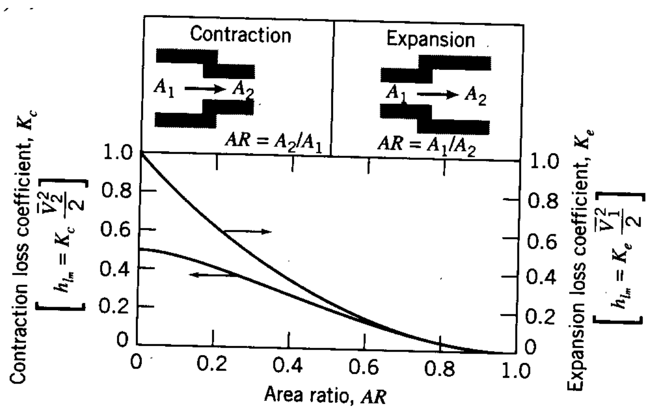

Fig. 8.15 Loss coefficients for flow through sudden area changes. (Data from [1].)

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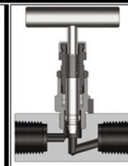
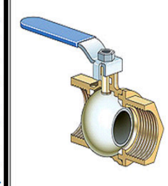
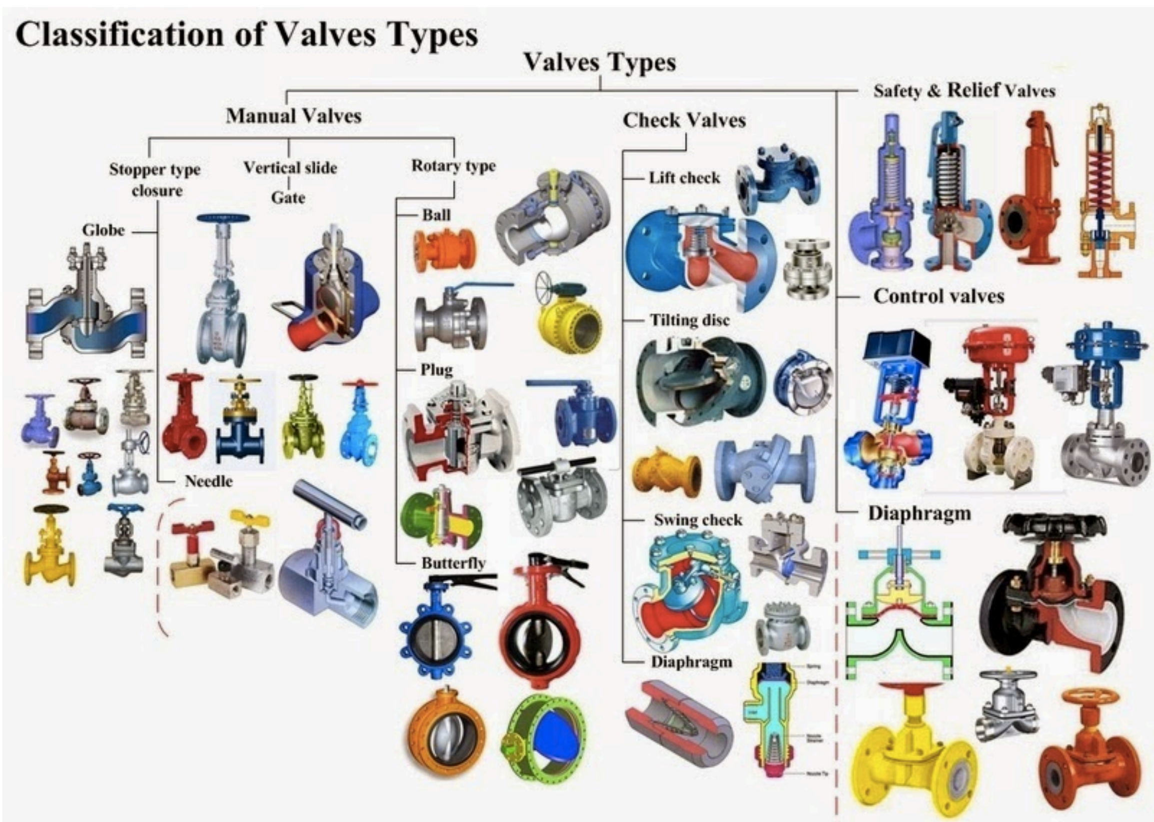
 **Butterfly Valve:** The closing mechanism is provided by a disc that sits in the middle of the valve body, and rotates. When the lever is turned, this disc moves sideways or upright to allow or block the flow of water.	 **Plug Valve:** There is a cone-shaped plug inside the valve body that moves aside when the handle or wheel is turned.	 **Needle Valve:** there is a sliding needle to regulate flow of water. It is a very precise valve, and is used in engines, such as central heating systems and automotive carburetors.
 **Gate Valve:** A metal gate raises and lowers, regulating flow of water. Most valves of this design are kept either fully closed or fully open. Its purpose is not for partial closing/opening. It is mostly used in water supply pipes.	 **Globe Valve:** The most common example of a globe valve is your water faucet. When the handle or wheel is turned, the blocking part moves upward to allow for water flow. Unlike a gate valve, you can easily regulate the flow of water.	 **Ball Valve:** A rotating ball that has hollow core operates in the center of valve body. When the hole is parallel to the pipe, water will flow. When the hole is perpendicular to the pipe, water will stop flowing.



http://mechanicstips.blogspot.com/2016/02/types-of-valves.html