4.6. The First Law of Thermodynamics for a Control Volume

The reader should review the Introductory Thermodynamics chapter (Chapter 3) before continuing with this section.

To write the First Law for a control volume, we utilize the Reynolds Transport Theorem (RTT) to convert our system expression to a control volume expression. Let's first rewrite Eq. (3.32) using the Lagrangian derivative notation (we're interested in how things change with respect to time as we follow a particular system of fluid) and write the total energy of a system in terms of an integral,

$$\underbrace{\frac{D}{Dt}\int_{V_{\text{sys}}} e\rho dV}_{=E_{\text{sys}}} = \dot{Q}_{\text{into sys}} + \dot{W}_{\text{on sys}}. \tag{4.104}$$

Applying the Reynolds Transport Theorem and noting that the system and control volume are coincident at the time we apply the theorem gives,

$$\boxed{\frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{on CV}}}. \tag{4.105}$$

This is the First Law of Thermodynamics for a control volume!

Notes:

(1) The specific total energy is $e = u + \frac{1}{2}V^2 + G$ where G is a conservative potential energy function with the specific gravitational force given by $\mathbf{f}_{\text{gravity}} = -\boldsymbol{\nabla}G$. For the remainder of these notes, G will be assumed to be $G = gz$ ($\implies$ $\mathbf{f}_{\text{gravity}} = -g\hat{\mathbf{e}}_z$) where g is the acceleration due to gravity.

Now let's expand the rate of work (power) term into rate of pressure work (pdV power) and the power due to other effects such as shaft work, viscous work, electric work, etc.,

$$\dot{W}_{\text{on CV}} = \dot{W}_{p,\text{on CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.106}$$

In particular, we can write the rate of pressure work term for fluid crossing the boundary in the following

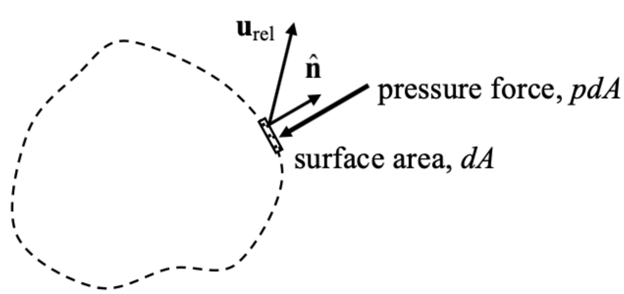


FIGURE 4.14. A schematic illustrating the rate of pressure work at the control surface.

way (Figure 4.14),

$$\begin{aligned} d\dot{W}_{p,\text{on CV}} &= d\mathbf{F}_{\text{p,on CV}}\cdot\mathbf{u}_{\text{rel}}, & (4.107)\\ &= (-pd\mathbf{A})\cdot\mathbf{u}_{\text{rel}}, & (4.108)\\ &= -p(\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}), & (4.109)\\ &= -\frac{p}{\rho}\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right). & (4.110)\end{aligned}$$

The rate of pressure work as fluid crosses the boundary over the entire CS is,

$$\boxed{\dot{W}_{p,\text{on CV}} = \int_{CS} -\frac{p}{\rho}\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right)}. \tag{4.111}$$

Equation (4.111) is the rate at which pressure work is performed on the fluid flowing through the control surface.

Substituting Eqs. (4.111) and (4.106) into Eq. (4.105), expanding the specific total energy term in the surface integral, and bringing the rate of pressure work term to the left-hand side gives,

$$\frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(u + \frac{p}{\rho} + \frac{1}{2}V^2 + gz\right)(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.112}$$

The quantity $(u + p/\rho)$ appears often in thermal-fluid systems and is given the special name of specific enthalpy, h,

$$h := u + \frac{p}{\rho} = u + pv, \tag{4.113}$$

where $v = 1/\rho$ is the specific volume. Note that just as with internal energy, tables of thermodynamic properties typically list the value of the specific enthalpy for various substances at various conditions.

Substituting Eq. (4.113) into Eq. (4.112) gives,

$$\boxed{\frac{d}{dt}\int_{CV} e\rho dV + \int_{CS}\left(h + \frac{1}{2}V^2 + gz\right)(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}}. \tag{4.114}$$

Notes:

(1) An alternate way to write Eq. (4.114) is,

$$\frac{dE_{CV}}{dt} = \sum_{\text{all inlets}} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) - \sum_{\text{all outlets}} \dot{m}\left(h + \frac{1}{2}V^2 + gz\right) + \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.115}$$

The previous equation can be integrated in time,

$$\Delta E_{CV} = \sum_{\text{all inlets}} m\left(h + \frac{1}{2}V^2 + gz\right) - \sum_{\text{all outlets}} m\left(h + \frac{1}{2}V^2 + gz\right) + Q_{\text{into CV}} + W_{\text{other,on CV}}, \tag{4.116}$$

where m is the total mass entering/leaving the control volume. It has been assumed that the specific enthalpies, kinetic energy, and potential energies at the inlets and outlets don't change with time. This form of the First Law is useful for evaluating conditions at the end of an unsteady process. Note that if there are no inlets and outlets, then Eq. (4.116) simplifies to the system form of the First Law (Eq. (3.32)).

(2) The specific enthalpy term in Eq. (4.114) accounts for the rate of pressure work as fluid crosses the control surface, e.g., at inlets and outlets of the control volume. If there is pressure work caused by a moving, solid boundary through which no fluid flows, e.g., a moving piston, then that work would be included in the $\dot{W}_{\text{other,on CV}}$ term.

(3) During problem solving, we often must estimate the relative magnitudes of the terms in the total specific enthalpy term, i.e., $h_T = h + \frac{1}{2}V^2 + gz$. For example, consider a simple system operating at steady state with a single inlet and a single outlet. The inlet and outlet mass flow rates will be the same. The change in the total enthalpy between the inlet and outlet is (refer to Eq. (4.115)),

$$\dot{m}\Delta h_T = \dot{m}\left[\Delta h + \Delta\left(\frac{1}{2}V^2\right) + g\Delta z\right]. \tag{4.117}$$

Let's assume that $\Delta h \sim 1\,\text{kJ}\,\text{kg}^{-1}$. To have an equivalent change in the kinetic energy, we would need $\Delta V \sim 45\,\text{m}\,\text{s}^{-1}$. An equivalent change in the potential energy would require $\Delta z \sim 100\,\text{m}$. Thus, it is often reasonable to neglect changes in kinetic and potential energies if the change in specific enthalpy is large and the changes in velocity and elevation are small.

(4) Let's examine the "other" work term more closely. This term includes work due to anything other than pressure work, such as work due to viscous forces, shaft work, electrical work, etc. In this note, let's examine the work done by viscous stresses. Consider the rate of viscous work done on the CV shown in Figure 4.15,

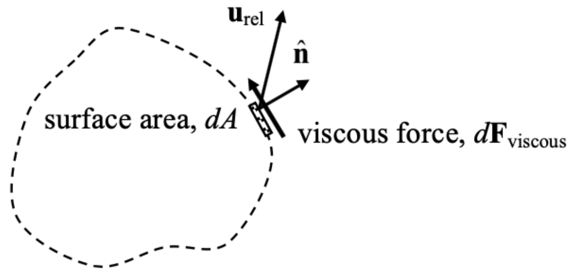


Figure 4.15. A schematic illustrating the rate of viscous work at the control surface.

$$d\dot{W}_{\text{viscous,on CV}} = d\mathbf{F}_{\text{viscous,on CV}} \cdot \mathbf{u}_{\text{rel}}, \tag{4.118}$$

so that the total rate of viscous work acting on the CS is,

$$\dot{W}_{\text{viscous,on CV}} = \int_{CS} d\mathbf{F}_{\text{viscous,on CV}} \cdot \mathbf{u}_{\text{rel}}. \tag{4.119}$$

(a) Note that at a solid boundary, $\mathbf{u}_{\text{rel}} = \mathbf{0}$ due to the no-slip condition so that the rate of viscous work is zero at solid surfaces. If the flow is inviscid, then $\mathbf{u}_{\text{rel}} \neq \mathbf{0}$, but $d\mathbf{F}_{\text{viscous,on CV}} = \mathbf{0}$ and so the rate of viscous work is zero for that case too.

(b) If the control volume is oriented such that the velocity vectors are perpendicular to the normal vectors of the CS, then the rate of viscous work done on the CV will be zero,

$$d\mathbf{F}_{\text{viscous,on CV}} \cdot \mathbf{u}_{\text{rel}} = 0, \tag{4.120}$$

since the viscous force will be perpendicular to the velocity vector. Thus, orienting the control surface so that it cuts perpendicularly across streamlines eliminates viscous work on the control volume.

(c) The rate of viscous work may not be negligible if the control volume is chosen as shown in Figure 4.16. Viscous forces along streamline surfaces may be significant if the shear stress there isn't negligible.

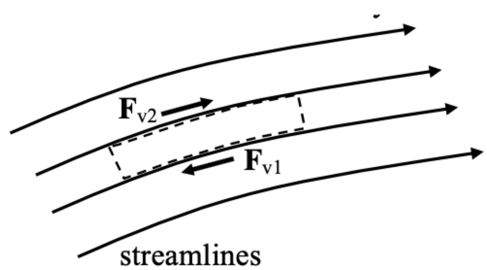


Figure 4.16. A schematic illustrating the viscous forces at the control surface if the control surface is tangential to the streamlines.

For the remainder of these notes, it will be assumed that the work on the CV due to viscous stresses is zero since our control surfaces will be chosen such that the surfaces are along solid boundaries or boundaries where viscous stresses are negligible (e.g., negligible velocity gradients), or with normal vectors perpendicular to the flow velocities.

(5) For a flow where the total energy within the CV does not change with time (steady state), Eq. (4.114) simplifies to,

$$\int_{CS} \left(h + \frac{1}{2}V^2 + gz\right)(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}) = \dot{Q}_{\text{into CV}} + \dot{W}_{\text{other,on CV}}. \tag{4.121}$$

Note that flows may be unsteady at the local level, e.g., the localized flow within a pump, but may be steady at a larger scale, e.g., the average conditions within the pump housing.

(6) For a steady state, steady flow (meaning that the mass flow rate remains constant) with a single inlet (call it state 1) and outlet (call it state 2), we can write Eq. (4.121) as,

$$\left(h + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_2 - \left(h + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_1 = \dot{q}_{\text{into CV}} + \dot{w}_{\text{other,on CV}}. \tag{4.122}$$

where $q = \dot{Q}/\dot{m}$ and $w = \dot{W}/\dot{m}$ are the specific heat, i.e., the heat transfer per unit mass, and the specific work, i.e., the work per unit mass, respectively. Note that from COM the mass flow rate into the CV equals the mass flow rate out of the CV, i.e., $\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m}$.

(a) The average velocity through an area is,

$$\bar{V} := \frac{1}{A}\int_A (\mathbf{V} \cdot d\mathbf{A}). \tag{4.123}$$

(b) The quantity, α, is known as the kinetic energy correction factor. It is a correction factor accounting for the fact that an average velocity profile, $\bar{V}$, may not contain the same kinetic energy as a non-uniform velocity profile. For example, consider the kinetic energy contained in the two flow profiles shown in Figure 4.17. The average flow rate of specific kinetic energy is,

$$\overline{ke} = \int_A \frac{1}{2}V^2 \left(\rho\mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) \neq \frac{1}{2}\dot{m}\bar{V}^2, \tag{4.124}$$

in general. We define the kinetic energy correction factor, α, as,

$$\boxed{\alpha := \frac{\int_A \frac{1}{2}V^2 \left(\rho\mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)}{\frac{1}{2}\dot{m}\bar{V}^2}}. \tag{4.125}$$

so that,

$$\overline{ke} = \alpha\frac{1}{2}\dot{m}\bar{V}^2. \tag{4.126}$$

For a laminar flow in a circular pipe, the velocity profile is parabolic (discussed in a different chapter) resulting in $\alpha = 2$. For a turbulent flow, $\alpha \to 1$ as increasing turbulent mixing causes the velocity profile to become more uniform.

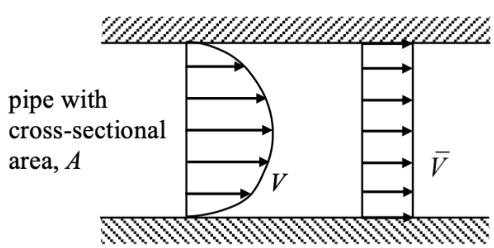


FIGURE 4.17. A schematic of a pipe flow with two different velocity profiles.

(c) The quantity,

$$h_T = h_0 := h + \alpha\frac{1}{2}\bar{V}^2 + gz, \tag{4.127}$$

is referred to as the total specific enthalpy, h_T or the stagnation specific enthalpy, h_0. Note that for gases, the gz term is much smaller than the other terms and, thus, is often neglected.

(d) If the flow is adiabatic ($q = 0$) and the rate of work by forces other than pressure can be neglected ($w_{\text{other}} = 0$), then,

$$h_T = h_0 = \text{constant}. \tag{4.128}$$

(7) Now let's re-write Eq. (4.122) but expand the specific enthalpy terms,

$$\left(u + \frac{p}{\rho} + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_2 - \left(u + \frac{p}{\rho} + \alpha\frac{1}{2}\bar{V}^2 + gz\right)_1 = q_{\text{into CV}} + w_{\text{other,on CV}}. \tag{4.129}$$

Re-arranging terms and dividing through by the gravitational acceleration gives,

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - \frac{u_2 - u_1 - q_{\text{into CV}}}{g} + \frac{\dot{W}_{\text{other,on CV}}}{\dot{m}g}. \tag{4.130}$$

Each term in this equation is referred to as a head quantity and has the dimensions of length:

$$\frac{p}{\rho g} := \text{pressure head} \tag{4.131}$$

$$\frac{\bar{V}^2}{2g} := \text{velocity head} \tag{4.132}$$

$$z := \text{elevation head} \tag{4.133}$$

$$\frac{u_2 - u_1 - q_{\text{into CV}}}{g} = H_L := \text{head loss} \tag{4.134}$$

$$\frac{\dot{W}_{\text{shaft,on CV}}}{\dot{m}g} = H_S := \text{shaft head} \tag{4.135}$$

The head loss is the head lost due to mechanical energy being converted to thermal energy and energy lost via heat transfer to the surroundings. The "other" work term frequently only includes shaft work, particularly in pipe flow systems, and so the shaft head is a convenient definition. It is the head added to the flow due to shaft work.

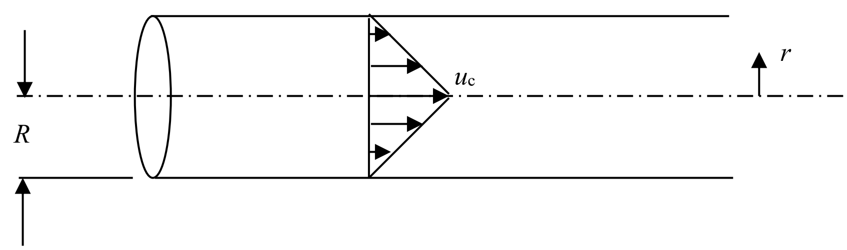
The equation in this form is known as the Extended Bernoulli Equation,

$$\boxed{\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_L + H_S}, \tag{4.136}$$

where it has been assumed that the only form of "other" work is shaft work.

Let's consider some examples to see how the First Law is applied to control volumes.

The velocity profile for a particular pipe flow is linear from zero at the wall to a maximum of u_c at the centerline. Determine the average velocity and the kinetic energy correction factor.



SOLUTION:
The average velocity is found by setting the volumetric flow rate using the average velocity profile equal to the volumetric flow rate using the real profile,

$$Q_{\substack{\text{avg}\\\text{profile}}} = Q_{\substack{\text{real}\\\text{profile}}} \tag{1}$$

$$\bar{u}\pi R^2 = \int_{r=0}^{r=R} u_C\left(1-\frac{r}{R}\right)(2\pi r dr)$$

$$= 2\pi u_C \int_{r=0}^{r=R}\left(r-\frac{r^2}{R}\right)dr$$

$$= 2\pi u_C\left[\frac{1}{2}r^2-\frac{1}{3}\frac{r^3}{R}\right]_{r=0}^{r=R}$$

$$\boxed{\therefore \bar{u} = \tfrac{1}{3}u_C} \tag{2}$$

The kinetic energy correction factor, α, is found by equating the kinetic energy flow rate using the average velocity with the kinetic energy flow rate using the actual velocity profile,

$$\alpha\tfrac{1}{2}\underbrace{\left(\rho\bar{u}\pi R^2\right)}_{=\dot{m}}\bar{u}^2 = \int_{r=0}^{r=R}\tfrac{1}{2}\underbrace{\left[\rho u_C\left(1-\frac{r}{R}\right)(2\pi r dr)\right]}_{=d\dot{m}}\left[u_C\left(1-\frac{r}{R}\right)\right]^2$$

$$= \int_{r=0}^{r=R}\tfrac{1}{2}\rho u_C^3\left(1-\frac{r}{R}\right)^3(2\pi r dr) \tag{3}$$

$$= \pi\rho u_C^3\int_{r=0}^{r=R}\left(1-\frac{r}{R}\right)^3(r dr)$$

where $\bar{u} = \tfrac{1}{3}u_C$. Solving the previous equation for α gives,

$$\boxed{\alpha = \frac{27}{10} = 2.7} \tag{4}$$

11.6. The Extended Bernoulli Equation

Recall from Section 4.6 that the First Law of Thermodynamics may be written as,

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}, \tag{11.48}$$

where each of the terms in the equation has dimensions of length. The "1" and "2" subscripts refer to the inlet and outlet conditions, respectively. Note that in writing this form of the Extended Bernoulli Equation (EBE), it has been assumed that z points in the direction opposite to gravitational acceleration. If z pointed in the same direction as gravity, then there would be a $-$ sign in front of the z in the EBE.

The various terms in the equation are referred to as "head" quantities:

$$\frac{p}{\rho g} := \underline{\text{pressure head}}, \tag{11.49}$$

$$\alpha\frac{\bar{V}}{2g} := \underline{\text{velocity or dynamic head}}, \tag{11.50}$$

$$z := \underline{\text{elevation head}}, \tag{11.51}$$

$$H_L := \underline{\text{head loss}}, \tag{11.52}$$

$$H_S := \underline{\text{shaft head}}. \tag{11.53}$$

$$\tag{11.54}$$

Recall that the α in the velocity head term is the kinetic energy correction factor, which accounts for the fact that an average speed is used in the EBE rather than the real velocity profile (again, refer to Section 4.6). A value of $\alpha = 2$ is used for laminar flows while $\alpha = 1$ is typically assumed for turbulent flows (actually, $\alpha \to 1$ as $\mathrm{Re}_D \to \infty$).

The head loss term (H_L) accounts for both major and minor losses and may be written as,

$$H_{L,12}=\sum_{\forall i}k_i\frac{\bar{V}_i^2}{2g}, \tag{11.55}$$

where the subscript "i" accounts for every loss in the pipe system. Recall that the major loss coefficient may be written as,

$$k_{\text{major}}=f_D\left(\frac{L}{D}\right). \tag{11.56}$$

The shaft head term (H_S) accounts for the pressure addition (or reduction) resulting from the inclusion of devices such as pumps, compressors, fans, turbines, and windmills. Those devices that add head to the flow are positive (e.g., pumps), while those that extract head are negative (e.g., turbines). The shaft head term may be written in terms of shaft power, $\dot{W}_S$, as,

$$H_{S,12}=\frac{\dot{W}_{S,12}}{\dot{m}g}, \tag{11.57}$$

where $\dot{m}$ is the mass flow rate through the device.

Notes:

(1) One often must make a number of assumptions at the beginning of a pipe flow solution, e.g., the flow is laminar, the flow is turbulent, or the flow is in the fully rough zone. For example, for flow through a hypodermic needle, it's reasonable to assume that the flow will be laminar since the needle diameter is so small. Having experience with pipe flow systems helps one to make good assumptions. Regardless of what assumptions are made, it is important that one verifies that the calculated flow conditions are consistent with the assumptions that were made. For example, if one assumes laminar flow in the hypodermic needle then solves for the flow velocity, then the Reynolds number should be checked to verify that a laminar flow assumption was correct. If so, then the

solution procedure is consistent. If not, then the laminar flow assumption was incorrect and a turbulent flow assumption should be made and the problem re-solved.

11.7. Pipe Systems

Pipe flow systems can be classified as being of one of three types:

- Type I: The desired flow rate is specified and the required pressure drop must be determined.
- Type II: The desired pressure drop is specified and the required flow rate must be determined.
- Type III: The desired flow rate and pressure drop are specified and the required pipe diameter must be determined.

Type I pipe systems are the easiest to solve. Since the flow velocity and diameter are known, calculation of the major loss coefficient, and the friction factor in particular, is straightforward. Type II and Type III problems are more challenging to solve since the friction factor is unknown. These types of pipe systems usually require iteration to solve.

Notes:

(1) There is no unique iterative scheme that must be used to solve Type II and Type III pipe flow problems. Different people may propose different algorithms. In addition, there is no guarantee that a particular iterative scheme will converge to a solution.

(2) When using an iterative scheme, choose an initial flow rate or diameter that is reasonable. Don't start with an exceedingly small or large value. For example, for a Type II pipe system, choose a starting flow rate that corresponds to the fully turbulent zone region.

(3) It's often worthwhile to first assume that a Type II and Type III flow system is operating in the fully rough zone of the Moody plot. Using this assumption will often avoid the need for iteration. However, one must verify at the end of the solution that the assumption of fully rough flow was correct. If not, then an iterative solution should be considered with the fully rough zone conditions used as a starting point for the iterations.