Now use Eq. (4.25) to write the net rate at which B leaves the control volume,

$$\frac{d\left(B_{\text{out}} - B_{\text{in}}\right)}{dt} = \int_{CS} \beta \rho dQ = \int_{CS} \beta \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right). \tag{4.27}$$

Combining Eq. (4.27) with Eq. (4.22) gives,

$$\boxed{\underbrace{\frac{D}{Dt}\left(\int_{V_{\text{sys}}} \beta \rho dV\right)}_{\substack{\text{rate of increase of} \\ B \text{ within the system}}} = \underbrace{\frac{d}{dt}\left(\int_{V_{CV}} \beta \rho dV\right)}_{\substack{\text{rate of increase of} \\ B \text{ within the CV}}} + \underbrace{\int_{CS} \beta \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)}_{\substack{\text{net rate at which } B \text{ leaves} \\ \text{the CV through the CS}}}}. \tag{4.28}$$

This is the Reynolds Transport Theorem!

Notes:

(1) Often when using the Reynolds Transport Theorem, or relations derived from it, velocity vectors expressed in one coordinate system are expressed in another coordinate system for convenience. This conversion can be performed using a Galilean transformation, which consists of a vector translation and rotation. For example, consider Figure 4.8, which shows a jet of water that travels with velocity $V\hat{\boldsymbol{I}}$ and a cart with velocity $U\hat{\boldsymbol{I}}$, both measured relative to the ground. To view the jet velocity relative to the cart, we subtract $U\hat{\boldsymbol{I}}$ from all of the velocity vectors so the cart now appears stationary and the jet moves at velocity $V\hat{\boldsymbol{i}} - U\hat{\boldsymbol{i}} = (V-U)\hat{\boldsymbol{i}}$. Note that for this case no rotation is needed to express the velocities in the new coordinate system.

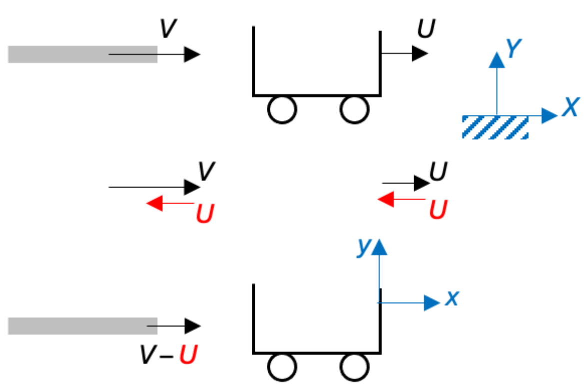

Figure 4.8. Transforming the velocity vector $V\hat{\boldsymbol{I}}$ from a coordinate system fixed to the ground to one fixed to the cart, which moves at speed $U\hat{\boldsymbol{I}}$ relative to the ground.

Now consider Figure 4.9, which shows a cart moving up an incline at velocity $U(\cos\theta\hat{\boldsymbol{I}} + \sin\theta\hat{\boldsymbol{J}})$ relative to the ground and a stream of water with velocity $-V\hat{\boldsymbol{J}}$ relative to the ground. To express the cart and water stream velocities in a coordinate system fixed to the cart, first subtract the cart velocity from the cart velocity (giving a zero vector) and from the water stream velocity. The latter gives the vector $-V\hat{\boldsymbol{J}} - U(\cos\theta\hat{\boldsymbol{I}} + \sin\theta\hat{\boldsymbol{J}}) = -U\cos\theta\hat{\boldsymbol{I}} - (U\sin\theta + V)\hat{\boldsymbol{J}}$. Next express the ground-based unit vectors in terms of the cart based unit vectors using a rotation, i.e., $\hat{\boldsymbol{I}} = \cos\theta\hat{\boldsymbol{i}} - \sin\theta\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{J}} = \sin\theta\hat{\boldsymbol{i}} + \cos\theta\hat{\boldsymbol{j}}$. Thus, the water stream vector can be written in the cart-based coordinate system as,

$$
\begin{aligned}
-U\cos\theta\hat{\boldsymbol{I}} - (U\sin\theta + V)\hat{\boldsymbol{J}} = \\
&= -U\cos\theta(\cos\theta\hat{\boldsymbol{i}} - \sin\theta\hat{\boldsymbol{j}}) - (U\sin\theta + V)(\sin\theta\hat{\boldsymbol{i}} + \cos\theta\hat{\boldsymbol{j}}), \\
&= -U\cos^2\theta\hat{\boldsymbol{i}} + U\cos\theta\sin\theta\hat{\boldsymbol{j}} - U\sin^2\theta\hat{\boldsymbol{i}} - U\sin\theta\cos\theta\hat{\boldsymbol{j}} - V\sin\theta\hat{\boldsymbol{i}} - V\cos\theta\hat{\boldsymbol{j}}, \\
&= -U(\cos^2\theta + \sin^2\theta)\hat{\boldsymbol{i}} - V\sin\theta\hat{\boldsymbol{i}} - V\cos\theta\hat{\boldsymbol{j}}, \\
&= -(U + V\sin\theta)\hat{\boldsymbol{i}} - V\cos\theta\hat{\boldsymbol{j}}.
\end{aligned} \tag{4.29}
$$

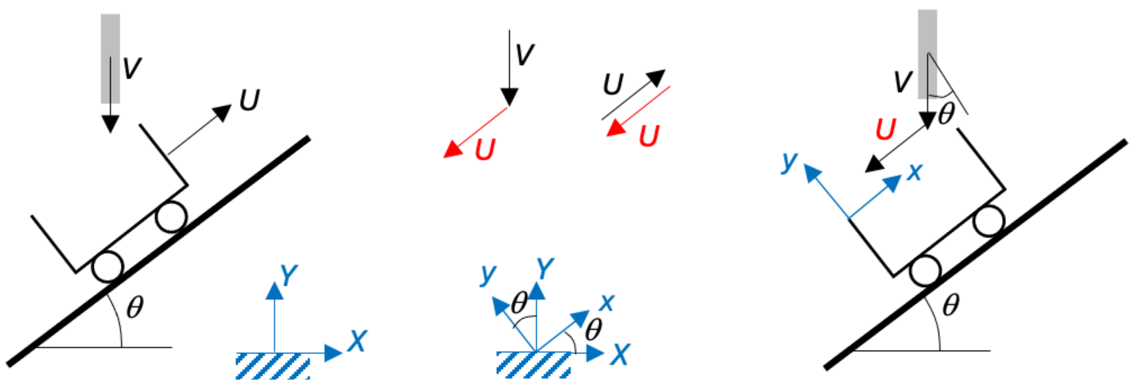

Figure 4.9. Transforming the velocity vector $V\hat{\boldsymbol{J}}$ from a coordinate system fixed to the ground to one fixed to the cart, which moves at speed $U(\cos\theta\hat{\boldsymbol{I}} + \sin\theta\hat{\boldsymbol{J}})$ relative to the ground.