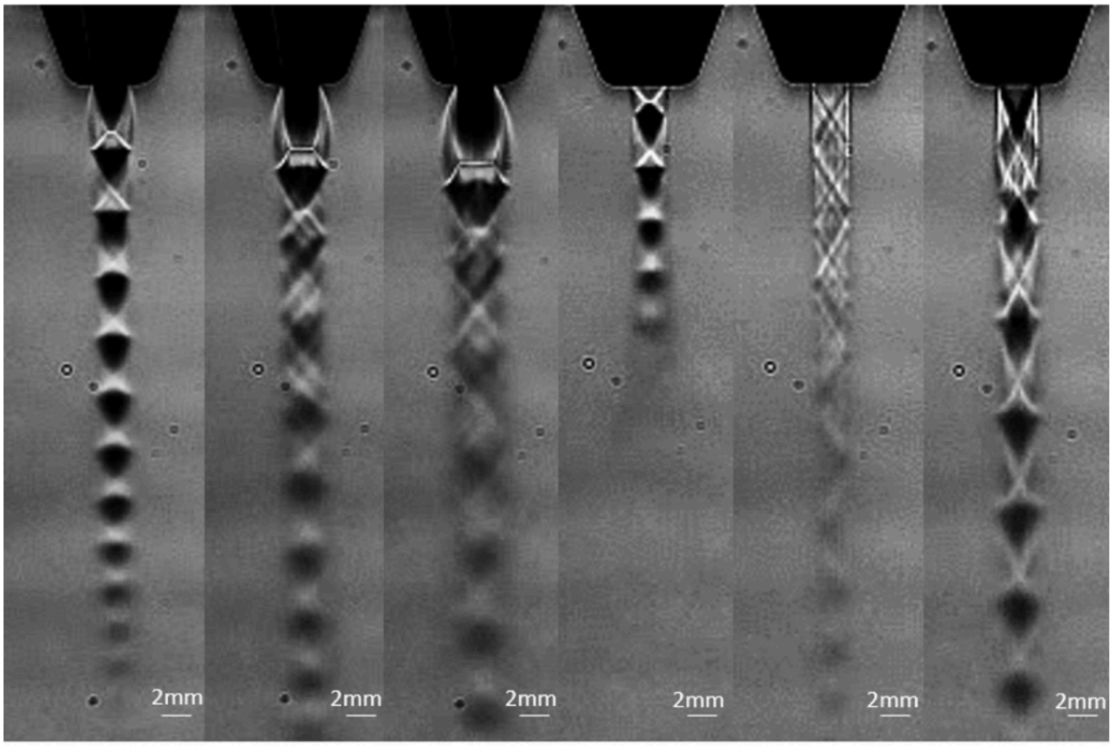
Compressible Flow – Flow through a Converging Nozzle



(a) (b) (c) (d) (e) (f)

FIG. 3. Free jet Schlieren results under different inlet pressures with different nozzles: (a) conical nozzle: 4 bars, (b) conical nozzle: 7 bars, (c) conical nozzle: 10 bars, (d) MLN nozzle: 4 bars, (e) MLN nozzle: 7 bars, and (f) MLN nozzle: 10 bars.

Image from: Zhang, C., Wen, P., Yao, Z., Yuan, Y., and Fan, X., 2016, “Visualization of flow separation inside cut kerf during laser cutting of thick sections”, *Journal of Laser Applications*, Vol. 28, No. 2, Article 022204.

Compressible Flow – Flow through a Converging Nozzle

1D, steady, adiabatic flow of a perfect gas with no work other than pressure work

$$\frac{T}{T_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{-1} \qquad \frac{T^*}{T_0} = \left(1 + \frac{k-1}{2}\right)^{-1}$$

$$\frac{c}{c_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{-\frac{1}{2}} \qquad \frac{c^*}{c_0} = \left(1 + \frac{k-1}{2}\right)^{-\frac{1}{2}}$$

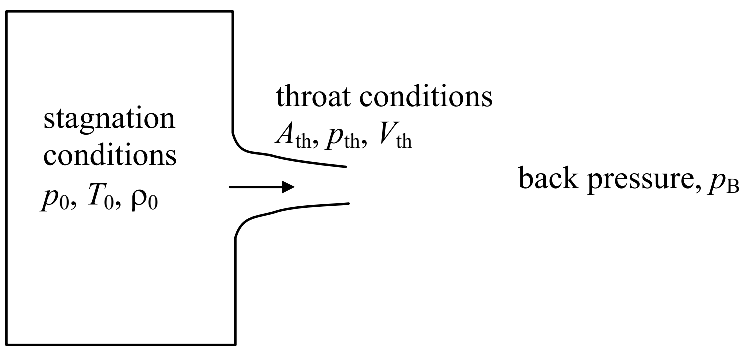
1D, steady, isentropic flow of a perfect gas with no work other than pressure work

$$\frac{p}{p_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{k}{1-k}} \qquad \frac{p^*}{p_0} = \left(1 + \frac{k-1}{2}\right)^{\frac{k}{1-k}} \qquad \left.\frac{p^*}{p_0}\right|_{\substack{\text{for air}\\(k=1.4)}} = 0.5283$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{1}{1-k}} \qquad \frac{\rho^*}{\rho_0} = \left(1 + \frac{k-1}{2}\right)^{\frac{1}{1-k}}$$

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left(\frac{1 + \frac{k-1}{2}\mathrm{Ma}^2}{1 + \frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$

Choked Flow



Combine the mass flow rate at sonic conditions with the isentropic relations at sonic conditions:

$$\dot{m}_{\text{choked}} = \left(1 + \frac{k-1}{2}\right)^{\frac{k+1}{2(1-k)}} p_0 \sqrt{\frac{k}{RT_0}}A^*$$

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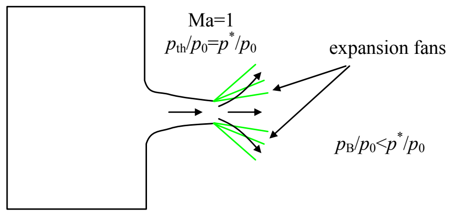

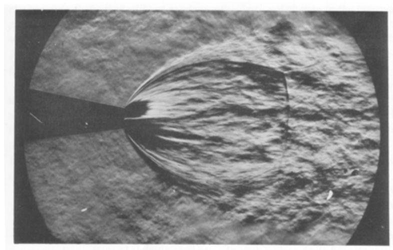

Figure 6.8
External expansion of the jet from a choked converging nozzle, $P_e/P_a \approx 105$.
(*Courtesy of E. S. Love, NASA*).