Compressible Flow – Converging-Diverging Nozzles

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For air:

$k = 1.4$, $R = 287$ J/(kg.K) = 53.3 (ft.lb$_f$)/(lb$_m$.degR) = 1716 ft^2/(s^2.degR)

1D, steady, <u>adiabatic</u> flow of a perfect gas with no work other than pressure work

$$T\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)=\text{constant}$$

$$\frac{T}{T_0}=\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1} \quad \text{and} \quad \frac{T^*}{T_0}=\left(1+\frac{k-1}{2}\right)^{-1}$$

$$\frac{c}{c_0}=\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-\frac{1}{2}} \quad \text{and} \quad \frac{c^*}{c_0}=\left(1+\frac{k-1}{2}\right)^{-\frac{1}{2}}$$

1D, steady, <u>isentropic</u> flow of a perfect gas with no work other than pressure work

$$\frac{p}{p_0}=\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{k}{1-k}} \quad \text{and} \quad \frac{p^*}{p_0}=\left(1+\frac{k-1}{2}\right)^{\frac{k}{1-k}} \quad \text{(for air } (k_{\text{air}} = 1.4),\ p^*/p_0 = 0.5283)$$

$$\frac{\rho}{\rho_0}=\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{1}{1-k}} \quad \text{and} \quad \frac{\rho^*}{\rho_0}=\left(1+\frac{k-1}{2}\right)^{\frac{1}{1-k}}$$

$$\frac{A}{A^*}=\frac{1}{\mathrm{Ma}}\left(\frac{1+\frac{k-1}{2}\mathrm{Ma}^2}{1+\frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$

$$\dot{m}_{\text{choked}}=\left(1+\frac{k-1}{2}\right)^{\frac{k+1}{2(1-k)}} p_0\sqrt{\frac{k}{RT_0}}A^*$$

Normal Shock Relations

$$\mathrm{Ma}_2^2=\frac{(k-1)\mathrm{Ma}_1^2+2}{2k\mathrm{Ma}_1^2-(k-1)}$$

$$\frac{T_2}{T_1}=\left[2+(k-1)\mathrm{Ma}_1^2\right]\left[\frac{2k\mathrm{Ma}_1^2-(k-1)}{(k+1)^2\,\mathrm{Ma}_1^2}\right]$$

$$\frac{\rho_2}{\rho_1}=\frac{V_1}{V_2}=\frac{(k+1)\mathrm{Ma}_1^2}{(k-1)\mathrm{Ma}_1^2+2}$$

$$\frac{p_2}{p_1}=\frac{2k\mathrm{Ma}_1^2}{k+1}-\frac{k-1}{k+1}$$

$$\frac{T_{02}}{T_{01}}=1$$

$$\frac{p_{02}}{p_{01}}=\frac{A_1^*}{A_2^*}=\frac{\rho_{02}}{\rho_{01}}=\left[\frac{\frac{k+1}{2}\mathrm{Ma}_1^2}{1+\frac{k-1}{2}\mathrm{Ma}_1^2}\right]^{\frac{k}{k-1}}\left[\frac{2k\mathrm{Ma}_1^2}{k+1}-\frac{k-1}{k+1}\right]^{\frac{1}{1-k}}$$

$$s_2-s_1=c_p\ln\frac{T_2}{T_1}-R\ln\frac{p_2}{p_1}$$

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