

Compressible Flow – Converging-Diverging Nozzles



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For air:

$$k = 1.4, R = 287 \text{ J/(kg}\cdot\text{K)} = 53.3 \text{ (ft}\cdot\text{lb}_f\text{)/(lb}_m\cdot\text{degR)} = 1716 \text{ ft}^2\text{/(s}^2\cdot\text{degR)}$$

1D, steady, adiabatic flow of a perfect gas with no work other than pressure work

$$T \left(1 + \frac{k-1}{2} \text{Ma}^2 \right) = \text{constant}$$

$$\frac{T}{T_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1} \quad \text{and} \quad \frac{T^*}{T_0} = \left(1 + \frac{k-1}{2} \right)^{-1}$$

$$\frac{c}{c_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-\frac{1}{2}} \quad \text{and} \quad \frac{c^*}{c_0} = \left(1 + \frac{k-1}{2} \right)^{-\frac{1}{2}}$$

1D, steady, isentropic flow of a perfect gas with no work other than pressure work

$$\frac{p}{p_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{\frac{k}{1-k}} \quad \text{and} \quad \frac{p^*}{p_0} = \left(1 + \frac{k-1}{2} \right)^{\frac{k}{1-k}} \quad (\text{for air } (k_{\text{air}} = 1.4), p^*/p_0 = 0.5283)$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{k-1}{2} \text{Ma}^2 \right)^{\frac{1}{1-k}} \quad \text{and} \quad \frac{\rho^*}{\rho_0} = \left(1 + \frac{k-1}{2} \right)^{\frac{1}{1-k}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}} \left(\frac{1 + \frac{k-1}{2} \text{Ma}^2}{1 + \frac{k-1}{2}} \right)^{\frac{k+1}{2(k-1)}}$$

$$\dot{m}_{\text{choked}} = \left(1 + \frac{k-1}{2} \right)^{\frac{k+1}{2(1-k)}} p_0 \sqrt{\frac{k}{RT_0}} A^*$$

Normal Shock Relations

$$\text{Ma}_2^2 = \frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - (k-1)}$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)\text{Ma}_1^2 \right] \left[\frac{2k\text{Ma}_1^2 - (k-1)}{(k+1)^2 \text{Ma}_1^2} \right]$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)\text{Ma}_1^2}{(k-1)\text{Ma}_1^2 + 2}$$

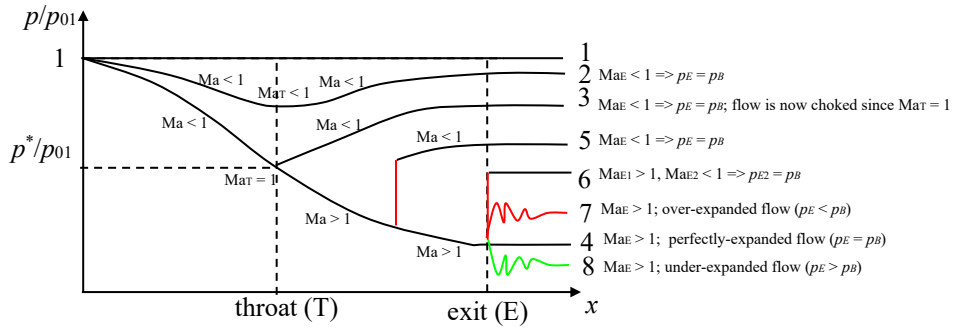
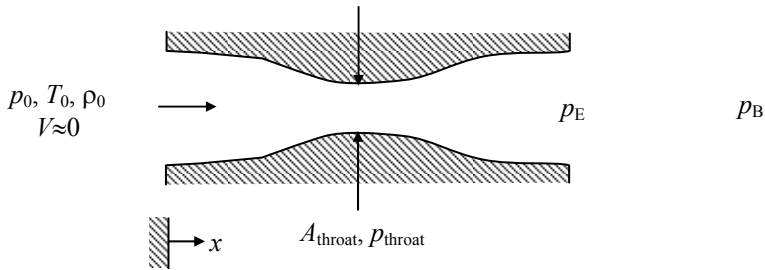
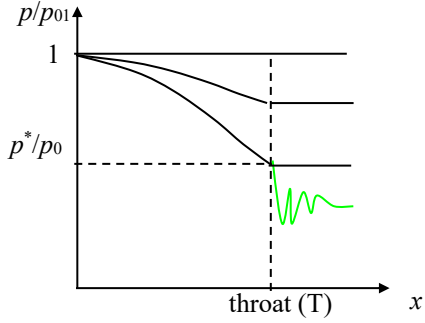
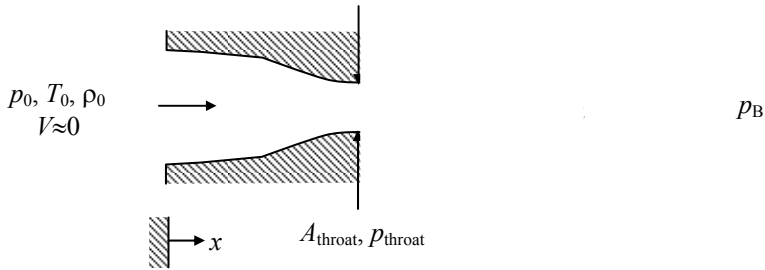
$$\frac{p_2}{p_1} = \frac{2k\text{Ma}_1^2}{k+1} - \frac{k-1}{k+1}$$

$$\frac{T_{02}}{T_{01}} = 1$$

$$\frac{p_{02}}{p_{01}} = \frac{A_1^*}{A_2^*} = \frac{\rho_{02}}{\rho_{01}} = \left[\frac{\frac{k+1}{2} \text{Ma}_1^2}{1 + \frac{k-1}{2} \text{Ma}_1^2} \right]^{\frac{k}{k-1}} \left[\frac{2k\text{Ma}_1^2}{k+1} - \frac{k-1}{k+1} \right]^{\frac{1}{1-k}}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

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$p_b \downarrow$