9.8. Turbulent Boundary Layer over a Flat Plate with No Pressure Gradient

To analyze a turbulent boundary layer we must use the momentum integral approach coupled with experimental data since no exact solutions are known. To approximate the velocity profile in a turbulent boundary layer, recall the Law of the Wall (refer to Chapter 10),

$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu} \qquad \text{for } \frac{yu^*}{\nu} \leq 5, \tag{9.166}$$

$$\frac{\bar{u}}{u^*} = \frac{1}{K'} \ln\left(\frac{yu^*}{\nu}\right) + c \quad \text{for } \frac{yu^*}{\nu} > 5, \tag{9.167}$$

where $u^* = \sqrt{\tau_w/\rho}$ is the "friction velocity". We could substitute this velocity profile into the KMIE and solve. This velocity profile is cumbersome to use, however. Instead, Prandtl suggested approximating the logarithmic turbulent velocity profile using a 1/7th power-law curve fit,

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \quad \text{for } \frac{y}{\delta} \leq 1, \tag{9.168}$$

$$\frac{u}{U_\infty} = 1 \qquad \text{for } \frac{y}{\delta} > 1. \tag{9.169}$$

Using this velocity profile, the momentum thickness becomes,

$$\delta_M = \int_0^\infty \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \left[1 - \left(\frac{y}{\delta}\right)^{\frac{1}{2}}\right] dy, \tag{9.170}$$

$$\delta_M = \frac{7}{72}\delta. \tag{9.171}$$

To determine the shear stress, recall that from the Kármán momentum integral equation,

$$\tau_w = \rho U_\infty^2 \frac{d\delta_M}{dx} = \frac{7}{72}\rho U_\infty^2 \frac{d\delta}{dx}, \tag{9.172}$$

so the friction coefficient becomes,

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{7}{36}\frac{d\delta}{dx}. \tag{9.173}$$

Experimental wall friction data for turbulent boundary layers can be fit using,

$$c_f \approx 0.020 \mathrm{Re}_\delta^{-\frac{1}{6}}, \tag{9.174}$$

where, $\mathrm{Re}_\delta = U_\infty \delta/\nu$. Note that experimental data for the wall shear stress is used instead of $\tau_w = \mu(du/dy)_{y=0}$, which was used for laminar boundary layers. The reason for the difference is that turbulent boundary layers use time-averaged data rather than instantaneous data. Equating the two friction coefficients gives,

$$\frac{7}{36}\frac{d\delta}{dx} = 0.020\left(\frac{U_\infty \delta}{\nu}\right)^{-\frac{1}{6}}, \tag{9.175}$$

$$\int_{\delta_0}^{\delta} \delta^{\frac{1}{6}} d\delta = 0.103\left(\frac{U_\infty}{\nu}\right)^{\frac{1}{6}} \int_{x_0}^{x} dx, \tag{9.176}$$

$$\delta^{\frac{7}{6}} - \delta_{)}^{\frac{7}{6}} = 0.120\left(\frac{U_\infty}{\nu}\right)^{-\frac{1}{6}} (x - x_0). \tag{9.177}$$

Assuming $\delta_0 = 0$ at $x_0 = 0$, meaning that the boundary layer starts off turbulent at the leading edge (refer to Figure 9.14), the previous equation becomes,

$$\left(\frac{\delta}{x}\right)^{\frac{7}{6}} = 0.120 \left(\frac{\nu}{U_\infty x}\right)^{\frac{1}{6}}, \tag{9.178}$$

$$\boxed{\frac{\delta}{x} \approx \frac{0.163}{\mathrm{Re}_x^{\frac{1}{7}}}}. \tag{9.179}$$

From this relation we can also determine the displacement thickness, momentum thickness, friction factor,

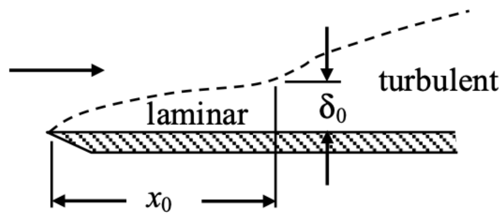


FIGURE 9.14. A sketch showing the boundary condition used in integrating the turbulent boundary layer thickness equation. The shear stress relationship holds strictly for the part of the boundary layer that is turbulent. If the laminar boundary layer thickness and distance downstream are small in comparison to the current thickness and location, then we may assume that the turbulent boundary layer starts approximately at the leading edge, i.e., $\delta_0 = 0$ at $x_0 = 0$.

and drag coefficient. These relations are summarized below.

$$\boxed{\frac{\delta}{x} \approx \frac{0.16}{\mathrm{Re}_x^{\frac{1}{7}}}} \quad \boxed{\frac{\delta_D}{x} \approx \frac{0.02}{\mathrm{Re}_x^{\frac{1}{7}}}} \quad \boxed{\frac{\delta_M}{x} \approx \frac{0.016}{\mathrm{Re}_x^{\frac{1}{7}}}}, \tag{9.180}$$

$$\boxed{c_f \approx \frac{0.027}{\mathrm{Re}_x^{\frac{1}{7}}}} \quad \boxed{c_D \approx \frac{0.031}{\mathrm{Re}_L^{\frac{1}{7}}}} \quad \mathrm{Re}_x = \frac{U_\infty x}{\nu}. \tag{9.181}$$

Notes:

(1) The boundary layer thickness grows as $\delta \sim x^{\frac{6}{7}}$ for a turbulent boundary layer whereas it grows as $\delta \sim x^{\frac{1}{2}}$ for a laminar boundary layer. Hence, a boundary layer grows more rapidly with downstream distance for turbulent flow than for a laminar flow. The momentum and displacement thicknesses also increase more rapidly for turbulent boundary layers.

(2) The shear stress decreases more rapidly for laminar flow than for a turbulent flow. The drag does not increase as rapidly in a laminar flow as compared to a turbulent flow.

(3) Another experimental friction curve fit that is commonly used is:

$$c_f \approx \frac{0.0466}{\mathrm{Re}_\delta^{\frac{1}{4}}}, \tag{9.182}$$

which gives,

$$\boxed{\frac{\delta}{x} \approx \frac{0.382}{\mathrm{Re}_x^{\frac{1}{5}}}} \quad \boxed{\frac{\delta_D}{x} \approx \frac{0.0478}{\mathrm{Re}_x^{\frac{1}{5}}}} \quad \boxed{\frac{\delta_M}{x} \approx \frac{0.0371}{\mathrm{Re}_x^{\frac{1}{5}}}}, \tag{9.183}$$

$$\boxed{c_f \approx \frac{0.0594}{\mathrm{Re}_x^{\frac{1}{5}}}} \quad \boxed{c_D \approx \frac{0.0742}{\mathrm{Re}_L^{\frac{1}{5}}}} \quad \mathrm{Re}_x = \frac{U_\infty x}{\nu}. \tag{9.184}$$

White (in White, F.M., *Viscous Fluid Flow*, 2nd ed., McGraw-Hill) states that the experimental curve fit given by Eq. (9.182) is based on limited data and is not as accurate as the curve fit given by Eq. (9.174). This argument is supported by the plot shown in Figure 9.15.

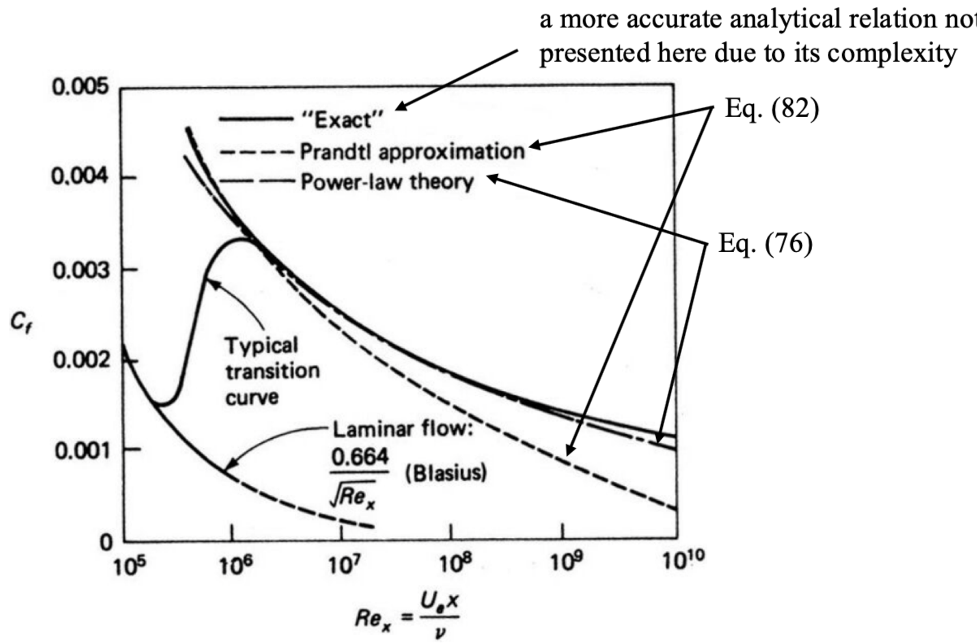
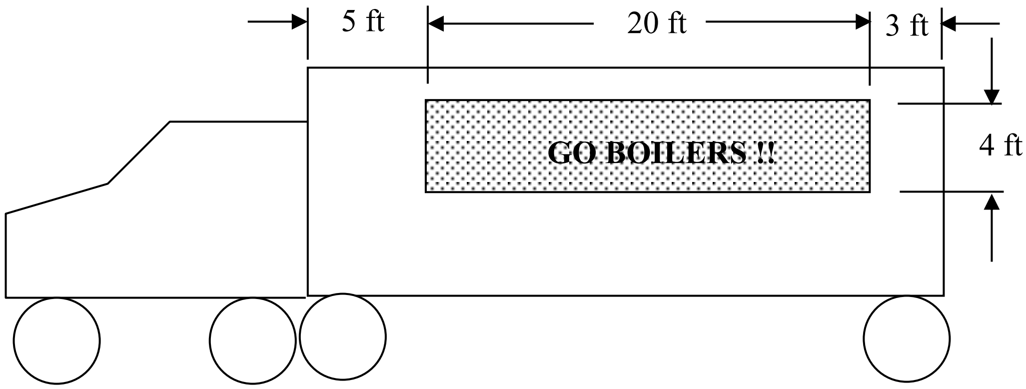


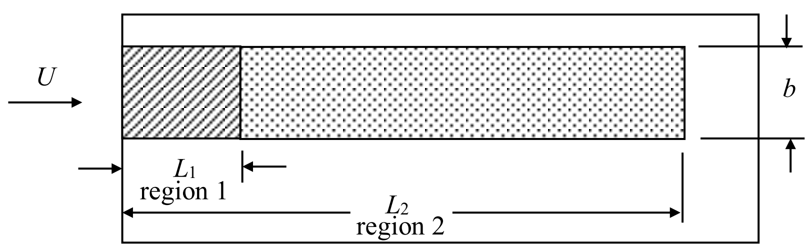
Figure 9.15. Boundary layer friction coefficients plotted against the Reynolds number based on the boundary layer thickness. Note that in the plot, Eq. (82) is actually Eq. (9.181) and Eq. (76) is actually Eq. (9.184)). Plot from White, F.M., *Viscous Fluid Flow*, 2nd ed., McGraw-Hill.

A thin smooth sign is attached to the side of a truck as shown. Estimate the skin friction drag on the sign when the truck speed is 55 mph.



SOLUTION:

Assume that the boundary layer forms at the front of the trailer.



$L_1 = 5$ ft
$L_2 = 25$ ft
$b = 4$ ft
$U = 55$ mph $= 80.7$ ft/s
$\nu_{air} = 1.57*10^{-4}$ ft^2/s
$\rho_{air} = 2.38*10^{-3}$ slugs/ft^3

To find the drag on the sign, determine the drag on region 2 and subtract the drag from region 1.

$$D_{\text{sign}} = D_2 - D_1 \tag{1}$$

where

$$D_i = c_{Di}\tfrac{1}{2}\rho U^2 L_i b \quad (i = 1 \text{ or } 2) \tag{2}$$

Substitute and simplify.

$$D_{\text{sign}} = \tfrac{1}{2}\rho U^2 b\left(c_{D2}L_2 - c_{D1}L_1\right) \tag{3}$$

The drag coefficients are determined from the Reynolds numbers at each region's trailing edge.

$$\text{Re}_1 = \frac{UL_1}{\nu} = \frac{(80.7\text{ ft/s})(5\text{ ft})}{\left(1.57*10^{-4}\text{ ft}^2/\text{s}\right)} = 2.6*10^6 \quad \text{(turbulent!)} \tag{4}$$

$$\text{Re}_2 = \frac{UL_2}{\nu} = \frac{(80.7\text{ ft/s})(25\text{ ft})}{\left(1.57*10^{-4}\text{ ft}^2/\text{s}\right)} = 1.3*10^7 \quad \text{(turbulent!)} \tag{5}$$

Assume that the flow is fully turbulent throughout regions 1 and 2 (neglect any laminar flow contribution) so that:

$$c_{D1} = \frac{0.0742}{\text{Re}_1^{1/5}} = \frac{0.0742}{\left(2.6*10^6\right)^{1/5}} = 3.87*10^{-3} \tag{6}$$

$$c_{D2} = \frac{0.0742}{\text{Re}_2^{1/5}} = \frac{0.0742}{\left(1.3*10^7\right)^{1/5}} = 2.80*10^{-3} \tag{7}$$

Substitute into Eqn. (3) and evaluate.

$$D_{\text{sign}} = \tfrac{1}{2}\left(2.38*10^{-3}\ \text{slugs/ft}^3\right)\left(80.7\ \text{ft/s}\right)^2\left(4\ \text{ft}\right)\left[\left(2.80*10^{-3}\right)\left(25\ \text{ft}\right)-\left(3.87*10^{-3}\right)\left(5\ \text{ft}\right)\right]$$

$$\boxed{\therefore D_{\text{sign}} = 1.57\ \text{lb}_\text{f}} \tag{8}$$

A vertical stabilizing fin on a land-speed-record car is 1.65 m long and 0.785 m tall. The automobile is to be driven at the Bonneville Salt Flats in Utah, where the elevation is 1340 m and the summer temperature reaches 50 degC. The car speed is 560 km/hr. Calculate the power required to overcome skin friction drag on the fin.

SOLUTION:

At a temperature of 313 K, the kinematic viscosity of air is $\nu = 2.0*10^{-5}$ m^2/s. Thus, the Reynolds number at the trailing edge of the vertical fin is:

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{\left(560*10^3\ \frac{\text{m}}{\text{hr}}\cdot\frac{\text{hr}}{3600\text{ s}}\right)(1.65\text{ m})}{\left(1.7*10^{-5}\ \frac{\text{m}^2}{\text{s}}\right)} \Rightarrow \mathrm{Re}_L = 1.51*10^7 \tag{1}$$

Clearly the flow is turbulent at the trailing edge of the vertical fin. At what distance from the leading edge of the fin does the flow transition from laminar to turbulent? To answer this question, calculate the distance at the transition Reynolds number,

$$\mathrm{Re}_{\text{crit}} = 500,000 = \frac{Ux}{\nu} = \frac{\left(560*10^3\ \frac{\text{m}}{\text{hr}}\cdot\frac{\text{hr}}{3600\text{ s}}\right)x}{\left(1.7*10^{-5}\ \frac{\text{m}^2}{\text{s}}\right)} \Rightarrow x = 5.5\text{ cm} \tag{2}$$

Thus, most of the flow over the fin is turbulent. Since this is the case, approximate the entire flow over the fin as being turbulent. The drag coefficient for a turbulent boundary layer over a flat plate is,

$$C_D = \frac{0.0742}{\mathrm{Re}_L^{1/5}} \Rightarrow \frac{D_{\text{1-side}}}{\frac{1}{2}\rho U^2 LH} = \frac{0.0742}{\mathrm{Re}_L^{1/5}} \Rightarrow D_{\text{2-sides}} = 2D_{\text{1-side}} = 2\cdot\frac{0.0742}{\mathrm{Re}_L^{1/5}}\cdot\frac{1}{2}\rho U^2 LH \tag{3}$$

The power is given by,

$$P = UD_{\text{2-sides}} \Rightarrow \boxed{P = \frac{0.0742}{\mathrm{Re}_L^{1/5}}\cdot\rho U^3 LH} \tag{4}$$

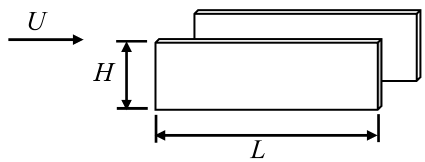
Using the given data,

ρ = 1.075 kg/m^3 (standard atmosphere at an altitude of 1340 m)
Re_L = $1.51*10^7$
U = 560 km/hr = 155.6 m/s
L = 1.65 m
H = 0.785 m
$\Rightarrow$ $\boxed{P = 14.3\text{ kW}}$

Note that a speed of 540 km/hr at a temperature of 50 degC result in a Mach number of 0.43. Thus, a more accurate approach to solving this problem would assume relations for a compressible boundary layer, rather than the incompressible relations assumed in the previous solution.

The U.S. Navy has built the *Sea Shadow*, which is a *small waterplane area twin-hull* (SWATH) ship with a reduced radar profile. This catamaran is 160 ft long and its twin hulls have a draft of 14 ft. Assume that ocean turbulence triggers a fully turbulent boundary layer on the sides of each hull. Treat these as flat plate boundary layers and calculate the drag on the ship and power required to overcome this drag for speeds ranging from 5 to 13 knots.

SOLUTION:
Model the twin hulls as two flat plates with turbulent boundary layers as shown in the figure below.



Assuming turbulent boundary layer over the full length of the hull, the drag force on one side of a hull is,

$$C_D \equiv \frac{D}{\frac{1}{2}\rho U^2 LH} = \frac{0.0742}{\mathrm{Re}_L^{1/5}}, \tag{1}$$

$$D_{\substack{\text{one side}\\ \text{of hull}}} = \frac{0.0742}{\mathrm{Re}_L^{1/5}}\left(\tfrac{1}{2}\rho U^2 LH\right) \text{ where } \mathrm{Re}_L = \frac{UL}{\nu}. \tag{2}$$

The total drag acting on the ship will be four times the drag in Eq. (2) since there are two hulls, each with two sides,

$$D_{\text{total}} = 4D_{\substack{\text{one side}\\ \text{of hull}}}. \tag{3}$$

Using the given numbers,

ρ_{seawater} = 1025 kg/m^3 = 63.99 lb$_m$/ft^3,
U = 5 to 13 kn = 8.44 ft/s to 21.94 ft/s (1 kt = 1.15 mph = 1.688 ft/s),
L = 160 ft,
H = 14 ft,
μ_{seawater} = 1.08*10^{-3} Pa.s,
=> $\nu_{\text{seawater}} = \mu/\rho$ = 1.05*10^{-6} m^2/s = 1.13*10^{-5} ft^2/s,
=> Re_L = 1.19*10^8 – 3.10*10^8 (clearly in the turbulent regime),
=> $D_{\text{one side of hull}}$ = 285 – 1590 lb$_f$ (1 lb$_f$ = 32.2 lb$_m$.ft/s^2),
=> $\boxed{D_{\text{total}} = 1140 - 6370 \text{ lb}_f}$.

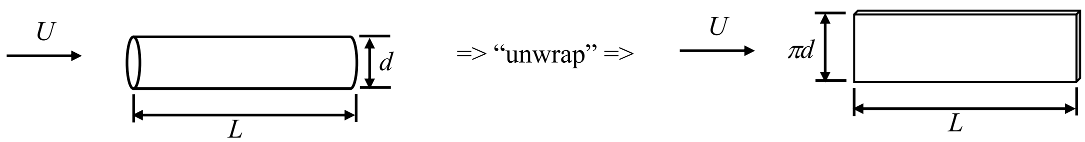
The power to overcome this total drag is,
$P = D_{\text{total}}U$,
=> $\boxed{P = 17.5 - 254 \text{ hp}}$ (1 hp = 550 lb$_f$.ft/s)

Note that the hulls for the *Sea Shadow* are more complex than the flat plates described in this simple problem. The actual hulls have cylindrical elements, which are tapered at the ends, as shown in the figure to the side.

The U.S. Navy's *Ohio*-class guided-missile submarines have a length of 170.69 m (560 ft) and a beam, i.e., width, of 12.8 m (42 ft). Assume the submarine travels at 37.0 kph (= 20 kn) when fully submerged.

a. What percentage of the submarine's surface has a laminar boundary layer for these conditions?
b. Estimate the power required for the submarine to overcome skin friction drag for these conditions.

SOLUTION:
Model the submarine as a cylinder with a diameter of d = 12.8 m and a length of L = 170.69 m. "Unwrap" the cylinder and model the flow along its length of the cylinder as flow adjacent to a flat plate as shown in the figures below.



First calculate the distance from the leading edge at which the boundary layer transitions from laminar to turbulent flow,

$$\mathrm{Re}_{x_{\text{crit}}} = \frac{Ux_{\text{crit}}}{\nu} = 500{,}000 \quad \Rightarrow \quad x_{\text{crit}} = 500{,}000\left(\frac{\nu}{U}\right). \tag{1}$$

Using the given numbers,

U = 37.0 kph = 10.3 m/s,
$\rho_{\text{seawater}} = 1025\ \text{kg/m}^3$,
$\mu_{\text{seawater}} = 1.08\times10^{-3}$ Pa.s,
=> $\nu = \mu/\rho = 1.05\times10^{-6}\ \text{m}^2\text{/s}$, (2)
=> $x_{\text{crit}} = 5.10\times10^{-2}$ m = 5.1 cm!

Thus, the fraction of the length that's laminar is,

$\boxed{x_{\text{crit}}/L = (5.10\times10^{-2}\ \text{m})/(170.69\ \text{m}) = 0.030\%.}$ (3)

Clearly, the flow over the submarine can be assumed turbulent over the entire length without much error.

Assuming a turbulent boundary layer over the full length of the hull, the drag force is,

$$C_D \equiv \frac{D}{\frac{1}{2}\rho U^2 L(\pi d)} = \frac{0.0742}{\mathrm{Re}_L^{1/5}}, \tag{4}$$

$$D = \frac{0.0742}{\mathrm{Re}_L^{1/5}}\left(\tfrac{1}{2}\rho U^2 LH\right) \text{ where } \mathrm{Re}_L = \frac{UL}{\nu}. \tag{5}$$

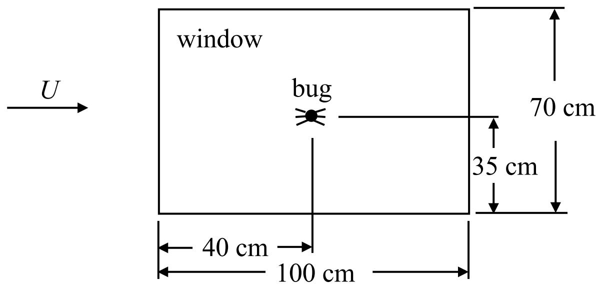
Using the given numbers,

L = 170.69 m,
$\pi d = \pi(12.8\ \text{m}) = 40.2$ m,
=> $\mathrm{Re}_L = 1.67\times10^9$,
=> $\boxed{D = 3.96\times10^5\ \text{N}}$ (= 89,000 lb_f!)

The power to overcome this skin friction drag is,

$P = DU$,
=> $\boxed{P = 4.08\times10^6\ \text{W}}$ (= 5470 hp!)

A small bug rests on the outside of a car side window as shown in the figure below. The surrounding air has a density of 1.2 kg/m^3 and kinematic viscosity of 1.5*10^{-5} m^2/s. To first order, we can approximate the flow as flat plate flow with no pressure gradient and the start of the boundary layer begins at the leading edge of the window. Also assume that the flow is turbulent over the entire length of the window (this isn't a good assumption, but for simplicity, we'll make it here).



a. Determine the minimum speed at which the bug will be sheared off of the car window if the bug can resist a shear stress of up to 1 N/m^2.
b. What is the total skin friction drag acting on the window at a speed of U = 20 m/s?
c. Ignoring the presence of the bug, at what streamwise location will the boundary layer separation point occur on the window? Justify your answer.

SOLUTION:

Assume that the flow over the window is turbulent at the bug location so that:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{\mathrm{Re}_x^{1/5}}, \tag{1}$$

$$\tau_w = (0.0594)\tfrac{1}{2}\rho U^2 \left(\frac{\nu}{Ux}\right)^{1/5} = (0.0594)\tfrac{1}{2}\rho U^{9/5}\left(\frac{\nu}{x}\right)^{1/5}, \tag{2}$$

$$\therefore U = \left[\frac{\tau_w}{\frac{1}{2}(0.0594)\rho}\left(\frac{x}{\nu}\right)^{1/5}\right]^{5/9}. \tag{3}$$

Using the given data:

τ_w = 1 N/m^2
ρ = 1.2 kg/m^3
x = 0.4 m
ν = 1.5*10^{-5} m^2/s
$\Rightarrow$ U = 20 m/s

Check the Reynolds number to verify the turbulent flow assumption.

$$\mathrm{Re}_x = \frac{Ux}{\nu} \approx 530{,}000 \;\Rightarrow\; \text{Turbulent flow assumption is ok!} \tag{4}$$

Hence, the minimum required speed to shear off the bug is 20 m/s.

The total skin friction drag acting on the car window (assuming turbulent flow throughout) at the given velocity is:

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 (LW)} = \frac{0.0742}{\mathrm{Re}_L^{1/5}} \quad \text{where } \mathrm{Re}_L = \frac{UL}{\nu}, \tag{5}$$

$$\therefore D = \tfrac{1}{2}\rho U^2 (LW)\frac{0.0742}{\mathrm{Re}_L^{1/5}}. \tag{6}$$

Using the given data:

U = 20 m/s
ρ = 1.2 kg/m^3
L = 1 m
W = 0.7 m
ν = $1.5*10^{-5}$ m^2/s
$\Rightarrow$ $\mathrm{Re}_L = 1.3*10^6$
$\Rightarrow$ $C_D = 4.4*10^{-3}$
$\Rightarrow$ $\boxed{D = 0.7 \text{ N}}$

Boundary layer separation will not occur since there is no adverse pressure gradient in the flow (zero pressure gradient was assumed).

Note that the distance from the leading edge where the flow transitions from laminar to turbulent flow at U = 20 m/s is,

$$\mathrm{Re}_{x_{\text{crit}}} = \frac{Ux_{\text{crit}}}{\nu} = 500{,}000, \tag{7}$$

$$\Rightarrow x_{\text{crit}} = 500{,}000\left(\frac{\nu}{U}\right), \tag{8}$$

$x_{\text{crit}} =$ 37.5 cm,

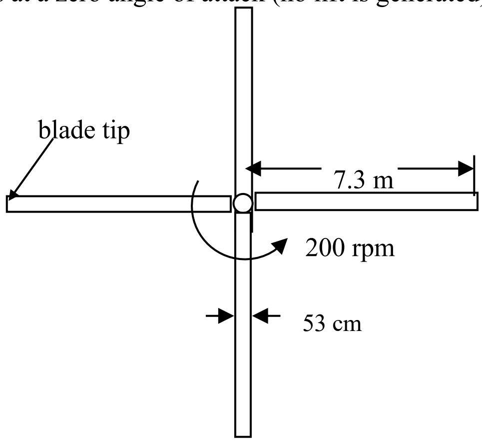
This distance is a considerable portion of the window length. Hence, a better approach to solving this problem would be to include both the laminar and turbulent portions in the analysis rather than neglecting the laminar portion as was done in the previous analysis.

Use the drag coefficient given in Pritchard et al. (8th ed., Eq. 9.37a), which takes into account the skin friction drag of the laminar part and the turbulent part,

$$C_D = \frac{0.0742}{\mathrm{Re}_L^{1/5}} - \frac{1740}{\mathrm{Re}_L}. \tag{9}$$

For $\mathrm{Re}_L = 1.3*10^6$ (calculated previously), $C_D = 3.1*10^{-3}$, which gives a drag force of D = 0.5 N. This more accurate value for the drag is approximately 42% less than drag calculated assuming turbulent flow over the full length.

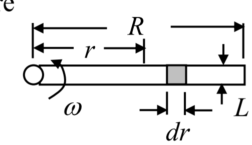
A four-bladed Apache helicopter rotor rotates at 200 rpm in air (with a density of 1.2 kg/m^3 and kinematic viscosity $1.5*10^{-5}$ m^2/s). Each blade has a chord length of 53 cm and extends a distance of 7.3 m from the center of the rotor hub. To greatly simplify the problem, assume that the blades can be modeled as very thin flat plates at a zero angle of attack (no lift is generated).



a. At what radial distance from the hub center is the flow at the blade trailing edge turbulent?
b. What is the (99%) boundary layer thickness at the blade tip trailing edge?
c. Assuming that the flow over the entire length of the four blades is turbulent, estimate the power required to drive the helicopter rotor (neglecting all other effects besides aerodynamic drag).

SOLUTION:

The transition to turbulence occurs when Re_{crit} = 500,000 where



$$\text{Re}_{\text{crit}} = \frac{UL}{\nu} = \frac{(r_{\text{crit}}\omega)L}{\nu} \tag{1}$$

$$r_{\text{crit}} = \frac{\nu\,\text{Re}_{\text{crit}}}{\omega L} = \frac{\left(1.5*10^{-5}\ \text{m}^2/\text{s}\right)(500{,}000)}{\left(20.94\ \text{rad}/\text{s}\right)\left(53*10^{-2}\ \text{m}\right)}$$

$$\text{(Note: } \omega = \left(200\ \text{rot}/\text{min}\right)\left(\frac{2\pi\ \text{rad}}{\text{rot}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right) = 20.94\ \text{rad}/\text{s}\ \text{)}$$

$$\boxed{\therefore r_{\text{crit}} = 0.68\ \text{m}} \tag{2}$$

To determine the boundary layer thickness at the blade tip trailing edge, first calculate the Reynolds number there.

$$\text{Re}_L = \frac{UL}{\nu} = \frac{(R\omega)L}{\nu} = \frac{(7.3\ \text{m})\left(20.94\ \text{rad}/\text{s}\right)\left(53*10^{-2}\ \text{m}\right)}{\left(1.5*10^{-5}\ \text{m}^2/\text{s}\right)} = 5.40*10^6 \tag{3}$$

$$\therefore \text{Re}_L = 5.40*10^6 \quad \Rightarrow \text{ the flow is turbulent}$$

Using the turbulent boundary layer correlations:

$$\frac{\delta}{L} = \frac{0.382}{\text{Re}_L^{1/5}} \tag{4}$$

$$\delta = \frac{0.382\left(53*10^{-2}\ \text{m}\right)}{\left(5.40*10^6\right)^{1/5}}$$

$$\boxed{\therefore \delta = 9.1*10^{-3}\ \text{m} = 9.1\ \text{mm}} \tag{5}$$

In order to determine the power required to drive the rotor, first determine the torque resulting from the skin friction drag acting on the blades.

$$dF_{\text{1-blade}} = \underbrace{2}_{\substack{\text{two}\\\text{sides}}} C_D \tfrac{1}{2}\rho \underbrace{U}_{=r\omega}^{2} \underbrace{dA}_{=Ldr} \quad \text{where } C_D = \frac{0.0742}{\text{Re}_L^{1/5}} \quad \text{(turbulent BL correlation)} \tag{6}$$

$$dT_{\text{1-blade}} = r dF_{\text{1-blade}} \tag{7}$$

$$T_{\text{4-blades}} = 4\int_{r=0}^{r=R} dT_{\text{1-blade}} = 4\int_{r=0}^{r=R} r dF_{\text{1-blade}}$$

$$= 4\int_{r=0}^{r=R} r2\frac{0.0742}{\text{Re}_L^{1/5}}\tfrac{1}{2}\rho(r\omega)^2 Ldr$$

$$= 4\int_{r=0}^{r=R} r\frac{0.0742}{\left(\frac{r\omega L}{\nu}\right)^{1/5}}\rho(r\omega)^2 Ldr$$

$$= 2.97*10^{-1}\rho\nu^{1/5}\omega^{9/5}L^{4/5}\int_{r=0}^{r=R} r^{14/5}dr$$

$$\therefore T_{\text{4-blades}} = 7.81*10^{-2}\rho\nu^{1/5}\omega^{9/5}L^{4/5}R^{19/5} \tag{8}$$

$$\boxed{P_{\text{4-blades}} = \omega T_{\text{4-blades}} = 7.81*10^{-2}\rho\nu^{1/5}\omega^{14/5}L^{4/5}R^{19/5}} \tag{9}$$

For:

ρ = 1.2 kg/m^3
ν = 1.5*10^{-5} m^2/s
ω = 20.94 rad/s
L = 53*10^{-2} m
R = 7.3 m
$\Rightarrow$ $T_{\text{4-blades}}$ = 2790 N·m $\Rightarrow$ $\boxed{P_{\text{4-blades}} = 58.3 \text{ kW}}$

A wind tunnel has a test section 1 m square by 6 m long with air at 20°C moving at an average velocity of 30 m/s. To account for the growing boundary layer, the walls are slanted slightly outward. At what angle should the walls be slanted between $x = 2$ m and $x = 4$ m to keep the test-section velocity constant?

SOLUTION:

Determine the displacement boundary layer thickness assuming flat plate flow. First check the flow Reynolds number to determine whether or not the flow is laminar.

$$\mathrm{Re}_{x=2\text{ m}} = \frac{Ux}{\nu} = \frac{(30\text{ m/s})(2\text{ m})}{(1.5e\text{-}5\text{ m}^2/\text{s})} = 4.0e6 \quad (1)$$

$$\mathrm{Re}_{x=4\text{ m}} = \frac{Ux}{\nu} = \frac{(30\text{ m/s})(4\text{ m})}{(1.5e\text{-}5\text{ m}^2/\text{s})} = 8.0e6$$

Thus the flow in the tunnel is turbulent in the range of interest.

Use the following correlation for turbulent flat plate flow to determine the displacement boundary layer thickness.

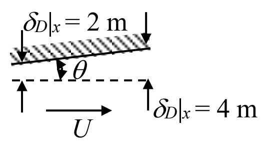
$$\frac{\delta_D}{x} = \frac{0.0478}{\mathrm{Re}_x^{1/5}} \quad (2)$$

$x = 2$ m: $\mathrm{Re}_x = 4.0e6 \Rightarrow \delta_D = 4.6e\text{-}3$ m

$x = 4$ m: $\mathrm{Re}_x = 8.0e6 \Rightarrow \delta_D = 8.0e\text{-}3$ m

As an approximation, assume that the boundary layer grows linearly between $x = 2$ m and $x = 4$ m so that the angle the walls need to be slanted outward is:

$$\tan\theta = \frac{\delta_D\big|_{x=4\text{ m}} - \delta_D\big|_{x=4\text{ m}}}{4\text{ m} - 2\text{ m}} \quad (3)$$



$$\boxed{\therefore \theta = 0.1^\circ} \quad (4)$$