9.9. Boundary Layer Separation

Consider flow around a cylinder as shown in Figure 9.16. On the front side of the cylinder the boundary layer

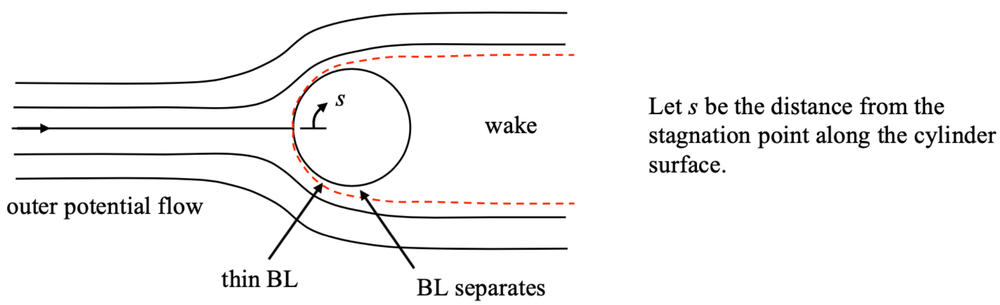


Figure 9.16. A sketch of the flow around a cylinder showing boundary layer separation.

grows with increasing distance along the surface, as we might expect. Moving toward the back half of the cylinder, however, the boundary layer no longer remains "attached" to the surface and, instead of following the cylinder contour, forms a wake behind the cylinder. The point where the boundary layer no longer follows the cylinder surface is termed the boundary layer separation point. As part of our examination of boundary layer separation, it's helpful to define the following:

$$\text{an adverse pressure gradient is one in which } dp/ds > 0, \text{ and} \tag{9.185}$$

$$\text{a favorable pressure gradient is one in which } dp/ds < 0. \tag{9.186}$$

In an adverse pressure gradient, the pressure increases moving downstream. Thus, the pressure is acting to decelerate the flow. In a favorable pressure gradient, the pressure decreases moving downstream and the pressure acts to accelerate the flow.

Now let's plot the pressure as a function of position on the cylinder surface. Figure 9.17 plots the pressure determined from potential flow theory as a function of the angle measured from the leading stagnation point. Imagine a fluid particle near the cylinder surface. It experiences the same pressure as that in the outer potential flow (recall that the pressure doesn't vary much in the spanwise direction within a boundary layer). As the fluid particle moves from point A to point B, it accelerates and gains energy as it's accelerated by the favorable pressure gradient. In going from point B to C, however, the fluid particle experiences an adverse pressure gradient that acts to decelerate the fluid particle. If the flow is inviscid, as is the case in a potential flow, then the flow on the back half of the cylinder will be symmetric to the flow on the front half and the fluid particle decelerates to zero speed at the trailing edge stagnation point located at point C. In a real flow, however, the fluid particle is subject to viscous stresses, which constantly remove energy from the flow. As a result, on the back half of the cylinder the fluid particle decelerates rapidly due to the adverse pressure gradient and viscous dissipation. At some point between B and C, the fluid particle loses its forward momentum and is no longer able to move further into the adverse pressure gradient. It is at this point the flow separates, with the fluid particle moving downstream and forming a wake rather than following the cylinder surface.

Figure 9.18 plots the dimensionless pressure on the cylinder surface for a potential (inviscid) flow along with representative curves for viscous flows with laminar and turbulent boundary layers. The inviscid analysis works well at predicting the pressure on the upstream side of the cylinder near the leading stagnation point, but becomes increasingly inaccurate moving toward the downstream side of the cylinder. The reason for the increasing inaccuracy is due to boundary layer separation, which distorts the flow streamlines so they no longer match those from a potential flow analysis. Once the boundary layer separates, a wake forms, within which the pressure remains nearly constant and equal to the pressure on the cylinder surface where the boundary layer separated.

Notes:

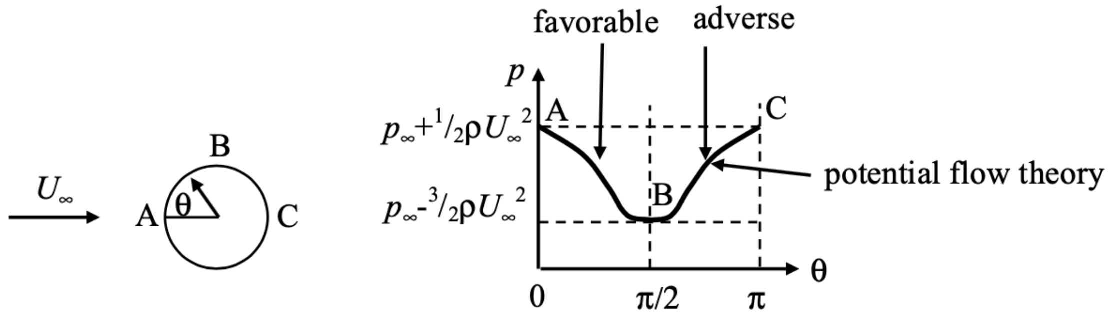
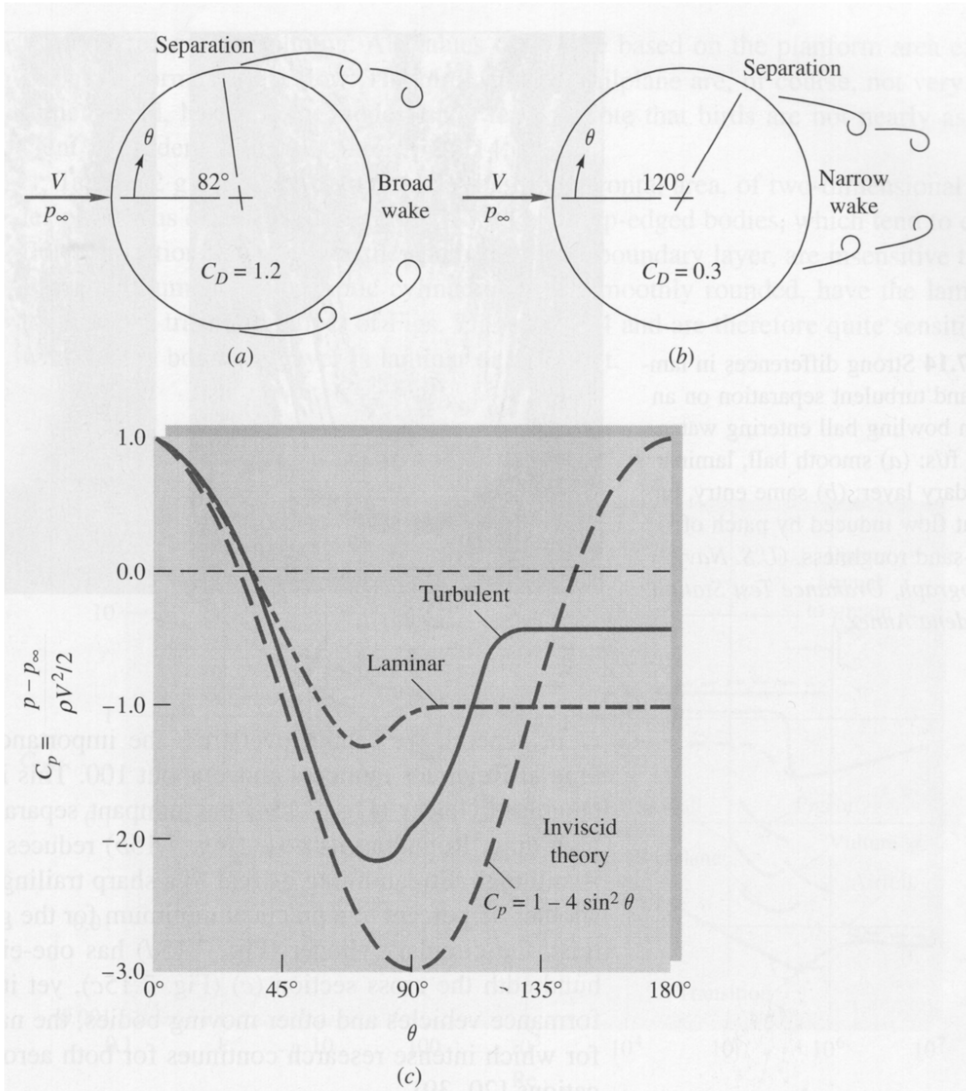


Figure 9.17. The pressure at the surface of the cylinder plotted as a function of the angle from the leading stagnation point, assuming potential flow around the cylinder. Over the front half of the cylinder the pressure gradient is favorable. Over the back half the pressure gradient is adverse.

(1) Boundary layer separation requires an adverse pressure gradient; however, an adverse pressure gradient does not necessarily cause boundary layer separation. It's possible for the boundary layer to have enough momentum to carry it through the adverse pressure gradient region.
(2) Delaying boundary layer separation results in a wake that is both smaller and with a more symmetric pressure profile (front and back halves of the cylinder). Both effects act to reduce the cylinder's form drag, i.e., the drag due to pressure forces.
(3) A turbulent boundary layer separates later than a laminar boundary layer (and, thus, has smaller form drag). The reason for the delayed separation is that a turbulent boundary layer has more momentum than a laminar boundary layer, as shown in Figure 9.19. The larger momentum in the turbulent boundary layer means that the boundary layer flow can travel further into the adverse pressure gradient region before separating. The larger momentum in the turbulent boundary layer is the result of turbulent eddies mixing free stream air, which has large momentum, into the boundary layer.
(4) Although a turbulent boundary layer results in a smaller form drag, i.e., pressure drag, due to delayed boundary layer separation, it does increase the skin friction drag, i.e., the drag due to viscous wall shear stresses on the surface. From Figure 9.19 one can see that the velocity gradient at the surface is larger for a turbulent boundary layer than for a laminar boundary layer and, thus, the shear stress will be larger. As is discussed in the following section, one must consider both form drag and skin friction drag when determining the total drag on an object.
(5) A laminar boundary layer can be "tripped" into becoming a turbulent boundary layer. Common techniques for inducing a turbulent boundary layer are to add roughness or bumps to a surface.
(6) Examples of flows with boundary layer separation are shown in Figures 9.20 - 9.23. Figure 9.20 provides a schematic of the boundary layer flow in a diverging channel. From Conservation of Mass, the velocity in the diverging channel decreases moving downstream. From Bernoulli's equation, the pressure must increase. Thus, the pressure gradient is adverse and boundary layer separation can occur. Figure 9.21 shows photographs from a corresponding flow.
    Figure 9.22 shows a photograph of a laminar boundary layer separating over the top of a cylinder (top-most image). This separation point is further upstream than when the boundary layer is turbulent (image second from the top). The bottom two photographs show flow over a sharp, obtuse angle. When the boundary layer is laminar (second from the bottom), the flow separates immediately at the apex; however, when the boundary layer is turbulent, it has enough momentum to remain attached (bottom). This last photograph demonstrates the point made in the first note: Just because there's an adverse pressure gradient, it doesn't mean the boundary layer must separate. In this case the boundary layer has enough momentum to flow into the adverse pressure gradient region.



(From White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.)

Figure 9.18. The pressure coefficient on the surface of a cylinder plotted as a function of angle from the leading edge stagnation point. The figure includes curves assuming potential flow (inviscid theory), viscous laminar flow, and viscous turbulent flow. This figure is from White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.

The images shown in Figure 9.23 show bowling balls dropped into water. The ball in the left image is smooth and the boundary layer is laminar, resulting in separation near the ball's equator. The ball in the left image has a roughened surface at the leading edge, inducing a turbulent boundary layer and delayed separation.

Let's look at the velocity profile at different points along a flat plate for a flow with an adverse pressure gradient ($dp/dx > 0$), as shown in Figure 9.24. In an adverse pressure gradient flow the boundary layer velocity profile will always have an inflection point. This behavior can be shown by considering the boundary layer momentum equation,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2}. \tag{9.187}$$

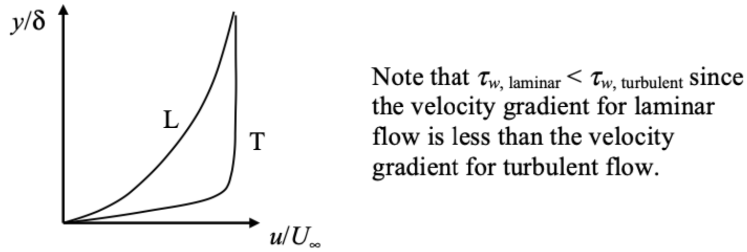
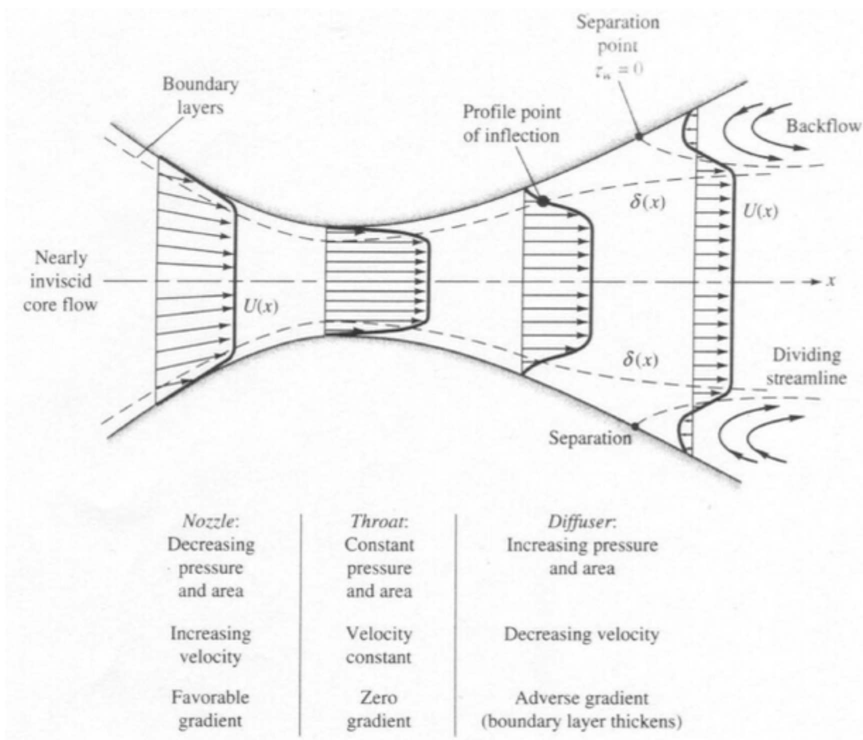


Figure 9.19. Sketches of laminar and turbulent boundary layer velocity profiles. Due to turbulent mixing, the turbulent boundary layer has more momentum than the laminar boundary layer.



(From White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.)

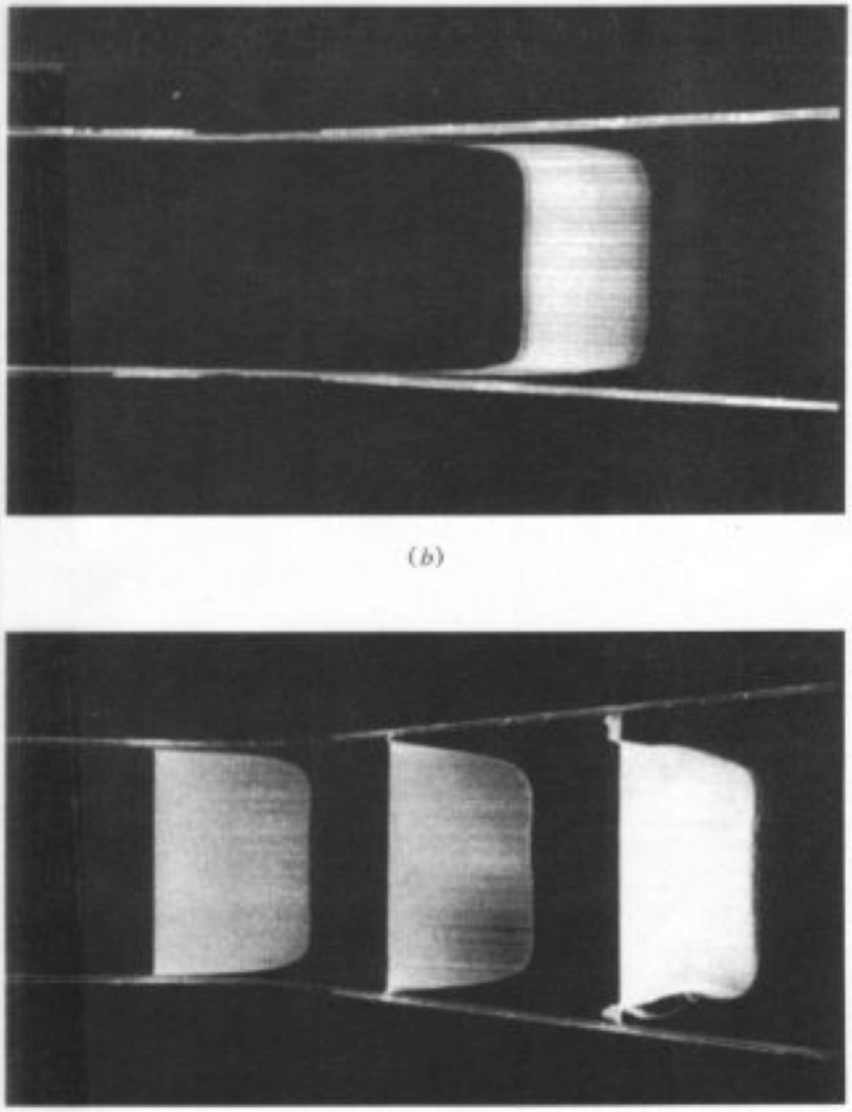
Figure 9.20. An illustration of flow in a diverging, planar channel. This figure is from White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.

Note that at the wall boundary ($y = 0$), $u = v = 0$ so that,

$$\frac{dp}{dx} = \mu \left. \frac{\partial^2 u}{\partial y^2} \right|_{y=0}. \tag{9.188}$$

Thus, at the wall boundary in an adverse pressure gradient ($dp/dx > 0$),

$$\left. \frac{\partial^2 u}{\partial y^2} \right|_{y=0} > 0. \tag{9.189}$$



(b)

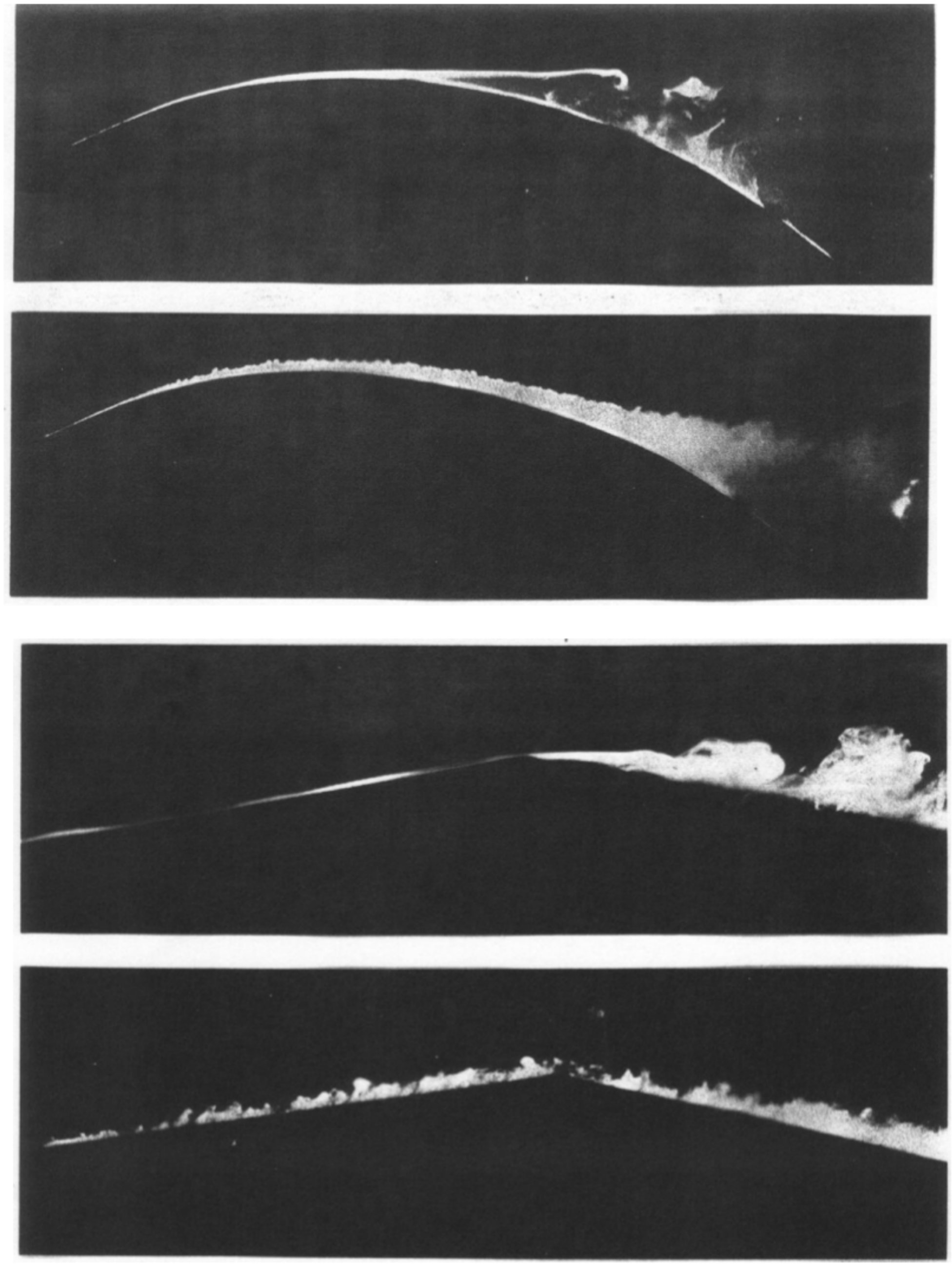
(c)

(b) Boundary-layer growth in a small-angle diffuser. (c) Boundary-layer separation in a large-angle diffuser. [Parts (b) and (c) from the film "Fundamental of Boundary Layers," by the National Committee for Fluid Mechanics Films and the Educational Development Center.]

Figure 9.21. Flow in a diverging, planar channel. In the top figure, the diverging angle is small and the adverse pressure gradient is sufficiently small so that boundary layer separation doesn't occur. In the bottom figure, the diverging angle is larger resulting in a larger adverse pressure gradient and separation.

At the free stream ($y = \delta$), however, we have,

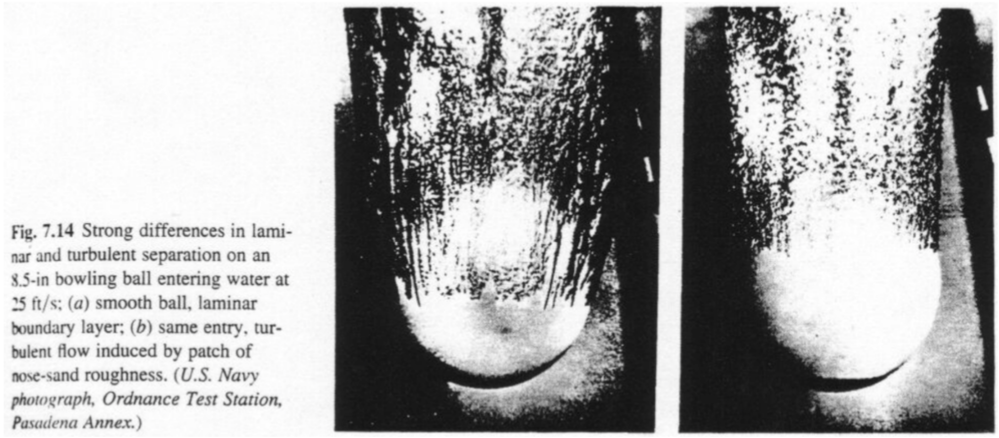
$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=\delta} < 0, \tag{9.190}$$



(From Van Dyke, M., *An Album of Fluid Motion*, Parabolic Press.)

Figure 9.22. Photographs of a laminar boundary flowing over a cylinder (top), a turbulent boundary layer flowing over a cylinder (second from top), a laminar boundary layer flowing over a sharp angle (second from bottom), and a turbulent boundary layer flowing over a sharp angle (bottom).

in order for the boundary layer profile to merge smoothly with the outer flow velocity. The change in sign of the boundary layer curvature indicates that somewhere within the boundary layer there must be a point of



(From White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.)

Figure 9.23. Two bowling balls dropped into water. The left image is a smooth bowling ball and the right image is a bowling ball with the leading edge surface roughened by attaching sand paper in order to induce a turbulent boundary layer. This figure is from White, F.M., *Fluid Mechanics*, 3rd ed., McGraw-Hill.

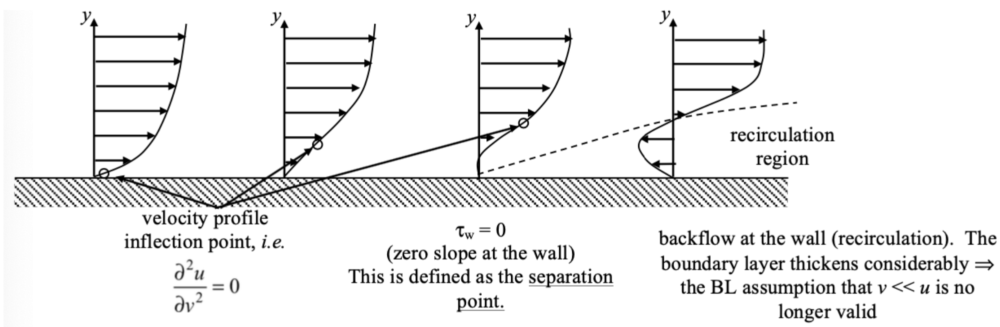


Figure 9.24. The boundary layer velocity profile at different downstream locations. Boundary layer separation is defined as occurring where the wall shear stress is zero. Downstream of this point there is flow recirculation.

inflection. The inflection point moves toward the outer flow boundary as the flow moves downstream in an adverse pressure gradient flow.

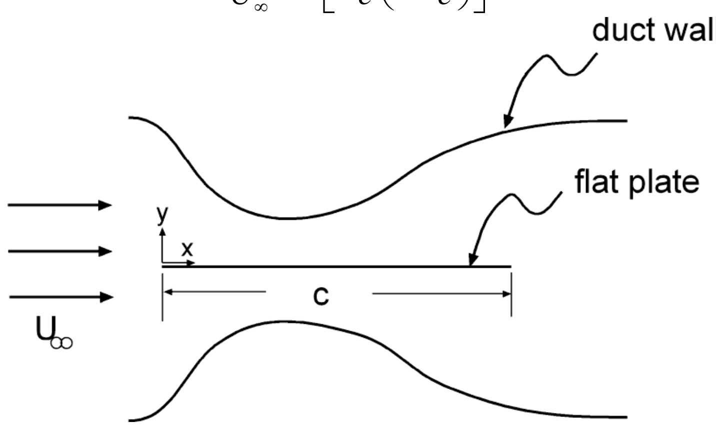
Boundary layer separation is defined as occurring where the shear stress is zero at the wall, i.e.,

$$\tau_w = 0 \implies \mu \left.\frac{du}{dy}\right|_{y=0} = 0 \implies \underline{\text{boundary layer separation}} \tag{9.191}$$

Downstream of the separation point, a recirculation zone occurs where the flow reverses direction (being pushed by the adverse pressure gradient). When recirculation occurs, the boundary layer grows rapidly in thickness and the fundamental assumption that the boundary layer thickness is small compared to the downstream distance breaks down. Thus, the boundary layer is considered not to extend beyond the point of separation.

A flat plate of length c is placed inside a duct. By curving the walls of the duct, the pressure distribution on the flat plate can be set. Assume the walls of the duct are contoured in such a way that the outer flow over the plate gives the following velocity on the surface of the flat plate:

$$\frac{u_e(x)}{U_\infty} = \left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{1/5}$$



1. Write an expression for the streamwise pressure gradient as a function of x/c.
2. Determine which portions of the plate have a favorable pressure gradient and which portions have an adverse pressure gradient.

SOLUTION:

In the outer flow region (the inviscid core), we can use Bernoulli's equation,

$$p + \tfrac{1}{2}\rho U^2 = \text{constant} \;\Rightarrow\; \frac{dp}{dx} + \rho U\frac{dU}{dx} = 0 \;\Rightarrow\; \frac{dp}{dx} = -\rho U\frac{dU}{dx} \tag{1}$$

Here,

$$U = u_e(x) = U_\infty\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{1/5} \;\Rightarrow\; \frac{dU}{dx} = \frac{8}{5}U_\infty\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{4}{5}}\left(\frac{1}{c}-\frac{2x}{c^2}\right) \;\Rightarrow$$

$$\frac{dU}{dx} = \frac{8}{5}\frac{U_\infty}{c}\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{4}{5}}\left(1-2\frac{x}{c}\right) \tag{2}$$

Thus,

$$\frac{dp}{dx} = -\rho\left\{U_\infty\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{1/5}\right\}\left\{\frac{8}{5}\frac{U_\infty}{c}\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{4}{5}}\left(1-2\frac{x}{c}\right)\right\} \tag{3}$$

$$\boxed{\frac{dp}{dx} = -\frac{8}{5}\frac{\rho U_\infty^2}{c}\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{3}{5}}\left(1-2\frac{x}{c}\right)} \tag{4}$$

An adverse pressure gradient is one in which $dp/dx > 0$. A favorable pressure gradient is one in which $dp/dx < 0$. Note also that $0 \le x/c \le 1$.

$$-\frac{8}{5}\frac{\rho U_\infty^2}{c}\underbrace{\left[8\frac{x}{c}\left(1-\frac{x}{c}\right)\right]^{-\frac{3}{5}}}_{>0 \text{ for } 0<x/c<1}\underbrace{\left(1-2\frac{x}{c}\right)}_{>0 \text{ for } x/c<\frac{1}{2}}\begin{cases} <0 & x/c<\frac{1}{2} \\ >0 & x/c>\frac{1}{2}\end{cases} \tag{5}$$

Thus, there is a favorable pressure gradient for $x/c < ½$ and adverse pressure gradient for $x/c > ½$.