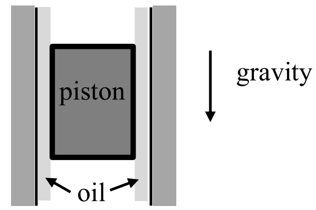
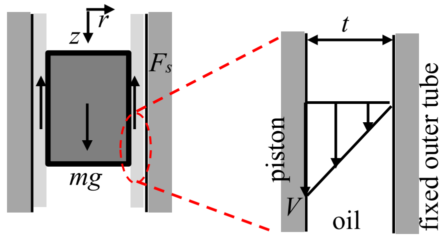
A 73 mm diameter aluminum (specific gravity of 2.64) piston of 100 mm length is centered in a stationary 75 mm inner diameter steel tube lined with SAE 10W-30 oil at 25 °C. The piston is set into motion by cutting a support cord. What is the terminal velocity of the piston? You may assume a linear velocity profile within the oil.



SOLUTION:

Draw a free body diagram of the piston and, since we're interested in the terminal speed of the piston, sum the forces in the z direction and set them equal to zero.



$$\sum F_z = 0 = mg - F_s, \tag{1}$$

where F_s is the shear force due to the viscous stress the fluid exerts on the piston. The mass of the piston is,

$$m = SG_{Al}\rho_{H20}\pi R^2 L, \tag{2}$$

where R is the radius of the piston.

The shear force acting on the surface of the piston is due to the viscous stress applied by the oil. To find this viscous stress, first assume a linear profile in the oil,

$$u_z = V\left(\frac{r-R}{t}\right), \tag{3}$$

where V is the speed of the piston. Hence, the gradient of the z velocity component in the r direction in the oil is,

$$\frac{du_z}{dr} = \frac{V}{t}. \tag{4}$$

The shear stress acting on the surface of the piston due to the oil, assuming Newtonian fluid behavior, is,

$$\tau_{rz}\big|_{r=R} = \mu \frac{du_z}{dr}\bigg|_{r=R} = \mu\frac{V}{t}. \tag{5}$$

Note that this shear stress is constant. The corresponding shear force due to this shear stress is determined by multiplying the shear stress by the area over which this stress acts,

$$F_s = \tau_{rz}\big|_{r=R}(2\pi RL) = \left(\mu\frac{V}{t}\right)(2\pi RL). \tag{6}$$

Substitute Eqs. (6) and (2) into Eq. (1) and solve for V,

$$0 = \left(SG_{Al}\rho_{H20}\pi R^2 L\right)g - \left(\mu\frac{V}{t}\right)(2\pi RL), \tag{7}$$

$$V = \frac{SG_{Al}\rho_{H20}\pi R^2 Lgt}{2\pi RL\mu}, \tag{8}$$

$$\boxed{V = \frac{SG_{Al}\rho_{H20}Rgt}{2\mu}}. \tag{9}$$

Note that the piston length isn't a factor.

Substitute the given parameters,

$SG_{Al} = 2.64$

$\rho_{H20} = 1000$ kg/m^3,

$R = 0.5*73*10^{-3}$ m $= 36.5*10^{-3}$ m,

$g = 9.81$ m/s^2,

$t = 0.5*(75 - 73)*10^{-3}$ m $= 1.0*10^{-3}$ m

$\mu = 0.13$ Pa.s (from Fig. A.2 of Pritchard et al.),

=> $\boxed{V = 3.64 \text{ m/s}}$.