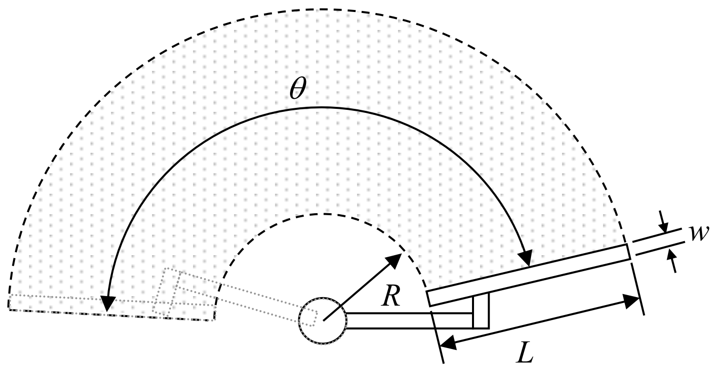
Derive an expression estimating the torque required to rotate a windshield wiper blade over the surface of a wet windshield in terms of the parameters given in the figure below,

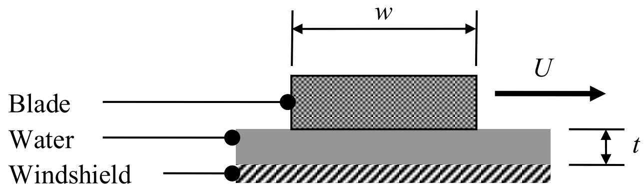


Here, R is the radius of the inner most point swept by the wiper blade, L is the length of the wiper blade, θ is the angle swept by the blade in a time T, w is the width of the part of the blade in contact with the windshield, t is the thickness of the water layer, and μ is the viscosity of water.

For a 1996 Toyota Rav 4, R = 6", L = 11", θ = 110°, T = 2 sec, w = ¼", and t = $^{1}/_{16}$". Evaluate your expression to estimate the torque.

SOLUTION:

Consider a differential element of the wiper along the radial direction. The cross-section for this element at radius r can be imagined as shown in the figure below.



If the wiper blade sweeps out an angle θ in a time T, the average speed U of the element over the windshield is,

$$U = \frac{\theta r}{T}. \tag{1}$$

Assuming Couette flow between the wiper blade and the windshield, the shear stress applied to the differential element is,

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{t} = \mu \frac{\theta r}{Tt}, \tag{2}$$

where t is the thickness of the water layer between the wiper and the windshield. The force on this differential element is the shear stress acting on this element multiplied by the area of the element,

$$dF = \tau dA = \mu \frac{\theta r}{Tt}(wdr). \tag{3}$$

The small moment on the wiper blade pivot caused by this small force is,

$$dM = rdF = r\mu \frac{\theta r}{Tt}(wdr). \tag{4}$$

Integrating over the length of the blade gives the total moment,

$$M = \int_{r=R}^{r=R+L} dM = \int_{r=R}^{r=R+L} \mu \frac{r^2\theta}{Tt} w dr, \tag{5}$$

$$\boxed{\therefore M = \frac{1}{3}\mu \frac{\theta}{T} R^3 \left[\left(1+\frac{L}{R}\right)^3 - 1\right]\frac{w}{t}}. \tag{6}$$

The numerical solution for the given parameters is,

$$M = \frac{1}{3}\left(2*10^{-5}\ \tfrac{\text{lb}_\text{f}\cdot\text{s}}{\text{ft}^2}\right)\left(\frac{110^\circ \cdot \frac{\pi}{180^\circ}}{2\text{ s}}\right)\left(\frac{6\text{ in}}{12\ \frac{\text{in}}{\text{ft}}}\right)^3\left[\left(1+\frac{11\text{ in}}{6\text{ in}}\right)^3 - 1^3\right]\left(\frac{\frac{1}{4}\text{ in}}{\frac{1}{16}\text{ in}}\right) \tag{7}$$

$$\approx 7*10^{-5}\text{ ft-lb}_\text{f}$$

This torque is quite small. An important factor contributing to this small value is that the actual wiper geometry is not parallel to window as given in this simple example, but instead has an inclined surface. This inclined surface will result in a pressure force acting on the blade surface in addition to a shear force. This pressure force will be much larger than the shear force and, hence, a larger torque will result. Wiper blade dynamics are actually quite complex and have been the study of a number of research projects.

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