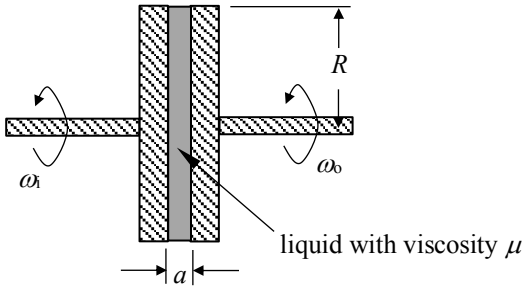
A viscous clutch is made from a pair of closely spaced parallel, circular disks enclosing a thin layer of viscous liquid.

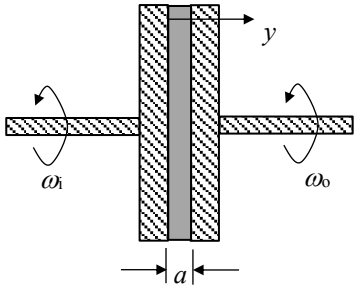


Develop an expression for the torque, T, transmitted by the disk pair, in terms of the liquid dynamic viscosity, μ, the disk radius, R, the disk spacing, a, and the angular speeds of the input disk, ω_i, and output disk, ω_o.

SOLUTION:

Since the disks are closely spaced, assume that the velocity profile in the liquid is linear, with the velocity gradient at a radius r being,

$$\frac{du}{dy} \overset{\text{since linear}}{=} \frac{\Delta u}{\Delta y} = \frac{\omega_o r - \omega_i r}{a} = \frac{(\omega_o - \omega_i) r}{a} . \tag{1}$$



The torque acting on the output disk due to the shear force exerted by the liquid is,

$$T = \int_{r=0}^{r=R} r\, dF = \int_{r=0}^{r=R} r \underbrace{\tau\big|_{y=a}\, dA}_{=dF} = \int_{r=0}^{r=R} r \underbrace{\left[\mu \frac{(\omega_o - \omega_i) r}{a} \right]}_{=\tau} \underbrace{(2\pi r dr)}_{=dA} = 2\pi\mu \frac{(\omega_o - \omega_i)}{a} \int_{r=0}^{r=R} r^3 dr , \tag{2}$$

$$\boxed{T = \frac{\pi}{2} \mu \frac{(\omega_o - \omega_i)}{a} R^4} . \tag{3}$$