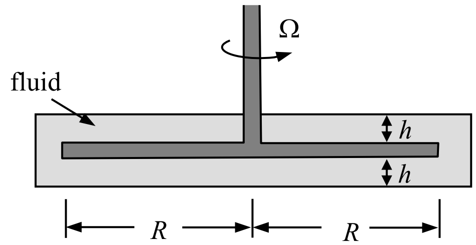
A rotating disk viscometer has a radius, R = 50 mm, and a clearance, h = 1 mm, as shown in the figure.



a. If the torque required to rotate the disk at Ω = 900 rpm is 0.537 N·m, determine the dynamic viscosity of the fluid. You may neglect the viscous forces acting on the rim of the disk and on the vertical shaft.
b. If the uncertainty in each parameter is ±1%, determine the uncertainty in the viscosity.

SOLUTION:

The torque acting on the rotating section is due to the fluid shear stresses acting on the rotating surface. Consider only the stresses acting on the upper and lower surfaces since the edge stresses act over a negligibly small area. Determine the force acting on a small area of the rotating disk.

$$dF = 2\tau_{y\theta}\big|_{y=0} dA = 2\tau_{y\theta}\big|_{y=0}\left(2\pi r dr\right) \tag{1}$$

(Note that the force acts on both the upper and lower surfaces.)

The shear stress acting on the upper and lower surfaces, assuming a Newtonian fluid and a linear velocity profile in the gap (a reasonable assumption if the gap width is narrow), is:

$$\tau_{y\theta} = \mu\frac{du}{dy} = \mu\frac{0-r\Omega}{0-h} = \mu\frac{r\Omega}{h} \quad \text{(Note } u(y=0)=0 \text{ and } u(y=h)=r\Omega.\text{)} \tag{2}$$

Substitute into Eqn. (1).

$$dF = 2\mu\frac{r\Omega}{h}\left(2\pi r dr\right) \tag{3}$$

The torque due to this force contribution is:

$$dT = rdF = r2\mu\frac{r\Omega}{h}\left(2\pi r dr\right) \tag{4}$$

The total torque is found by integrating over the disk radius.

$$T = \int_{r=0}^{r=R} dT = 4\pi\mu\frac{\Omega}{h}\int_{r=0}^{r=R} r^3 dr$$

$$\therefore T = \pi\mu\frac{\Omega R^4}{h} \tag{5}$$

Re-arrange to solve for the viscosity.

$$\boxed{\mu = \frac{Th}{\pi\Omega R^4}} \tag{6}$$

Using the given values:

T = 0.537 N·m
h = $1.0*10^{-3}$ m
Ω = 900 rpm = 94.2 rad/s
R = $50.0*10^{-3}$ m
$\Rightarrow$ $\boxed{\mu = 0.29 \text{ kg/(m·s)} \quad \text{or } 0.29 \text{ Pa·s or } 290 \text{ cP}}$

Perform an uncertainty analysis on Eqn. (6).

$$u_\mu = \left[u_{\mu,T}^2 + u_{\mu,h}^2 + u_{\mu,\Omega}^2 + u_{\mu,R}^2\right]^{1/2} \tag{7}$$

where

$$u_{\mu,T} = \frac{1}{\mu}\frac{\partial\mu}{\partial T}\delta T = \left(\frac{Th}{\pi\Omega R^4}\right)^{-1}\frac{h}{\pi\Omega R^4}\delta T = \frac{\delta T}{T} = u_T$$

$$u_{\mu,h} = \frac{1}{\mu}\frac{\partial\mu}{\partial h}\delta h = \left(\frac{Th}{\pi\Omega R^4}\right)^{-1}\frac{T}{\pi\Omega R^4}\delta h = \frac{\delta h}{h} = u_h$$

$$u_{\mu,\Omega} = \frac{1}{\mu}\frac{\partial\mu}{\partial \Omega}\delta\Omega = \left(\frac{Th}{\pi\Omega R^4}\right)^{-1}\frac{-Th}{\pi\Omega^2 R^4}\delta\Omega = -\frac{\delta\Omega}{\Omega} = -u_\Omega$$

$$u_{\mu,R} = \frac{1}{\mu}\frac{\partial\mu}{\partial R}\delta R = \left(\frac{Th}{\pi\Omega R^4}\right)^{-1}\frac{-4Th}{\pi\Omega R^5}\delta R = -4\frac{\delta R}{R} = -4u_R$$

Substitute into Eqn. (7).

$$\boxed{u_\mu = \left[u_T^2 + u_h^2 + u_\Omega^2 + 16u_R^2\right]^{1/2}} \tag{8}$$

Note that the relative uncertainty in R contributes the most to the uncertainty in the viscosity.

From the problem statement, the relative uncertainty in each of the measurements is ±1% so that:

$$\boxed{u_\mu = \pm 4.4\%} \tag{9}$$

$$\Rightarrow \boxed{\mu = 0.29 \pm 0.01\ \text{Pa}\cdot\text{s}}$$