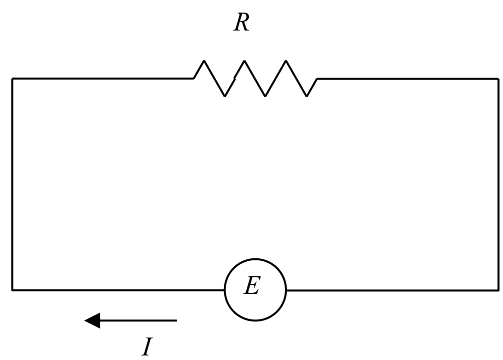
A resistor has a nominal stated value of 10±0.1 Ω. A voltage difference occurs across the resister and the power dissipation is to be calculated in two different ways:

a. from $P = E^2/R$
b. from $P = EI$

In (a) only a voltage measurement will be made while both current and voltage will be measured in (b). Calculate the uncertainty in the power for each case when the measured values of E and I are:

E = 100±1 V (for both cases)
I = 10±0.1 A



SOLUTION:

Perform an uncertainty analysis using the first formula for power.

$$P = E^2 / R \quad (1)$$

The relative uncertainty in P is:

$$u_P = \left[u_{P,E}^2 + u_{P,R}^2 \right]^{1/2} \quad (2)$$

where

$$u_{P,E} = \frac{1}{P}\frac{\partial P}{\partial E}\delta E = \frac{R}{E^2}\left(\frac{2E}{R}\right)\delta E = 2\frac{\delta E}{E} = 2u_E \quad (3)$$

$$u_{P,R} = \frac{1}{P}\frac{\partial P}{\partial R}\delta R = \frac{R}{E^2}\left(\frac{-E^2}{R^2}\right)\delta R = -\frac{\delta R}{R} = -u_R \quad (4)$$

Substitute into Eqn. (2).

$$u_P = \left[4u_E^2 + u_R^2 \right]^{1/2} \quad (5)$$

The relative uncertainties in the voltage and resistance are:

$$u_E = \frac{\delta E}{E} = \frac{1\text{ V}}{100\text{ V}} = 1\% \quad (6)$$

$$u_R = \frac{\delta R}{R} = \frac{0.1\ \Omega}{10\ \Omega} = 1\% \quad (7)$$

$$\Rightarrow \boxed{u_P = 2.24\%}$$

Now perform an uncertainty analysis using the second relation for power.

$$P = EI \tag{8}$$

The relative uncertainty in P is:

$$u_P = \left[u_{P,E}^2 + u_{P,I}^2\right]^{1/2}$$

where

$$u_{P,E} = \frac{1}{P}\frac{\partial P}{\partial E}\delta E = \frac{1}{EI}(I)\delta E = \frac{\delta E}{E} = u_E \tag{9}$$

$$u_{P,I} = \frac{1}{P}\frac{\partial P}{\partial R}\delta R = \frac{1}{EI}(E)\delta I = \frac{\delta I}{I} = u_I \tag{10}$$

Substitute into Eqn. (2).

$$u_P = \left[u_E^2 + u_I^2\right]^{1/2} \tag{11}$$

The relative uncertainties in the voltage and resistance are:

$$u_E = \frac{\delta E}{E} = \frac{1\text{ V}}{100\text{ V}} = 1\% \tag{12}$$

$$u_I = \frac{\delta I}{I} = \frac{0.1\text{ A}}{10\text{ A}} = 1\% \tag{13}$$

$$\Rightarrow \boxed{u_P = 1.41\%}$$

We observe that using the second relation ($P = EI$) gives a smaller uncertainty for the given values.