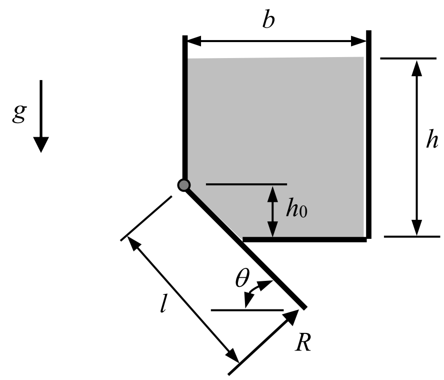
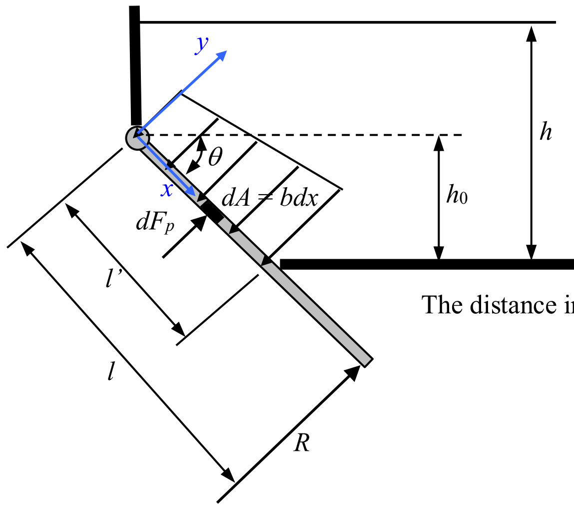
A hinged gate is used to hold back a liquid of density ρ in a square cross-sectioned tank as shown in the figure below. The weight of the gate is small compared to the pressure force acting on the gate by the water.

a. Calculate the minimum force R required to hold the gate in place, assuming $h > h_0$. You may assume h, h_0, b, θ, l, ρ, and g are known.



b. Now consider the case when $h < h_0$. What is the minimum force R required for this case?

SOLUTION:



The distance into the page is b.

Sum moments about the hinge at set them equal to zero,

$$\sum M_{\text{hinge}} = 0 = lR - \int_{x=0}^{x=l'} xp\underbrace{bdx}_{=dA}, \tag{1}$$

where,

$$l' = \frac{h_0}{\sin\theta}, \tag{2}$$

$$p = \rho g\left[\left(h - h_0\right) + x\sin\theta\right]. \tag{3}$$

Substitute and solve for R,

$$lR = \int_{x=0}^{x=l'} x\rho g\left[\left(h-h_0\right) + x\sin\theta\right]bdx, \tag{4}$$

$$R = \frac{\rho g b}{l}\int_{x=0}^{x=l'}\left[x\left(h-h_0\right) + x^2\sin\theta\right]dx, \tag{5}$$

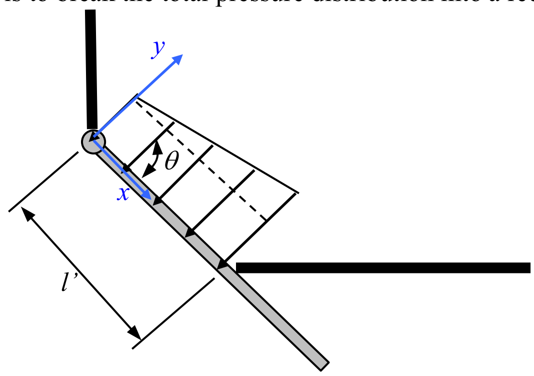
$$R = \frac{\rho g b}{l}\left[\frac{1}{2}l'^2\left(h-h_0\right) + \frac{1}{3}l'^3\sin\theta\right], \tag{6}$$

$$R = \frac{\rho g b l'^2}{l}\left[\frac{1}{2}\left(h-h_0\right) + \frac{1}{3}l'\sin\theta\right], \tag{7}$$

$$R = \frac{\rho g b h_0^2}{l\sin^2\theta}\left[\frac{1}{2}\left(h-h_0\right) + \frac{1}{3}h_0\right], \tag{8}$$

$$\boxed{R = \frac{\rho g b h_0^2}{2l\sin^2\theta}\left(h - \frac{1}{3}h_0\right)}. \tag{9}$$

Another approach to solving this problem is to break the total pressure distribution into a rectangular portion and a triangular portion.



$$F_{\text{rect}} = \rho g\left(h - h_0\right)l'b\,, \tag{10}$$

$$F_{\text{tri}} = \frac{1}{2}\rho g h_0 l b\,, \tag{11}$$

$$x_{\text{rect}} = \frac{1}{2}l'\,, \tag{12}$$

$$x_{\text{tri}} = \frac{2}{3}l'\,. \tag{13}$$

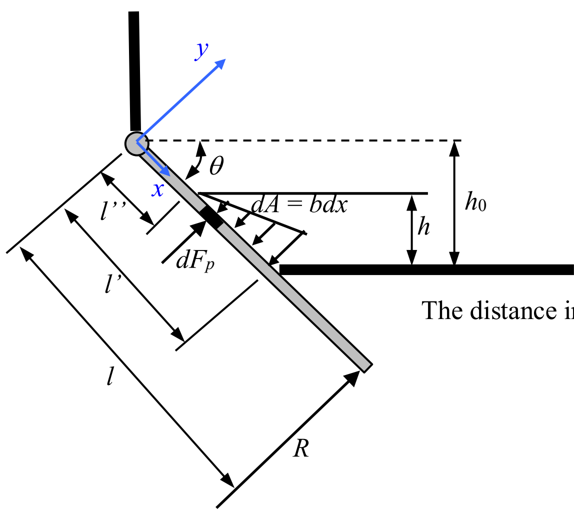
Balance moments about the hinge,

$$lR = x_{\text{rect}}F_{\text{rect}} + x_{\text{tri}}F_{\text{tri}} = \frac{1}{2}l'\rho g\left(h - h_0\right)l'b + \frac{2}{3}l'\rho g h_0 l'b\,, \tag{14}$$

$$R = \frac{\rho g b l'^2}{l}\left[\frac{1}{2}\left(h - h_0\right) + \frac{2}{3}h_0\right],$$

$$R = \frac{\rho g b h_0^2}{2l\sin^2\theta}\left(h - \frac{1}{3}h_0\right), \text{ which is the same answer found previously.} \tag{15}$$

Now solve for the case when $h < h_0$,



The distance into the page is b.

Sum moments about the hinge at set them equal to zero,

$$\sum M_{\text{hinge}} = 0 = lR - \int_{x=l''}^{x=l'} xp\underbrace{bdx}_{=dA}\,, \tag{16}$$

where,

$$l' = \frac{h_0}{\sin\theta}\,, \tag{17}$$

$$l'' = \frac{h_0 - h}{\sin\theta}\,, \tag{18}$$

$$p = \rho g x\sin\theta\,. \tag{19}$$

Substitute and solve for *R*,

$$lR=\int_{x=l''}^{x=l'} x\rho g x\sin\theta b\,dx\ , \tag{20}$$

$$R=\frac{\rho g b\sin\theta}{l}\int_{x=l''}^{x=l'} x^2\,dx\ , \tag{21}$$

$$R=\frac{\rho g b\sin\theta}{3l}\left(l'^3-l''^3\right), \tag{22}$$

$$\boxed{R=\frac{\rho g b}{3l\sin^2\theta}\left[h_0^3-\left(h_0-h\right)^3\right]}. \tag{23}$$