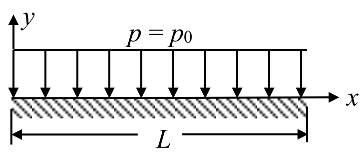
For each of the following pressure profiles,

a. Determine the magnitude of the total pressure force acting on the horizontal plate.

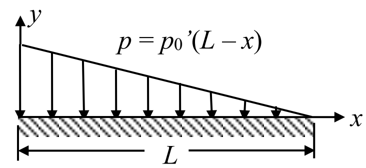
b. Determine the location of the center of pressure.

Assume the plate has unit depth in the z direction. Show all of your work.

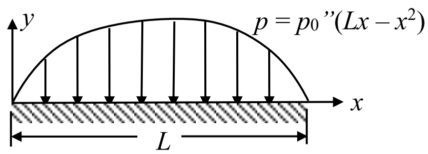
1.



2.



3.



SOLUTION:

The total pressure force may be found via integration of the differential pressure force.

$$\left|F_p\right| = \int_{x=0}^{x=L} p\,\underbrace{dx(1)}_{=dA} \qquad \text{(Note: The differential area is } dA = dx(1) \text{ since the plate has unit depth.)} \tag{1}$$

1. $$\left|F_p\right| = \int_{x=0}^{x=L} p_0\,dx \;\Rightarrow\; \boxed{\left|F_p\right| = p_0 L} \tag{2}$$

2. $$\left|F_p\right| = \int_{x=0}^{x=L} p_0'(L-x)\,dx = p_0'\left(L^2 - \tfrac{1}{2}L^2\right) \;\Rightarrow\; \boxed{\left|F_p\right| = \tfrac{1}{2}p_0'L^2} \tag{3}$$

3. $$\left|F_p\right| = \int_{x=0}^{x=L} p_0''\left(Lx - x^2\right)dx = p_0''\left(\tfrac{1}{2}L^3 - \tfrac{1}{3}L^3\right) \;\Rightarrow\; \boxed{\left|F_p\right| = \tfrac{1}{6}p_0''L^3} \tag{4}$$

The center of pressure may be found by equating the moment resulting from the pressure distribution to the moment caused by the total pressure force acting at the center of pressure.

$$\int_{x=0}^{x=L} xp\,\underbrace{dx(1)}_{=dA} = x_{CP}F_p \;\Rightarrow\; x_{CP} = \frac{1}{F_p}\int_{x=0}^{x=L} xp\,\underbrace{dx(1)}_{=dA} \tag{5}$$

1. $$x_{CP} = \frac{1}{p_0 L}\int_{x=0}^{x=L} xp_0\,dx = \frac{1}{p_0 L}\left(\tfrac{1}{2}p_0 L^2\right) \;\Rightarrow\; \boxed{x_{CP} = \tfrac{1}{2}L} \tag{6}$$

2. $$x_{CP} = \frac{1}{\tfrac{1}{2}p_0'L^2}\int_{x=0}^{x=L} xp_0'(L-x)\,dx = \frac{1}{\tfrac{1}{2}p_0'L^2}\left[p_0'\left(\tfrac{1}{2}L^3 - \tfrac{1}{3}L^3\right)\right] \;\Rightarrow\; \boxed{x_{CP} = \tfrac{1}{3}L} \tag{7}$$

3. $$x_{CP} = \frac{1}{\tfrac{1}{6}p_0''L^3}\int_{x=0}^{x=L} xp_0''\left(Lx - x^2\right)dx = \frac{1}{\tfrac{1}{6}p_0''L^3}\left[p_0''\left(\tfrac{1}{3}L^4 - \tfrac{1}{4}L^4\right)\right] \;\Rightarrow\; \boxed{x_{CP} = \tfrac{1}{2}L} \tag{8}$$