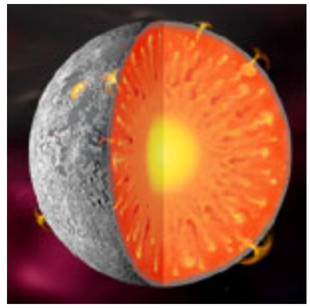
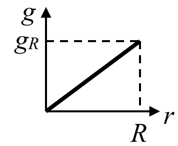
It is often conjectured that the Earth was, at one time, comprised of molten material. If the acceleration due to gravity within this fluid sphere (with a radius of 6440 km) varied linearly with distance, *r*, from the Earth's center, the acceleration due to gravity at $r = 6440$ km was 9.81 m/s^2, and the density of the fluid was uniformly 5600 kg/m^3, determine the gage pressure at the center of this fluid Earth.



SOLUTION:

Since the acceleration due to gravity, g, varies linearly with r:



$$g = cr \tag{1}$$

where c is a constant. Since $g(r = R = 6440\text{ km}) = g_R = 9.81\text{ m/s}^2$:

$$c = \frac{g_R}{R} = \frac{9.81\ \text{m/s}^2}{6440*10^3\ \text{m}} = 1.523*10^{-6}\ \text{s}^{-2} \tag{2}$$

From the hydrostatic pressure distribution:

$$\frac{dp}{dr} = -\rho g \tag{3}$$

Substitute Eq. (1) and solve the resulting differential equation.

$$\frac{dp}{dr} = -\rho c r \Rightarrow \int_{p=p_0}^{p=0} dp = -\rho c \int_{r=0}^{r=R} r dr \tag{4}$$

$$\boxed{\therefore p_0 = \tfrac{1}{2}\rho c R^2} \tag{5}$$

Using the given data:

ρ = 5600 kg/m^3
c = $1.523*10^{-6}$ s^{-2}
R = $6.440*10^6$ m
$\boxed{p_0 = 1.769*10^{11}\text{ Pa} = 1.769*10^{6}\text{ atm}}$