A rectangular block of concrete (SG=2.5) is used as a retaining wall or dam for a reservoir of water:

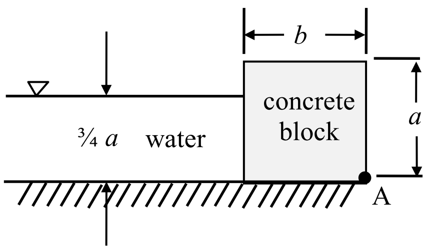


Figure (a)

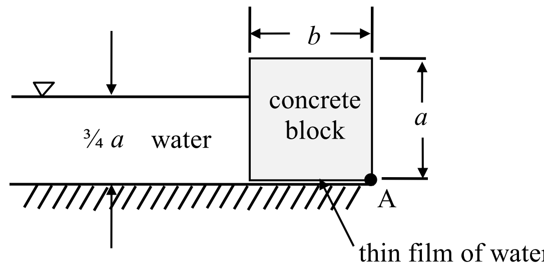


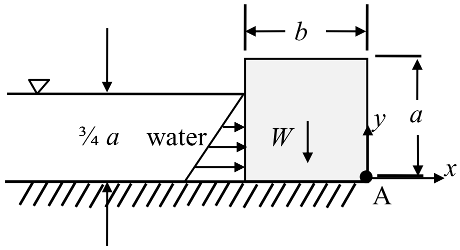
Figure (b)

The block has a height, *a*, a breadth, *b*, and unit depth into the page. The depth of the water is $3a/4$.

a. Determine the critical ratio, b/a, below which the block will be overturned by the water (figure a). Assume the block does not slide on the base but can rotate about the point A. For figure (a), there is no fluid underneath the block.
b. What is the critical ratio, b/a, if there is seepage and a thin film of water forms under the block (figure b)? Assume that a seal at point A prevents water from flowing out from underneath the block.

SOLUTION:

Draw a free body diagram of the block. Note that when the block is on the verge of tipping over, the vertical force the ground exerts on the block is zero.



Sum moments about point A.

$$\sum M_A = 0 = \left(\tfrac{1}{2}b\right)W - \int_{y=0}^{y=\frac{3}{4}a} y\, \underbrace{p\underbrace{\left(dy\cdot 1\right)}_{=dA}}_{=dF} \tag{1}$$

where

$$W = \rho_{\text{block}}\left(b\cdot a\cdot 1\right)g \tag{2}$$

$$p = \rho_{H20}\, g\left(\tfrac{3}{4}a - y\right) \quad \text{(note that this is a gage pressure)} \tag{3}$$

Substitute and simplify.

$$\left(\tfrac{1}{2}b\right)\left(\rho_{\text{block}}bag\right) - \int_{y=0}^{y=\frac{3}{4}a} y\rho_{H20}\, g\left(\tfrac{3}{4}a - y\right)dy = 0 \tag{4}$$

$$\tfrac{1}{2}\rho_{\text{block}}b^2 a - \rho_{H20}\int_{y=0}^{y=\frac{3}{4}a}\left(\tfrac{3}{4}ay - y^2\right)dy = 0 \tag{5}$$

$$\tfrac{1}{2}\rho_{\text{block}}b^2 a - \rho_{H20}\left(\tfrac{3}{8}ay^2 - \tfrac{1}{3}y^3\right)\Big|_{y=0}^{y=\frac{3}{4}a} = 0 \tag{6}$$

$$\tfrac{1}{2}\rho_{\text{block}}b^2 a - \rho_{H20}\left(\tfrac{3}{8}\tfrac{9}{16}a^3 - \tfrac{1}{3}\tfrac{27}{64}a^3\right) = 0 \tag{7}$$

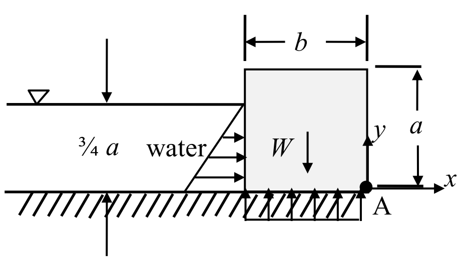
$$\tfrac{1}{2}SG_{\text{block}}b^2 a - \tfrac{9}{128}a^3 = 0 \tag{8}$$

$$\left(\frac{b}{a}\right)^2 = \frac{9}{64}\frac{1}{SG_{\text{block}}} \tag{9}$$

$$\therefore \frac{b}{a} = \frac{3}{8}\frac{1}{\sqrt{SG_{\text{block}}}} \quad \text{when the block is just about to tip over} \tag{10}$$

Thus, the block will tip over for $SG_{\text{block}} = 2.5$ if $\boxed{b/a < 0.237}$.

Now draw the free body diagram for a block with a thin liquid layer underneath it.



Sum moments about point A.

$$\sum M_A = 0 = \left(\tfrac{1}{2}b\right)W - \int_{y=0}^{y=\frac{3}{4}a} yp\left(dy\cdot 1\right) - \left(\tfrac{1}{2}b\right)\underbrace{\underbrace{\rho_{H20}g\tfrac{3}{4}a}_{=p}\underbrace{\left(b\cdot 1\right)}_{=A}}_{=F} \tag{11}$$

where the weight and pressure on the side are given in Eqs. (2) and (3). The last term in the previous equation is the (gage) pressure that the liquid layer on the bottom exerts on the block.

Substitute and simplify.

$$\tfrac{1}{2}SG_{\text{block}}b^2a - \tfrac{9}{128}a^3 - \tfrac{3}{8}ab^2 = 0 \tag{12}$$

$$SG_{\text{block}}b^2a - \tfrac{9}{64}a^3 - \tfrac{3}{4}ab^2 = 0 \tag{13}$$

$$SG_{\text{block}}\left(\frac{b}{a}\right)^2 - \frac{3}{4}\left(\frac{b}{a}\right)^2 = \frac{9}{64} \tag{14}$$

$$\therefore \frac{b}{a} = \frac{3}{8}\left(SG_{\text{block}} - \frac{3}{4}\right)^{-\frac{1}{2}} \text{ when the block is just about to tip over} \tag{15}$$

Thus, the block will tip over for $SG_{\text{block}} = 2.5$ if $\boxed{b/a < 0.283}$.