A tank is divided by a wall into two independent chambers. The left chamber is filled to a depth of $H_{\mathrm{L}}=6 \mathrm{~m}$ with water $\left(\rho_{\mathrm{H} 20}=1000 \mathrm{~kg} / \mathrm{m}^{3}\right)$ and the right side if filled to a depth of $H_{\mathrm{R}}=5 \mathrm{~m}$ with an unknown fluid. A wooden cube ( $S G_{\text {wood }}=0.6$ ) with a length of $L=0.20 \mathrm{~m}$ on each side floats half submerged in the unknown fluid. Air ( $\rho_{\text {air }}=1.2 \mathrm{~kg} / \mathrm{m}^{3}$ ) fills the remainder of the container above each fluid. The right container has a pipe that is vented to the atmosphere while the left container is sealed from the atmosphere. A manometer using mercury as the gage fluid ( $S G_{\mathrm{Hg}}=13.6$ ) connects the two chambers and indicates that $h=0.150 \mathrm{~m}$.
a. Determine the density of the unknown fluid.
b. Determine the magnitude of the force (per unit depth into the page) acting on the dividing wall due to the unknown fluid.
c. Determine the magnitude of the force (per unit depth into the page) acting on the dividing wall due to the water.


## SOLUTION:

Balance forces on the wooden cube.

$$
\begin{align*}
& \sum F_{y}=0=\rho_{\text {fluid }} g\left(\frac{1}{2} L\right) L^{2}-\rho_{\text {wood }} g L^{3}  \tag{1}\\
& \therefore \rho_{\text {fluid }}=2 \rho_{\text {wood }}=2 S G_{\text {wood }} \rho_{\mathrm{H}_{2} \mathrm{O}} \tag{2}
\end{align*}
$$

Using the given data:

$$
\begin{aligned}
S G_{\text {wood }} & =0.6 \\
\rho_{\mathrm{H} 2 \mathrm{O}} & =1000 \mathrm{~kg} / \mathrm{m}^{3} \\
\Rightarrow \rho_{\text {fluid }} & =1200 \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

Now determine the force acting on the wall due to the unknown fluid.

$$
\begin{align*}
& F_{p, R}=\int_{y=0}^{y=H_{R}}\left(p_{\mathrm{atm}}+\rho_{\text {fluid }} g y\right) d y(1)  \tag{3}\\
& =p(\mathrm{abs})  \tag{4}\\
& =d A
\end{align*}
$$

Using the given data:

$$
\begin{array}{ll}
p_{\text {atm }} & =101 \mathrm{kPa}(\mathrm{abs}) \\
H_{R} & =5 \mathrm{~m} \\
\rho_{\text {fluid }} & =1200 \mathrm{~kg} / \mathrm{m}^{3} \\
g & =9.81 \mathrm{~m} / \mathrm{s}^{2} \\
\Rightarrow F_{p, R} & =506 \mathrm{kN} / \mathrm{m}
\end{array}
$$

Now find the pressure force due to the water.

$$
\begin{align*}
& F_{p, L}=\int_{y=0}^{y=H_{L}}\left(p_{L}+\rho_{\mathrm{H}_{2} \mathrm{O}} g y\right) d y(1)  \tag{5}\\
& =p(\mathrm{abs})  \tag{6}\\
& \therefore F_{p, L}=p_{L} H_{L}+\frac{1}{2} \rho_{\mathrm{H}_{2} \mathrm{O}} g H_{L}^{2}
\end{align*}
$$

where $p_{L}$ is the (absolute) pressure acting on the free surface of the water. This pressure may be found using the manometer.

$$
\begin{equation*}
p_{L}=p_{\mathrm{atm}}+\rho_{\mathrm{Hg}} g h=p_{\mathrm{atm}}+S G_{\mathrm{Hg}} \rho_{\mathrm{H}_{2} \mathrm{O}} g h \tag{7}
\end{equation*}
$$

Substitute Eqn. (7) into Eqn. (6).

$$
\begin{equation*}
\therefore F_{p, L}=\left(p_{\mathrm{atm}}+S G_{\mathrm{Hg}^{2}} \rho_{\mathrm{H}_{2} \mathrm{O}} g h\right) H_{L}+\frac{1}{2} \rho_{\mathrm{H}_{2} \mathrm{O}} g H_{L}^{2} \tag{8}
\end{equation*}
$$

Using the given data:

$$
\begin{array}{ll}
p_{\mathrm{atm}} & =101 \mathrm{kPa}(\mathrm{abs}) \\
S G_{\mathrm{Hg}} & =13.6 \\
\rho_{\mathrm{H} 2 \mathrm{O}} & =1000 \mathrm{~kg} / \mathrm{m}^{3} \\
g & =9.81 \mathrm{~m} / \mathrm{s}^{2} \\
h & =0.150 \mathrm{~m} \\
H_{L} & =6 \mathrm{~m} \\
\Rightarrow F_{p, L} & =903 \mathrm{kN} / \mathrm{m}
\end{array}
$$

