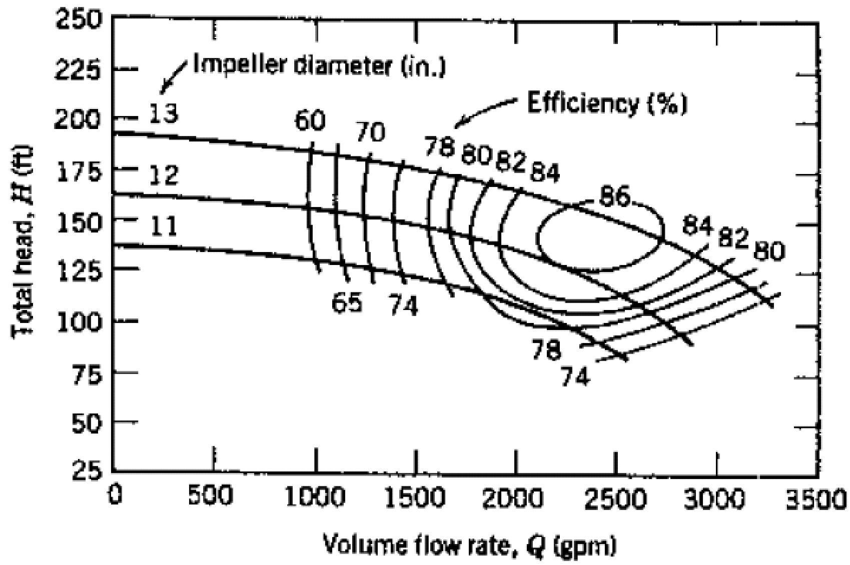
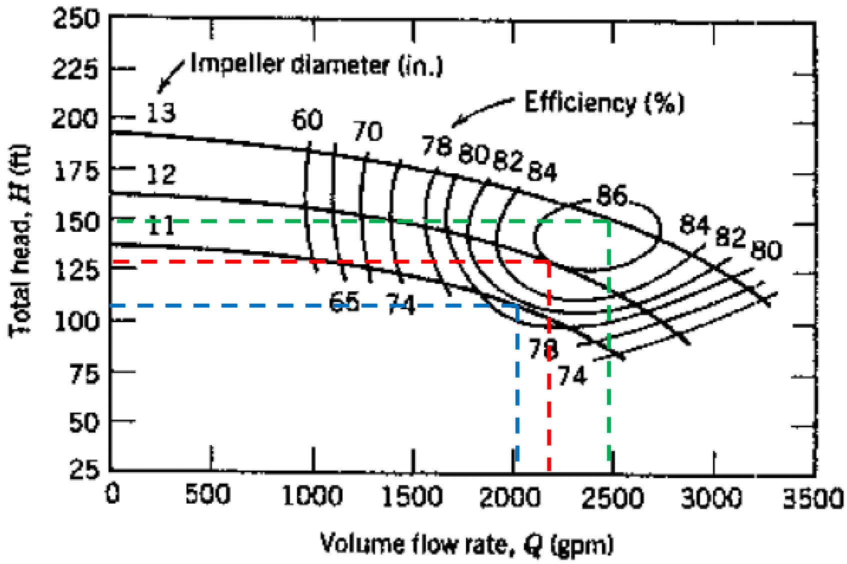
Typical performance curves for a centrifugal pump, tested with three different impeller diameters in a single casing, are shown in the figure below. Specify the flow rate and head produced by the pump at its best efficiency point with a 12 in. diameter impeller. Scale these data to predict the performance of this pump when tested with 11 in. and 13 in. impellers. Comment on the accuracy of the scaling procedure.



SOLUTION:



From the pump performance diagram,

$Q_{12\text{ in.},BEP} = 2200$ gpm
$H_{12\text{ in.},BEP} = 130$ ft
$\eta_{12\text{ in.},\,BEP} = 86\%$

Using the geometric scaling rule,

$$Q_2 = Q_1\left(\frac{D_2}{D_1}\right)^3 \quad (1)$$

For $D_2 = 11$ in. and $D_1 = 12$ in., $Q_{12\text{ in.}} = 2200$ gpm, $\underline{Q_{11\text{ in.}} = 1690\text{ gpm}}$.
For $D_2 = 13$ in. and $D_1 = 12$ in., $Q_{12\text{ in.}} = 2200$ gpm, $\underline{Q_{13\text{ in.}} = 2800\text{ gpm}}$.

Using the alternate scaling rule that takes into account imperfect geometric similarity,

$$Q_2 = Q_1\left(\frac{D_2}{D_1}\right)^2, \quad (2)$$

$\underline{Q_{11\text{ in.}} = 1850\text{ gpm}}$
$\underline{Q_{13\text{ in.}} = 2580\text{ gpm}}$

From the pump performance diagram, $Q_{11\text{ in.}} \approx 2000$ gpm.
From the pump performance diagram, $Q_{13\text{ in.}} \approx 2500$ gpm.

The alternate scaling predicts the volumetric flow rate much better than the geometric scaling rule. Indeed, the alternate scaling rule predictions are off by approximately 8% (11 in.) and 3% (13 in.) while the geometric scaling rule is off by 16% (11 in.) and 12% (13 in.).

From the pump scaling rules,

$$H_2 = H_1\left(\frac{D_2}{D_1}\right)^2 \tag{3}$$

For D_2 = 11 in. and D_1 = 12 in., H_1 = 130 ft, $\underline{H_{11\text{ in.}} = 109\text{ ft}}$.
For D_2 = 13 in. and D_1 = 12 in., H_1 = 130 ft, $\underline{H_{13\text{ in.}} = 153\text{ ft}}$.

From the pump performance diagram, $H_{11\text{ in.}} \approx 110$ ft.
From the pump performance diagram, $H_{13\text{ in.}} \approx 150$ ft.

The geometric scaling rule predictions are excellent with errors of only 1% (11 in.) and 2% (13 in.).

The best efficiencies may be found via the Moody empirical formula,

$$\frac{1-\eta_2}{1-\eta_1} = \left(\frac{D_1}{D_2}\right)^{1/5}, \tag{4}$$

For D_2 = 11 in. and D_1 = 12 in., $\eta_{1,BEP}$ = 86%, $\underline{\eta_{11\text{ in.},BEP} = 86\%}$.
For D_2 = 13 in. and D_1 = 12 in., $\eta_{1,BEP}$ = 86%, $\underline{\eta_{13\text{ in.},BEP} = 86\%}$.

From the pump performance diagram, $\eta_{11\text{ in.},BEP} \approx 82\%$ and $\eta_{11\text{ in.},BEP} \approx 87\%$. The Moody formula does a good job of predicting the best efficiency values for both impeller (5% and 1% relative errors for the 11 in. and 13 in. impellers, respectively).