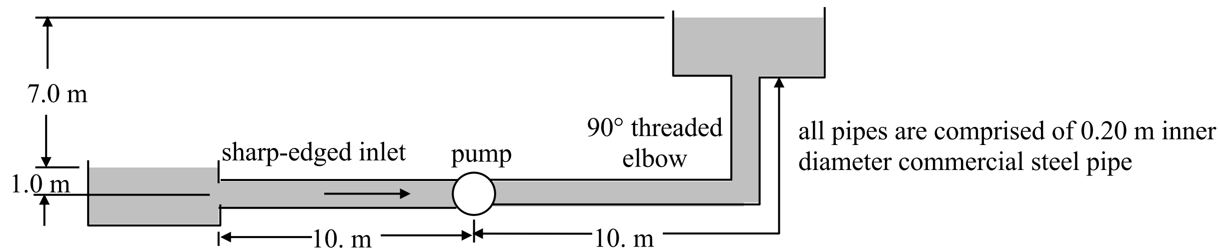
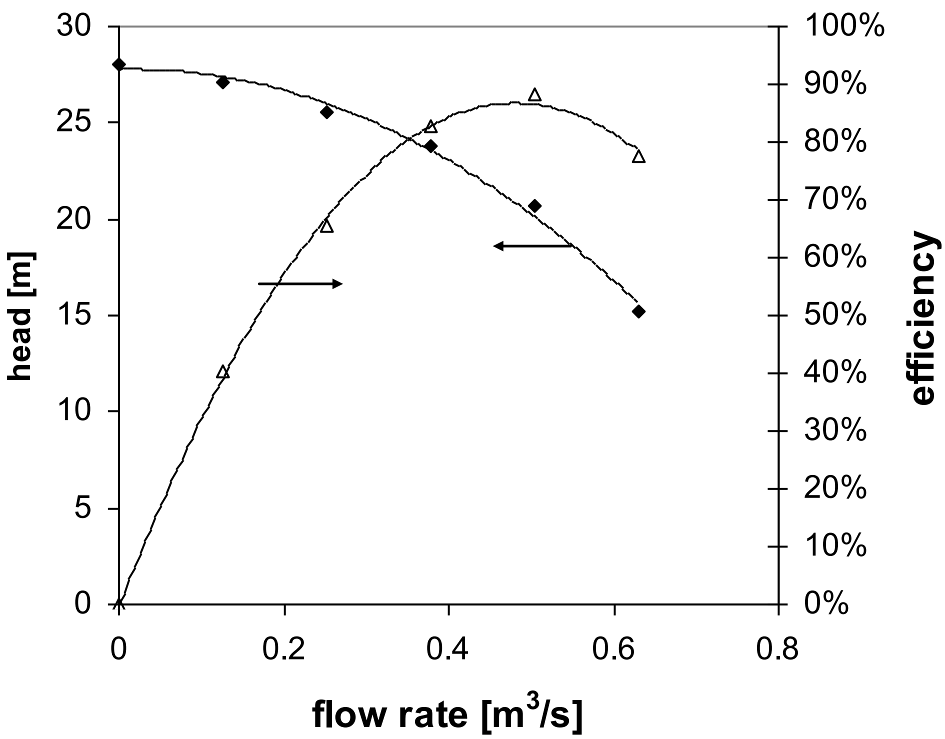
Consider the pipe system shown in the figure below. The fluid to be pumped is water with a density of 1000 kg/m^3, a kinematic viscosity of 1.0*10^{-6} m^2/s, and a vapor pressure of 2.3 kPa (abs).



The pump used in this system has the performance plot shown below.



Curve fits to the pump performance data are given below:

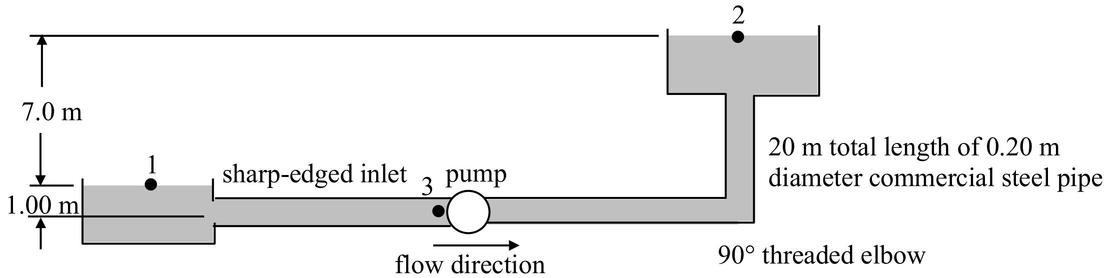
H [m] = (-32.5 s^2/m^5) Q^2 + (1.23 s/m^2) Q + (27.8 m)

η_P = (-3.74 s^2/m^6) Q^2 + (3.60 s/m^3) Q

a. Determine the operating volumetric flow rate of the system.
b. Is the given pump a good choice for this system? Explain your answer.
c. Determine the NPSHA to the pump for the flow rate determined in part (a).
d. Give one specific modification to the pipe system that could be employed to decrease the likelihood that cavitation will occur in the pump.

SOLUTION:

Apply the Extended Bernoulli Equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12} \qquad (1)$$

where

$p_1 = p_2 = p_{atm}$ (free surfaces exposed to the atmosphere)

$\bar{V}_1 \approx \bar{V}_2 \approx 0$ (large tanks)

$z_2 - z_1 = 7.00$ m (given)

$$H_{L,12}=f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}+K_{\substack{\text{sharp-edged}\\ \text{entrance}}}\frac{\bar{V}^2}{2g}+K_{\substack{90^\circ\ \text{threaded}\\ \text{elbow}}}\frac{\bar{V}^2}{2g}+K_{\text{exit}}\frac{\bar{V}^2}{2g}$$

$$=\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{sharp-edged}\\ \text{entrance}}}+K_{\substack{90^\circ\ \text{threaded}\\ \text{elbow}}}+K_{\text{exit}}\right]\frac{\bar{V}^2}{2g}$$

$$=\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{sharp-edged}\\ \text{entrance}}}+K_{\substack{90^\circ\ \text{threaded}\\ \text{elbow}}}+K_{\text{exit}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}$$

Re-arrange Eq. (1) to solve for $H_{S,12}$.

$$H_{S,12}=\left(z_2-z_1\right)+\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{sharp-edged}\\ \text{entrance}}}+K_{\substack{90^\circ\ \text{threaded}\\ \text{elbow}}}+K_{\text{exit}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2} \qquad (2)$$

Assume that the flow is in the fully rough zone where the friction factor is independent of the Reynolds number. The pipe roughness, ε, is $\varepsilon = 0.0450$ mm so that the relative roughness is,

$$\frac{\epsilon}{D}=\frac{4.50*10^{-5}\ \text{m}}{2.00*10^{-2}\ \text{m}}=2.25*10^{-4}. \qquad (3)$$

From the Moody chart in the fully rough zone,

$$f = 1.41*10^{-2} \qquad (4)$$

Substitute in the given data into Eqn. (2):

z_2-z_1 = 7.00 m
L = 20.0 m
D = 0.20 m
K_{inlet} = 0.5
K_{elbow} = 1.5
K_{exit} = 1
g = 9.81 m/s^2

$$H_{S,12} = (7.00\text{E}0 \text{ m}) + \left(2.28\text{E}2 \ {}^{\text{s}^2}\!/_{\text{m}^5}\right) Q^2 \tag{5}$$

Equate the system head curve (Eq. **Error! Reference source not found.**) to the given curve fit for the pump head curve to solve for the operating point flow rate.

$$\left(\text{-3.25E1 s}^2/\text{m}^5\right)Q^2 + \left(1.23\text{E0 s/m}^2\right)Q + \left(2.78\text{E1 m}\right) = \left(7.00\text{E0 m}\right) + \left(2.28\text{E2 } {}^{\text{s}^2}\!/_{\text{m}^5}\right)Q^2 \tag{6}$$

$$\left(\text{-2.60E2 s}^2/\text{m}^5\right)Q^2 + \left(1.23\text{E0 s/m}^2\right)Q + \left(2.08\text{E1 m}\right) = 0 \tag{7}$$

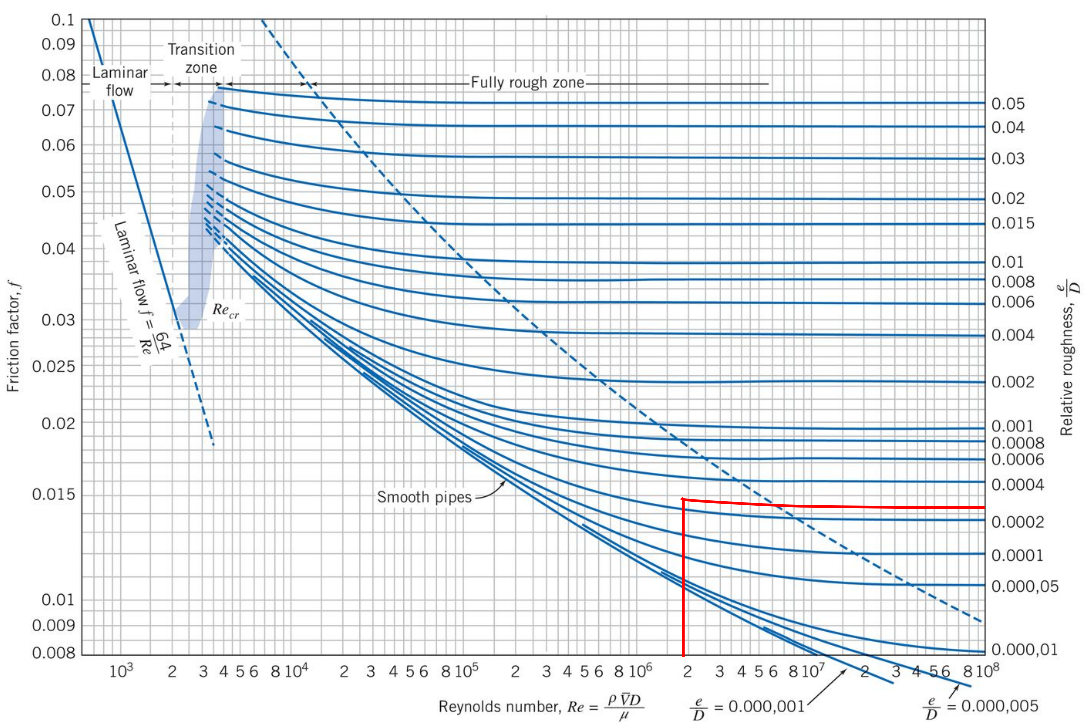
$Q = 0.285$ m^3/s.

Check that the Reynolds number is in the fully rough zone as assumed.

$$\bar{V} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{2.85\text{E-1 m}^3/\text{s}}{\frac{\pi}{4}\left(2.0\text{E-2 m}\right)} = 9.08 \text{ m/s} \tag{8}$$

$$\text{Re} = \frac{\bar{V}D}{\nu} = \frac{\left(9.08\text{E0 m/s}\right)\left(2.0\text{E-1 m}\right)}{\left(1.0\text{E-6 m}^2/\text{s}\right)} = 1.82\text{E6} \tag{9}$$

The Reynolds number and the relative roughness put the flow just outside the fully rough zone so the assumption was not a good one. Thus, an iterative approach must be used to determine the flow rate.



A Python program was used to perform the iterations (refer to the code at the end of this solution) to find the root to the equation,

$$H_{S,sys}(Q) - H_{S,pump}(Q) = 0, \tag{10}$$

where $H_{S,sys}(Q)$ is Eq. (2) and $H_{S,pump}(Q)$ is given in the problem statement. Using the program, the operating flow rate is,

$\boxed{Q = 2.84 * 10^{-1} \text{ m}^3/\text{s}}$.

Note that this result is nearly identical to the one found assuming that the flow is in the fully-rough zone. This result is to be expected since the relative roughness and Reynolds number assuming fully-rough flow were close to, but just outside the boundary for fully rough flow on the Moody plot. The corresponding Reynolds number is,

$\text{Re}_D = 1.81 * 10^6$,

and the friction factor is,

$f = 1.458 * 10^{-2}$.

The efficiency is determined using the given curve fit for the efficiency (in the problem statement) and the calculated volumetric flow rate,

$\eta = 0.720 = 72.0\%$.

The best efficiency for the pump is $\eta_{\text{BEP}} = 86.6\%$, which occurs at $Q_{\text{BEP}} = 0.481 \text{ m}^3/\text{s}$. The operating point efficiency isn't too far from the BEP so this would be an acceptable pump to use, at least as far as efficiency is concerned (other factors may affect the appropriateness of the pump, such as cost, flow rate, durability, etc.).

The NPSHA to the pump is found by applying the Extended Bernoulli Equation between points 1 and 3 and utilizing the definition of NPSH,

$$\left(\frac{p}{\rho g} + \alpha \frac{\bar{V}^2}{2g} + z\right)_3 = \left(\frac{p}{\rho g} + \alpha \frac{\bar{V}^2}{2g} + z\right)_1 - H_{L,13} + H_{S,13}, \tag{11}$$

where,

$p_1 = p_{\text{atm}}$ (free surface exposed to the atmosphere)

$\bar{V}_1 \approx 0$ (large tank)

$z_1 - z_3 = 1.00\text{E}0 \text{ m}$ (given)

$\alpha_3 \approx 1$ (turbulent flow based on the Reynolds number calculated previously)

$H_{S,13} = 0$

$$H_{S,13} = \left[f\left(\frac{L_{13}}{D}\right) + K_{\substack{\text{sharp-edged}\\ \text{entrance}}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}$$

Substitute into the definition of NPSH,

$$NPSH = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g}\right)_s - \frac{p_v}{\rho g}$$

$$NPSHA = \left(\frac{p_{\text{atm}} - p_v}{\rho g}\right) + (z_1 - z_3) - \left[f\left(\frac{L_{13}}{D}\right) + K_{\substack{\text{sharp-edged}\\ \text{entrance}}}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}$$

Using the given data,

p_{atm}	=	10100 Pa
p_v	=	2300 Pa
ρ	=	1000 kg/m^3
g	=	9.81 m/s^2
z_1-z_3	=	1.00 m
f	=	$1.458 * 10^{-2}$ (found previously)
L_{13}	=	10.0 m
D	=	0.20 m

K_{inlet} = 0.5
Q = 0.284 m^3/s (found previously)
$\Rightarrow$ NPSHA = 5.95 m

In order to avoid cavitating the pump, we would need to make sure that NPSHA > NPSHR for the pump.

If NPSHA < NPSHR, then the following could be easily implemented to increase NPSHA:
1. Decrease the elevation of the pump inlet so that $z_1 - z_3$ increases.
2. Decrease the losses from 1 to 3 by:
   a. decreasing the pipe length from 1 to 3 and
   b. using a rounded inlet into the pipe.

Note that increasing the pipe diameter from 1 to 3 or changing the pipe material from 1 to 3 might be difficult to implement and would also change the system operating point. Although they would be difficult to implement, increasing the pressure in tank 1 or decreasing the flow temperature to decrease the vapor pressure would also act to increase NPSHA.

The Python code used to perform the iterative calculations.

```
# pump_13.py

# Import some useful libraries.
import numpy as np
from scipy import optimize

# Initialize some parameters.
p_atm = 1.01e5  # Pa, atmospheric pressure
p_v = 2.30e3  # Pa, water vapor pressure
rho = 1.00e3  # kg/m^3, water density
nu = 1.00e-6  # m^2/s, water kinematic viscosity
g = 9.81e0  # m/s^2, gravitational acceleration
z1 = 1.00e0  # elevation of point 1
z2 = 8.00e0  # m, elevation of point 2
z3 = 0.00e0  # m, elevation of point 3
L13 = 1.00e1  # m, length of upstream pipe
L32 = 1.00e1  # m, length of downstream pipe
D = 2.00e-1  # m, pipe diameter
K_inlet = 5.00e-1  # -, inlet loss coefficient
K_elbow = 1.50e0  # -, elbow loss coefficient
K_exit = 1.00e0  # -, exit loss coefficient
e = 4.50e-5  # m, roughness of commercial steel

A = np.pi*D*D/4  # pipe cross-sectional area
e_D = e/D  # pipe relative roughness
print("e_D = %.3e" % e_D)


def Re(Q):  # Calculate the Reynolds number
    V = Q/A
    return (V*D/nu)

def f_Haaland(Re): # Calculate the friction factor using the Haaland formula
    return (-1.8*np.log10(6.9/Re+(e_D/3.7)**1.11))**-2

def f_Colebrook(Re): # Calculate the friction factor using the Colebrook formula
    fprime = f_Haaland(Re)
    freldiff = 1
    tol = 0.001
    while (freldiff > tol):
        f = fprime
        fprime = (-2.0*np.log10(e_D/3.7 + 2.51/Re/np.sqrt(f)))**-2
        freldiff = np.absolute((fprime-f)/f)
    return f

def H_sys(Q):  # Calculate the head rise for the system curve
    Re_value = Re(Q)
    f = f_Colebrook(Re_value)
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    return ((z2 - z1) + (f*(L13+L32)/D + K_inlet + K_elbow + K_exit)*Q*Q/2/g/A/A)

def H_pump(Q):  # Calculate the head rise for the pump curve, Q in m^3/s and H in m
    return (-3.25e1*Q*Q + 1.23e0*Q + 2.78e1)

def delta_H(Q):  # Calculate the difference between the system and pump heads
    return (H_sys(Q) - H_pump(Q))

def eta(Q):  # Calculate the pump efficiency, Q in m^3/s
    return (-3.74e0*Q*Q + 3.60e0*Q)

def NPSHA(Q):  # Calculate the system NPSHA, Q in m^3/s
    Re_value = Re(Q)
    f = f_Colebrook(Re_value)
    return ((p_atm - p_v)/(rho*g) + (z1 - z3) - ((f*L13/D)+K_inlet)*Q*Q/2/g/A/A)


# Use a built-in method to find the operating flow rate.
sol = optimize.root_scalar(delta_H, bracket=[1e-12, 1], method='brentq')
Q = sol.root

print("Re = %.3e" % Re(Q))
print("f = %.3e" % f_Colebrook(Re(Q)))
print("Q = %.3e m^3/s" % Q)
print("H_sys = %.3e m" % H_sys(Q))
print("H_pump = %.3e m" % H_pump(Q))
print("eta = %.3e" % eta(Q))
print("NPSHA = %.3e m" % NPSHA(Q))
```