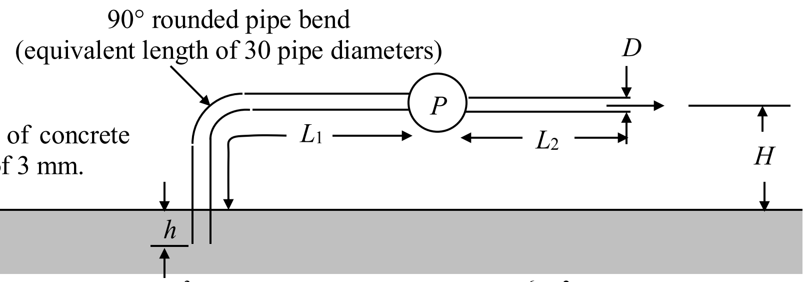
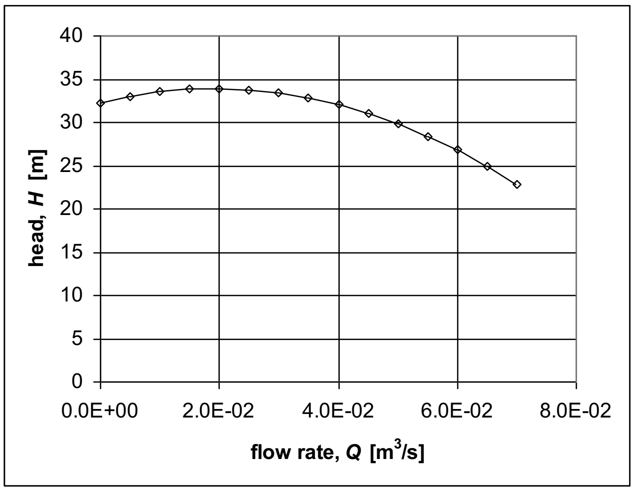
Consider the pipe/pump system shown in the figure below.

h = 0.5 m
H = 2 m
D = 0.2 m
L_1 = 10 m
L_2 = 20 m
The pipe is made of concrete with a roughness of 3 mm.



water with density of 1000 kg/m^3, kinematic viscosity of $1.0*10^{-6}$ m^2/s, and vapor pressure of 2.34 kPa



The pump performance head curve is approximated as:

$$H = (3.23*10^1 \text{ m}) + (1.65*10^2 \text{ s/m}^2)Q - (4.82*10^3 \text{ s}^2/\text{m}^5)Q^2$$

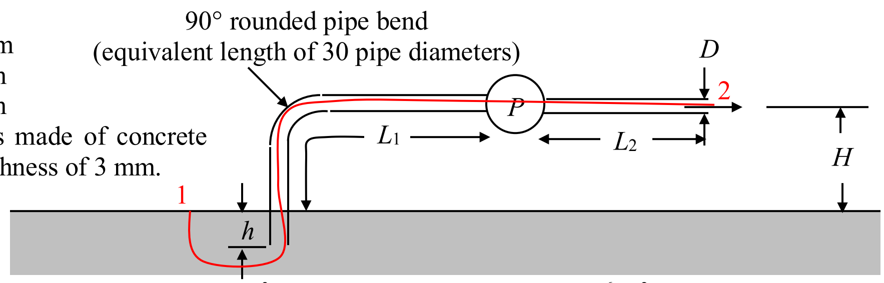
where $[H]$ = m and $[Q]$ = m^3/s.

a. Determine the system head curve for the pipe system.
b. Determine the operating point for the system.
c. How will the flow rate within the pipe change over time if the pipe carries "hard" water and lime deposits form on the interior pipe walls? Explain your answer. You should assume that the deposits do not significantly affect the pipe diameter.
d. Calculate the net positive suction head available at the pump inlet.
e. If we wanted to add a valve to control the flow rate in the pipe, would it be better to put the valve upstream or downstream of the pump? Explain your answer.

SOLUTION:

Apply the Extended Bernoulli Equation from points 1 to 2.

h = 0.5 m
H = 2 m
D = 0.2 m
L_1 = 10 m
L_2 = 20 m
The pipe is made of concrete with a roughness of 3 mm.



water with density of 1000 kg/m³, kinematic viscosity of $1.0*10^{-6}$ m²/s, and vapor pressure of 2.34 kPa

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \tag{1}$$

where

$$p_1=p_2=p_{atm} \tag{2}$$

$$\bar{V}_1\approx 0 \quad \text{and} \quad \bar{V}_2=\frac{4Q}{\pi D^2} \quad \text{(Also assume turbulent flow, } \alpha_2\approx 1.) \tag{3}$$

$$z_2-z_1=H \tag{4}$$

$$H_L=\left(K_{\text{major}}+K_{\text{inlet}}+K_{\text{elbow}}\right)\frac{\bar{V}_2^2}{2g} \tag{5}$$

Solve for H_S.

$$H_S=H+\left[1+f\left(\frac{L}{D}\right)+K_{\text{inlet}}+f\left(\frac{L_e}{D}\right)\right]\frac{8Q^2}{\pi^2 gD^4} \tag{6}$$

Here,

H = 2 m
L = $L_1 + L_2$ = 30 m
D = 0.2 m
K_{inlet} = 0.78 (re-entrant inlet)
L_e/D = 30
g = 9.81 m/s²

The relative roughness is:

$$e/D = (3*10^{-3}\text{ m})/(0.2\text{ m}) = 0.015 \tag{7}$$

Assume the flow Reynolds number is large enough so that it is in the fully rough zone and the friction factor is independent of the Reynolds number.

$$e/D = 0.015 \text{ in fully rough zone (Re > 70,000)} \Rightarrow f \approx 0.044 \tag{8}$$

Substitute and simplify.

$$H_S=2\text{ m}+\left[1+\underbrace{6.6}_{\text{major losses}}+\underbrace{0.78+1.32}_{\text{minor losses}}\right]\left(5.16*10^{1}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2 \tag{9}$$

(Note that the minor losses are not negligible compared to the major loss.)

$$\boxed{H_S=2\text{ m}+\left(5.01*10^{2}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2} \quad \text{(This is the system head curve.)} \tag{10}$$

The operating point occurs where the system and pump curves intersect.

$$\underbrace{2\text{ m}+\left(5.01*10^{2}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2}_{\text{system curve}}=\underbrace{\left(3.23*10^{1}\text{ m}\right)+\left(1.65*10^{2}\ \tfrac{\text{s}}{\text{m}^2}\right)Q-\left(4.82*10^{3}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2}_{\text{pump curve}} \tag{11}$$

$$\left(5.32*10^{3}\ \tfrac{\text{s}}{\text{m}^5}\right)Q^2-\left(1.65*10^{2}\ \tfrac{\text{s}}{\text{m}^2}\right)Q-\left(3.03*10^{1}\text{ m}\right)=0 \tag{12}$$

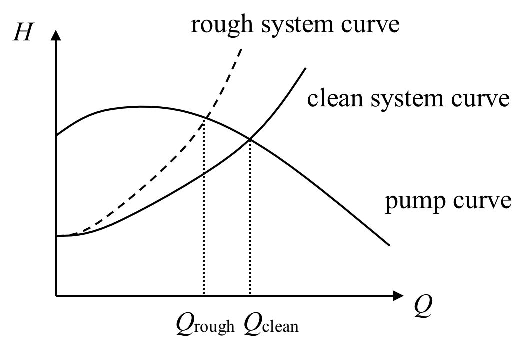
$$\boxed{Q=9.26*10^{-2}\ \tfrac{\text{m}^3}{\text{s}}} \tag{13}$$

Verify the Reynolds number assumption.

$$\bar{V}_2=\frac{4Q}{\pi D^2}=\frac{4\left(9.26*10^{-3}\ \frac{\text{m}^3}{\text{s}}\right)}{\pi\left(0.2\text{ m}\right)^2}=2.95\ \tfrac{\text{m}}{\text{s}} \tag{14}$$

$$\text{Re}=\frac{\bar{V}_2 D}{\nu}=\frac{\left(2.95\ \frac{\text{m}}{\text{s}}\right)\left(0.2\text{ m}\right)}{\left(1*10^{-6}\ \frac{\text{m}^2}{\text{s}}\right)}=590{,}000\ \Rightarrow\ \text{The assumption of fully turbulent flow is ok!} \tag{15}$$

As lime deposits collect, the relative roughness will increase resulting in an increase in the friction factor. Thus, the system curve will steepen over time and the operating flow rate will decrease.



Apply the Extended Bernoulli Equation from points 1 to 2 in the figure below.

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H = 2 m
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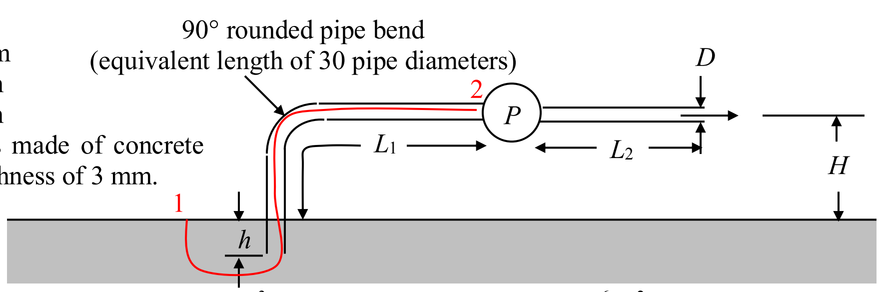

water with density of 1000 kg/m^3, kinematic viscosity of 1.0*10^{-6} m^2/s, and vapor pressure of 2.34 kPa

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \tag{16}$$

where

$p_1 = p_{atm}$ (17)

$\bar{V}_1 \approx 0$ (The flow has been shown to be turbulent $\Rightarrow \alpha_2 \approx 1$.) (18)

$z_2 - z_1 = H$ (19)

$$H_L=\left(K_{\text{major}}+K_{\text{inlet}}+K_{\text{elbow}}\right)\frac{\bar{V}_2^2}{2g} \tag{20}$$

$H_S = 0$ (There is no pump between points 1 and 2.) (21)

Re-arrange to put in terms of *NPSHA*.

$$NPSHA\equiv\left(\frac{p}{\rho g}+\frac{\bar{V}^2}{2g}\right)_2-\frac{p_v}{\rho g}=\frac{p_1-p_v}{\rho g}-H-\left[f\left(\frac{L_1}{D}\right)+K_{\text{inlet}}+f\left(\frac{L_e}{D}\right)\right]\frac{8Q^2}{\pi^2 g D^4} \tag{22}$$

(Note that the major loss is based on L_1.)

Here,

p_1	$= p_{atm} = 101$ kPa (abs)
p_v	$= 2.34$ kPa (abs)
ρ	$= 1000$ kg/m^3
g	$= 9.81$ m/s^2
H	$= 2$ m
f	≈ 0.044 (from previous work)
L_1	$= 10$ m
D	$= 0.2$ m
K_{inlet}	$= 0.78$ (re-entrant inlet)
L_e/D	$= 30$
Q	$= 9.26*10^{-2}$ m^3/s

Substitute and simplify.

$\boxed{NPSHA = 6.16\text{ m}}$ (23)

We would be better off putting the <u>valve downstream of the pump</u> so that the NPSHA remains as large as possible to avoid cavitation in the pump.