A volumetric flow rate of $0.3 \mathrm{~m}^{3} / \mathrm{s}$ of water flows through a 150 mm diameter constriction in a 300 mm diameter horizontal pipeline. The pressure at the inlet to the constriction is 345 kPa (gage) and the head lost between this point and the outlet of the constriction is 3 m . Calculate the pressure at the outlet of the constriction.


SOLUTION:
Apply the Extended Bernoulli Equation from the inlet (1) to the outlet (2) of the constriction,

$$
\begin{equation*}
\left(\frac{p}{\rho g}+\alpha \frac{\bar{v}^{2}}{2 g}+z\right)_{2}=\left(\frac{p}{\rho g}+\alpha \frac{\bar{v}^{2}}{2 g}+z\right)_{1}-H_{L, 12}+H_{S, 12} \tag{1}
\end{equation*}
$$

where,

$$
\begin{align*}
& p_{1}=345 \mathrm{kPa} \text { (gage) (given), } \\
& \bar{V}_{1}=\frac{Q}{\frac{\pi}{4} D_{1}^{2}} \text { and } \bar{V}_{2}=\frac{Q}{\frac{\pi}{4} D_{2}^{2}},  \tag{2}\\
& z_{2}=z_{1}, \\
& H_{L, 12}=3 \mathrm{~m} \text { (given), } \\
& H_{S, 12}=0 \text { (no fluid machinery between the two points). }
\end{align*}
$$

Using the given parameters,

$$
\begin{aligned}
& Q=0.3 \mathrm{~m}^{3} / \mathrm{s}, \\
& D_{1}=0.300 \mathrm{~m}, \\
& D_{2}=0.150 \mathrm{~m}, \\
& \Rightarrow \bar{V}_{1}=4.24 \mathrm{~m} / \mathrm{s} \text { and } \bar{V}_{2}=17.0 \mathrm{~m} / \mathrm{s}, \\
& v=1.8^{*} 10^{-6} \mathrm{~m}^{2} / \mathrm{s}, \\
& \Rightarrow \operatorname{Re}_{D}=\frac{\bar{V} D}{v} \Rightarrow \operatorname{Re}_{D 1}=7.07^{*} 10^{5} \text { and } \operatorname{Re}_{D 2}=1.41^{*} 10^{6} \text { (the flow is turbulent at both locations), } \\
& \Rightarrow \alpha_{1} \approx \alpha_{2} \approx 1 .
\end{aligned}
$$

Re-arrange Eq. (1) to solve for $p_{2}$,

$$
\begin{equation*}
p_{2}=p_{1}+\frac{1}{2} \rho\left(\bar{V}_{1}^{2}-\bar{V}_{2}^{2}\right)-\rho g H_{L, 12} . \tag{3}
\end{equation*}
$$

Using,
$\rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$ and $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$,
$\Rightarrow p_{2}=180 \mathrm{kPa}$ (gage).

