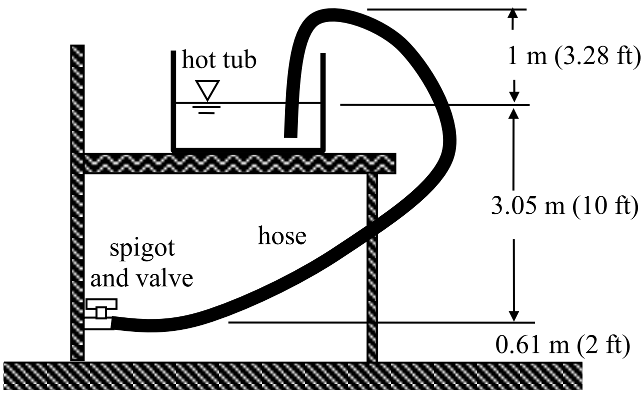
A hot tub sits on a deck as shown in the figure below. A homeowner plans to fill the hot tub with water from a 1.91 cm (0.75 in.) diameter, 7.62 m (25 ft) length of old garden hose attached to an outdoor spigot (aka faucet) located underneath the deck. The hose has an internal roughness of 0.5 mm ($1.97*10^{-2}$ in.) and the gage water pressure just upstream of the spigot valve is 379 kPa (55 psig).

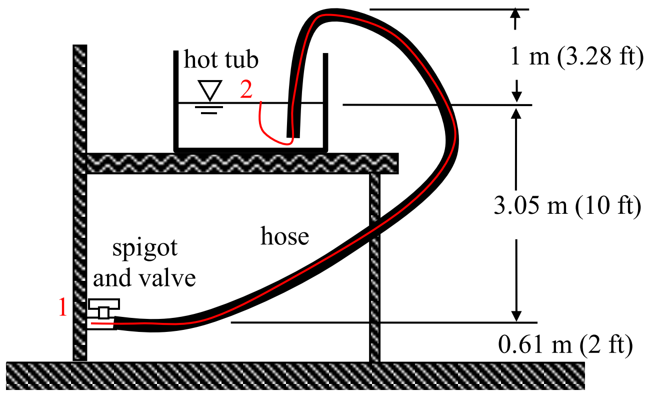


water density = 1000 kg/m^3 (62.4 lb$_m$/ft^3)
water kinematic viscosity = $1.00*10^{-6}$ m^2/s
($1.05*10^{-5}$ ft^2/s)

The minor losses due to the bends in the hose are much smaller than the minor loss due to the valve, which has a loss coefficient of 2.

Determine the volumetric flow rate of water into the hot tub. Clearly state and justify all significant assumptions.

SOLUTION:



Apply the Extended Bernoulli Equation from 1 to 2.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \quad (1)$$

where

$$p_{1,g}=3.79*10^3\text{ Pa}\ \text{ and }\ p_{2,g}=0 \quad (2)$$

$$\bar{V}_1=\bar{V}_h\ \text{ and }\ \bar{V}_2\approx 0 \quad (3)$$

$$z_2-z_1=\Delta z=3.05\text{ m} \quad (4)$$

$$H_L=\left[f\left(\frac{L}{D}\right)+K_{\text{valve}}+K_{\text{exit}}+K_{\text{bends}}\right]\frac{\bar{V}_h^2}{2g} \quad (5)$$

$$H_S=0 \quad (6)$$

Assume the flow in the hose is fully turbulent so that,

$$\alpha_1\approx 1, \quad (7)$$

and the friction factor is solely a function of the relative roughness,

$$\frac{e}{D}=\frac{0.5*10^{-3}\text{ m}}{1.91*10^{-2}\text{ m}}=2.62*10^{-2} \quad (8)$$

Using the Moody diagram, the friction factor is,

$$f\approx 0.054 \quad (9)$$

Substitute into Eq. (1) and solve for the average water speed in the hose.

$$\Delta z=\frac{p_{1,g}}{\rho g}+\frac{\bar{V}_h^2}{2g}-\left[f\left(\frac{L}{D}\right)+K_{\text{valve}}+K_{\text{exit}}+K_{\text{bends}}\right]\frac{\bar{V}_h^2}{2g} \quad (10)$$

Note that the major loss coefficient of,

$$K_{\text{major}}=f\left(\frac{L}{D}\right)=21.65, \quad (11)$$

is larger than the valve and exit losses (recall that from the problem statement, $K_{\text{bends}} \ll K_{\text{valve}}$, and thus the bend losses may be neglected), with $K_{\text{valve}}/K_{\text{major}} = 0.09$ and $K_{\text{exit}}/K_{\text{major}} = 0.05$. We should retain these minor losses since they are not small enough to be neglected.

Solving Eq. (10) for the hose average velocity gives,

$$\bar{V}_h = \sqrt{\frac{2g\left(\Delta z - \frac{p_{1,g}}{\rho g}\right)}{1 - f\left(\frac{L}{D}\right) - K_{\text{valve}} - K_{\text{exit}}}} \Rightarrow \bar{V}_h = 5.4 \text{ m/s} \tag{12}$$

The volumetric flow rate is,

$$Q = \bar{V}_h \tfrac{\pi}{4} D^2 \Rightarrow \boxed{Q = 1.6*10^{-3} \text{ m}^3\text{/s}} \tag{13}$$

Verify that the flow is indeed in the fully turbulent zone.

$$\text{Re}_D = \frac{\bar{V}_h D}{\nu} = \frac{(5.4 \text{ m/s})(1.91*10^{-2} \text{ m})}{1.0*10^{-6} \text{ m}^2\text{/s}} = 1.0*10^5 \tag{14}$$

This value of the Reynolds number is in the fully turbulent zone, so the assumption of fully turbulent flow was a good one.