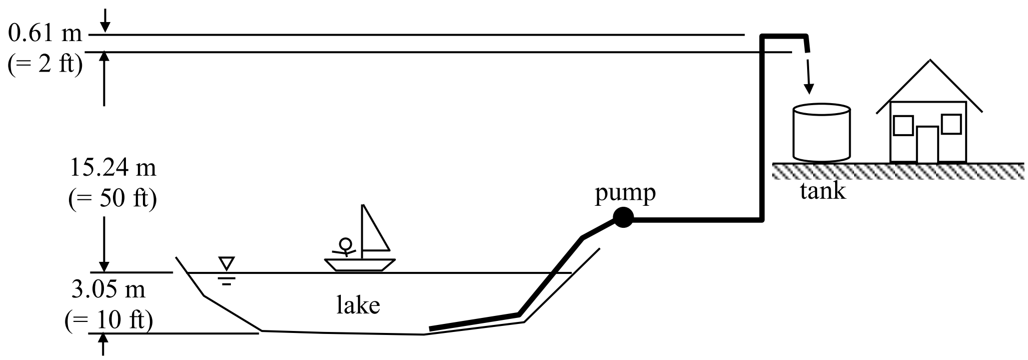
You purchase a cottage at a lake and need to install a pump to feed water to the house. You plan to pump water at night to fill a storage tank you've installed next to the cottage. The pipes and fittings you have chosen to use for the installation are listed in the table below.

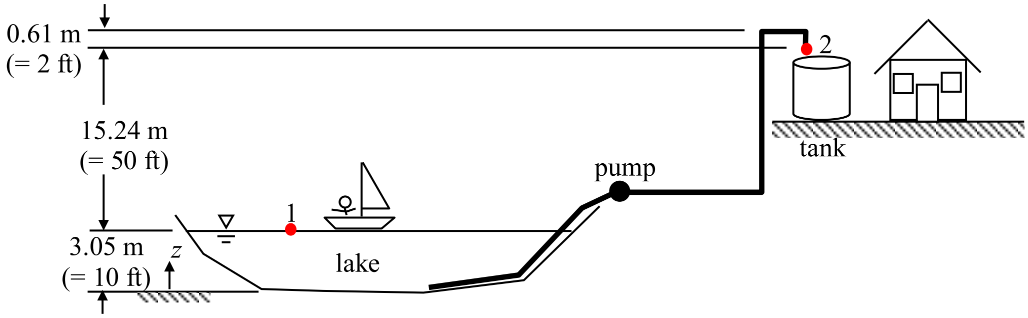
a. What is the minimum head rise across a pump that is capable of providing a flow rate of 18.93 liters per minute (= 5 gpm) of water to the tank?

b. What power should be supplied to the pump assuming the pump efficiency is 65%.



System Component	Amount
straight, smooth plastic pipe, 5.08 cm (= 2 in.) diameter	28.96 m (=95 ft)
re-entrant inlet	1
regular 90º flanged elbow	10
union	8
fully open globe valve	1
fully open gate valve	4

SOLUTION:



Apply the Extended Bernoulli Equation from point 1 to point 2.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2} \quad (1)$$

where

$$p_2=p_1=p_{\text{atm}} \quad (2)$$

$$\bar{V}_1\approx 0 \quad (3)$$

$$\bar{V}_2=\bar{V}_P=\frac{Q}{\frac{\pi D^2}{4}} \quad \text{and} \quad \alpha_2\approx 1 \text{ (assuming turbulent flow)} \quad (4)$$

$$z_2-z_1=15.24\text{ m} \quad (5)$$

$$H_{L,1\to 2}=\sum_i K_i\frac{\bar{V}_i^2}{2g}=K_{\text{major}}\frac{\bar{V}_P^2}{2g}+K_{\text{inlet}}\frac{\bar{V}_P^2}{2g}+10K_{\text{elbow}}\frac{\bar{V}_P^2}{2g}+8K_{\text{union}}\frac{\bar{V}_P^2}{2g}+K_{\substack{\text{globe}\\\text{valve}}}\frac{\bar{V}_P^2}{2g}+4K_{\substack{\text{gate}\\\text{valve}}}\frac{\bar{V}_P^2}{2g}$$
$$=\left(f\frac{L}{D}+K_{\text{inlet}}+10K_{\text{elbow}}+8K_{\text{union}}+K_{\substack{\text{globe}\\\text{valve}}}+4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{\bar{V}_P^2}{2g} \quad (6)$$

(where $\bar{V}_P=\bar{V}_2=\frac{Q}{\frac{\pi D^2}{4}}$)

$$H_{S,1\to 2}=? \quad (7)$$

Substitute into the Extended Bernoulli Equation and re-arrange.

$$\frac{\bar{V}_P^2}{2g}+z_2=z_1-\left(f\frac{L}{D}+K_{\text{inlet}}+10K_{\text{elbow}}+8K_{\text{union}}+K_{\substack{\text{globe}\\\text{valve}}}+4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{\bar{V}_P^2}{2g}+H_{S,1\to 2} \quad (8)$$

$$H_{S,1\to 2}=\left(z_2-z_1\right)+\left(1+f\frac{L}{D}+K_{\text{inlet}}+10K_{\text{elbow}}+8K_{\text{union}}+K_{\substack{\text{globe}\\\text{valve}}}+4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{\bar{V}_P^2}{2g} \quad (9)$$

$$\boxed{H_{S,1\to 2}=\left(z_2-z_1\right)+\left(1+f\frac{L}{D}+K_{\text{inlet}}+10K_{\text{elbow}}+8K_{\text{union}}+K_{\substack{\text{globe}\\\text{valve}}}+4K_{\substack{\text{gate}\\\text{valve}}}\right)\frac{8Q^2}{\pi^2 D^4 g}} \quad (10)$$

Use the given data to determine $H_{S,1\to 2}$.

$z_2 - z_1 = 15.24$ m
$L = 28.96$ m
$D = 5.08$ cm = 5.08e-2 m
$K_{\text{inlet}} = 0.8$
$K_{\text{elbow}} = 0.3$
$K_{\text{threaded union}} = 0.06$
$K_{\text{globe valve}} = 10$
$K_{\text{gate valve}} = 0.15$
$Q = 18.93$ L/min = 3.154e-4 m^3/s (= 5 gpm)
$g = 9.81$ m/s^2 (= 32.2 ft/s^2)

The friction factor is found using the Moody chart for a smooth pipe and a Reynolds number of:

$$\text{Re} = \frac{\bar{V}_P D}{\nu} = \frac{QD}{\frac{\pi D^2}{4}\nu} = \frac{4Q}{\pi D \nu} = \frac{4\left(3.154\text{e-4 m}^3/\text{s}\right)}{\pi\left(5.08\text{e-2 m}\right)\left(1\text{e-6 m}^2/\text{s}\right)} \approx 7900$$

(The turbulent flow assumption is valid!) (11)

$\Rightarrow f \approx 0.033$ (from the Moody diagram) (12)

$\boxed{\therefore H_{S,1\to 2} = 15.28 \text{ m}}$ (= 50.14 ft) (13)

Note that the head loss is much smaller than the elevation head.

The power is related to the shaft head by:

$$H_{S,1\to 2} = \frac{\dot{W}_{S,1\to 2}}{\dot{m}g} = \frac{\dot{W}_{S,1\to 2}}{\rho Q g} \tag{14}$$

$$\therefore \dot{W}_{S,1\to 2} = \rho Q g H_{S,1\to 2} \tag{15}$$

Using the given data ($\rho = 1000$ kg/m^3):

$\therefore \dot{W}_{S,1\to 2} = 47.3$ W (= 0.06 hp) (16)

Since the efficiency is $\eta = 65\%$, the power that must be supplied to the pump is:

$$\therefore \dot{W}_{\text{supply}} = \frac{\dot{W}_{S,1\to 2}}{\eta} \Rightarrow \boxed{\therefore \dot{W}_{\text{supply}} = 72.7 \text{ W}} \text{ (= 0.1 hp)} \tag{17}$$