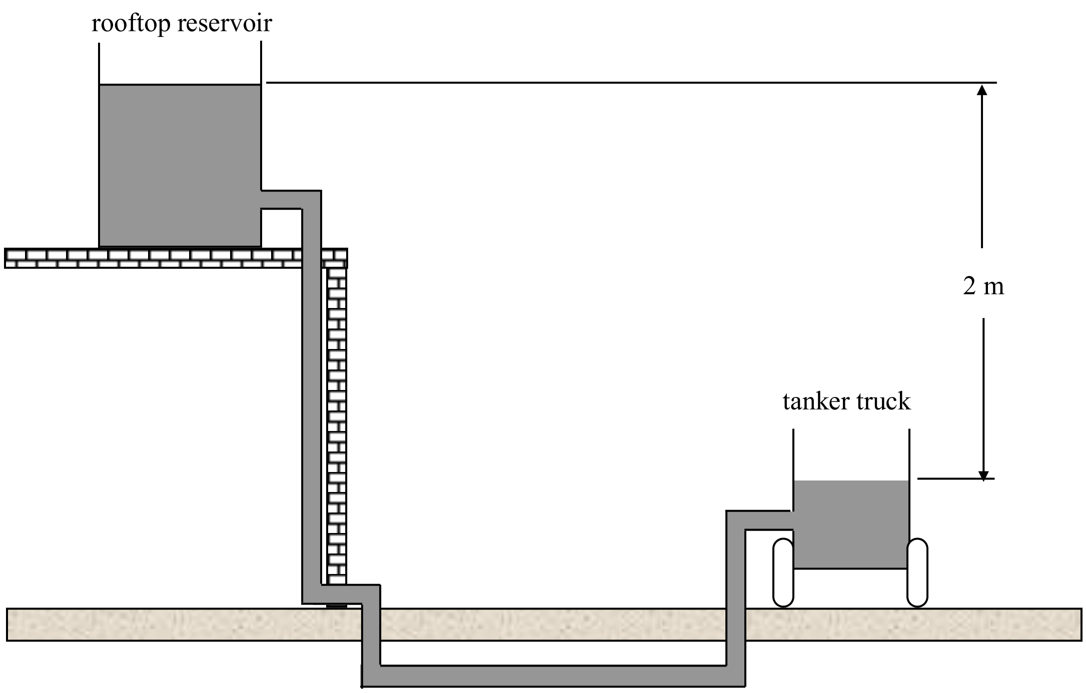
Water at 10 °C (kinematic viscosity of 1.307×10^{-6} m^2/s) is to flow from a roof-top reservoir to a tanker truck through a cast iron pipe (roughness of 0.26 mm) of length 20 m at a flow rate of 0.0020 m^3/s. The roof-top tank water level is located 2 m above the tanker truck fluid level. The system contains a sharp-edged entrance, six threaded 90° elbows, and a sharp-edged exit. Determine the required pipe diameter for the given flow conditions.



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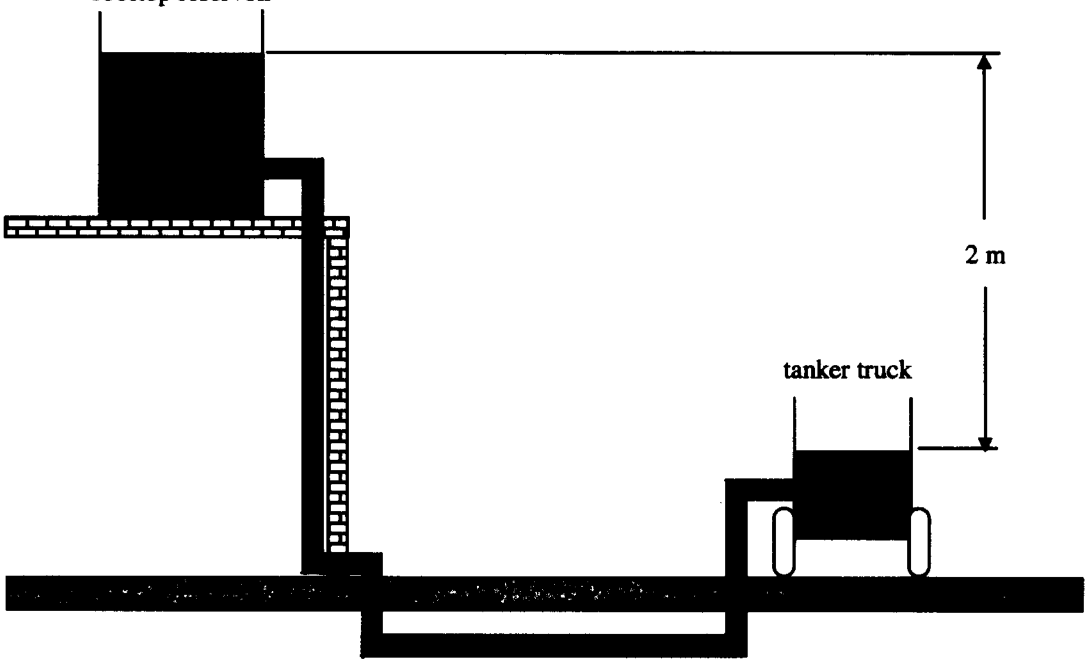
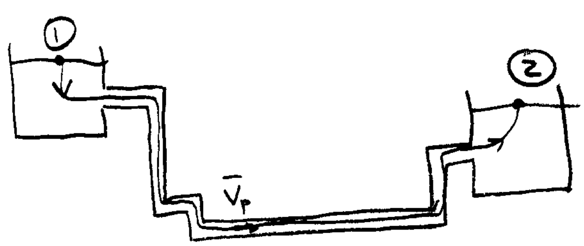


SOLUTION:

- Apply the Extended Bernoulli Eqn from ① to ②:



$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_{L_{1\to2}} + H_{S_{1\to2}}$$

where $p_1 = p_2 = p_{atm}$ $\qquad H_{S_{1\to2}} = 0$

$\bar{V}_1 = \bar{V}_2 \approx 0$

$$H_{L_{1\to2}} = \frac{\bar{V}_P^2}{2g}\left[\underbrace{K_{major}}_{=f\left(\frac{L}{D}\right)} + K_{entrance} + 6K_{elbow} + K_{exit}\right]$$

$z_1 - z_2 = 2\,m$

SOLUTION...

- Substitute

$$\Delta z = \frac{\bar{V}_p^2}{2g}\left[f\left(\frac{L}{D}\right) + K_{entrance} + 6K_{elbow} + K_{exit} \right]$$

- For $\Delta z = 2\,m$

$g = 9.81\,m/s^2$

$$\bar{V}_p = \frac{Q}{\left(\frac{\pi D^2}{4}\right)} = \frac{0.0020\,m^3/s}{\frac{\pi D^2}{4}} = \frac{2.55\times 10^{-3}\,m^3/s}{D^2}$$

$L = 20\,m$

$K_{entrance} = 0.5$

$K_{elbow} = 1.5$

$K_{exit} = 1$

$$\Rightarrow \quad 39.24\,m^2/s^2 = \frac{6.50\times 10^{-6}\,m^6/s^2}{D^4}\left[f\,\frac{20\,m}{D} + 0.5 + 6(1.5) + 1 \right]$$

$$\Rightarrow \quad (6.04\times 10^{6}\,1/m^4)D^5 - 10.5D - (20\,m)f = 0$$

NOTE: The friction factor, f, depends on both Re_D and ϵ/D:

$$Re_D = \frac{\bar{V}_p D}{\nu} = \frac{\left(\frac{2.55\times 10^{-3}\,m^3/s}{D^2}\right)D}{1.307\times 10^{-6}\,m^2/s} = \frac{(1950\,m)}{D}$$

$$\epsilon/D = \frac{(0.00026\,m)}{D}$$

SOLUTION...

- To solve for D we must iterate to a solution since f is also a function of D (because of the Re_D and ϵ/D dependance). The iterative procedure is as follows:

1) Choose a value of D
2) Calculate Re_D : $Re_D = \frac{(1950\,m)}{D}$
3) Calculate ϵ/D : $\epsilon/D = \frac{(2.6 \times 10^{-4}\,m)}{D}$
4) Determine f from Moody chart.
5) Determine f from : $(6.04 \times 10^{6}\,1/m^4) D^5 - 10.5\,D - (20\,m) f = 0$
6) Are the two f s the same?

No. Choose a new value of D.

Yes?

END

SOLUTION...

- Choose $D = 0.04\ m$

  $\Rightarrow Re_D = 48{,}800$

  $\Rightarrow \epsilon/D = 0.0065$

  $\Rightarrow f_{MOODY} = 0.038$

  $\Rightarrow f_{eqn} = 0.010$

  the two fs are not equal

- Choose $D = 0.05\ m$

  $\Rightarrow Re_D = 39{,}000$

  $\Rightarrow \epsilon/D = 0.0052$

  $\Rightarrow f_{MOODY} = 0.033$

  $\Rightarrow f_{eqn} = 0.068$

  the two fs are not equal

- Choose $D = 0.045\ m$

  $\Rightarrow Re_D = 43{,}300$

  $\Rightarrow \epsilon/D = 0.0058$

  $\Rightarrow f_{MOODY} = 0.032$

  $\Rightarrow f_{eqn} = 0.032$

  the two fs <u>are</u> equal

$$\boxed{\therefore\ D = 0.045\ m}$$

SOLUTION...

- An alternative, iterative method for finding D is:
  (and more computationally intensive)

1) Choose a value of D

2) Calculate Re_D: $Re_D = \frac{(1950\,m)}{D}$

3) Calculate ϵ/D: $\epsilon/D = \frac{(2.6\times10^{-4}\,m)}{D}$

4) Determine f from Moody chart.

5) Determine D from: $(6.04\times10^{6}\,m^{-4})\,D^5 - 10.5\,D - (20\,m)f = 0$

6) Are the two Ds the same?

Yes? → END

No? Use the value of D from step 5 and iterate.

SOLUTION...

- Choose $D = 0.040\,m$

  $\Rightarrow Re_D = 48{,}800$

  $\Rightarrow \epsilon/D = 0.0065$

  $\Rightarrow f_{MOODY} = 0.038$

  $\Rightarrow D_{eqn} = 0.046\,m$

  the guessed D and calculated D are not the same

- Choose $D = 0.046\,m$

  $\Rightarrow Re_D = 42{,}400$

  $\Rightarrow \epsilon/D = 0.0057$

  $\Rightarrow f_{MOODY} = 0.033$

  $\Rightarrow D_{eqn} = 0.045\,m$

  the two values of D are not the same

- Choose $D = 0.045\,m$

  $\Rightarrow Re_D = 43{,}300$

  $\Rightarrow \epsilon/D = 0.0058$

  $\Rightarrow f_{MOODY} = 0.032$

  $\Rightarrow D_{eqn} = 0.045\,m$

  the two values of D **ARE** the same

$$\boxed{\therefore\ D = 0.045\,m}$$