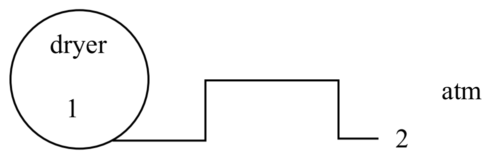
According to an appliance manufacturer, the 4 in diameter galvanized iron vent on a clothes dryer is not to contain more than 20 ft of pipe and four 90° elbows. Under these conditions, determine the air flow rate if the gage pressure within the dryer is 1.04 psf. You may assume the following:

kinematic viscosity of air: 1.79e-4 ft^2/s
density of air: 2.20e-3 $slugs/ft^3$
$K_{90° \text{ bend}} = 1.5$
$K_{\text{entrance}} = 0.5$

SOLUTION:

Apply the Extended Bernoulli Equation from point 1 to point 2 as shown in the figure below.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \quad (1)$$

where

$p_2 = 0$ (gage) and $p_1 = 1.04$ psfg

$\bar{V}_2 = ?$ (Assume turbulent flow so that $\alpha_2 \approx 1$.)

$\bar{V}_1 << \bar{V}_2$ (The air in the dryer is relatively stagnant compared to the outflowing air.)

$z_2 - z_1 \approx 0$

$H_S = 0$

$$H_L = f\left(\frac{L}{D}\right)\frac{\bar{V}_P^2}{2g}+K_{\text{entrance}}\frac{\bar{V}_P^2}{2g}+4K_{\text{bends}}\frac{\bar{V}_P^2}{2g} \quad (2)$$

Note that $\bar{V}_P = \bar{V}_2$ since the pipe and exit diameters are the same. Also, there is no exit loss since point 2 is located just at the exit of the pipe. The air has not undergone any exit losses at this point.

Substitute and simplify.

$$\frac{\bar{V}_2^2}{2}\left[f\left(\frac{L}{D}\right)+K_{\text{entrance}}+4K_{\text{bends}}+1\right]=\frac{p_{1,g}}{\rho} \quad (3)$$

It's given that

L = 20 ft

D = 4 in. = 0.33 ft

K_{elbow} = 1.5

$K_{entrance}$ = 0.5

$p_{1,g}$ = 1.04 lb_f/ft^2

ρ = $2.20*10^{-3}$ $slug/ft^3$

Using these parameters, Eq. (3) becomes,

$$\bar{V}_2^2\left[60f+7.5\right]=945.5\ \text{ft}^2/\text{s}^2 \quad (4)$$

Note that f is dependent on the Reynolds number and relative roughness,

$$\text{Re}_D=\frac{\bar{V}_P D}{\nu}=\left(1862\ \text{s/ft}\right)\bar{V}_2 \quad (5)$$

where $\nu_{air} = 1.79*10^{-4}$ ft^2/s and $\bar{V}_P = \bar{V}_2$. The roughness of galvanized iron pipe is $e = 0.0005$ ft so that the relative roughness is,

$$\frac{e}{D}=0.0015 \quad (6)$$

To solve for $\bar{V}_2$, we must iterate to a solution since *f* is also a (complex) function of $\bar{V}_2$ because of the Reynolds number dependence. One iterative procedure that can be used is given below.

1. Choose a value for *f*.
2. Calculate $\bar{V}_2$ using Eq. (4).
3. Calculate Re_D using Eq. (5).
4. Use the Moody diagram with the Re_D calculated from Step 3 and the relative roughness given in Eq. (6) to find *f'*.
5. Is $f' = f$? If so, then the iterations are complete and $\bar{V}_2$ is the value found in Step 2. Otherwise, use *f'* as the new value for *f* and go to Step 2.

Using this iterative algorithm and an initial guess of $f = 0.025$,

1. $f = 0.025$
   a. $\bar{V}_2 = 10.25$ ft/s
   b. $Re_D = 19{,}000$
   c. $f' = 0.029$ (This value is different than our original guess, must continue iterations.)
2. $f = 0.029$
   a. $\bar{V}_2 = 10.11$ ft/s
   b. $Re_D = 18{,}800$
   c. $f' = 0.029$ (This value matches our initial guess! Iterations complete!)

Note that the flow is turbulent, which is consistent with the assumption that $\alpha_2 \approx 1$.

The volumetric flow rate may be found using,

$$Q = \bar{V}_2 \frac{\pi D^2}{4} \quad \Rightarrow \quad \boxed{Q = 0.882 \text{ ft}^3\text{/s}} \tag{7}$$

where $\bar{V}_2 = 10.11$ ft/s and $D = 0.33$ ft.