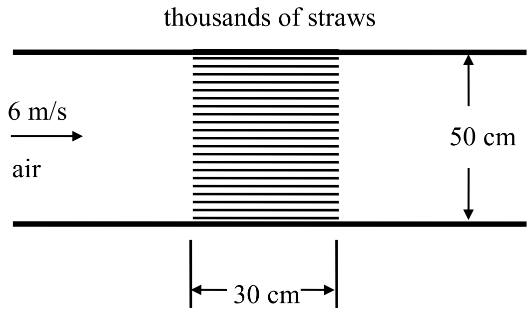
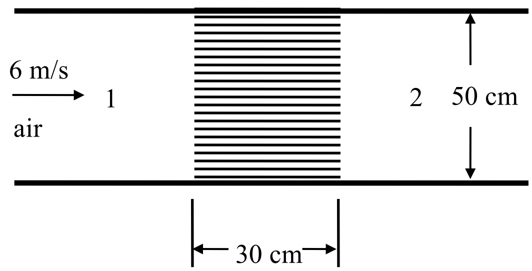
For straightening and smoothing an air flow in a 50 cm diameter duct, the duct is packed with a "honeycomb" of 30 cm long, 4 mm diameter thin straws. The inlet flow is air moving at an average velocity of 6 m/s. Estimate the pressure drop across the honeycomb. The density of the air is 1.2 kg/m^3 and the kinematic viscosity is 1.5e-5 m^2/s. You may neglect inlet and outlet minor losses.



SOLUTION:

Apply the Extended Bernoulli's Equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2} \quad (1)$$

where

$$\frac{p_2-p_1}{\rho g}=\frac{\Delta p}{\rho g}=? \quad \text{(This is what we're trying to find.)} \quad (2)$$

$$\left(\alpha\frac{\bar{V}^2}{2g}\right)_2=\left(\alpha\frac{\bar{V}^2}{2g}\right)_1 \quad (3)$$

$$z_2=z_1 \quad (4)$$

$$H_{S,1\to 2}=0 \quad \text{(There is no shaft work between points 1 and 2.)} \quad (5)$$

$$H_{L,1\to 2}=\left(f\frac{L_S}{D_S}\right)\frac{\bar{V}_S^2}{2g} \quad \text{(where the subscript "}S\text{" refers to the conditions in the straw)} \quad (6)$$

Note that minor losses have been neglected.

Substitute and re-arrange.

$$\frac{\Delta p}{\rho g}=-\left(f\frac{L_S}{D_S}\right)\frac{\bar{V}_S^2}{2g} \quad (7)$$

Now determine the average flow velocity in the straw, $\bar{V}_S$. Note that the flow rate through the pipe must be the same as the flow rate through all of the straws.

$$\bar{V}_P\frac{\pi}{4}D_P^2=N_S\bar{V}_S\frac{\pi}{4}D_S^2 \quad (8)$$

where the number of straws, N_S, is:

$$\frac{\pi}{4}D_P^2=N_S\frac{\pi}{4}D_S^2 \;\Rightarrow\; N_S=\left(\frac{D_P}{D_S}\right)^2 \quad (9)$$

and thus:

$$\bar{V}_S=\bar{V}_P \quad (10)$$

Now determine the friction factor for the flow through the straw. First calculate the straw's Reynolds number.

$$\mathrm{Re}_S = \frac{\bar{V}_S D_S}{\nu} = \frac{\left(6\ \mathrm{m/s}\right)\left(4e-3\ \mathrm{m}\right)}{\left(1.5e-5\ \mathrm{m^2/s}\right)} = 1600 \ \Rightarrow \text{ laminar flow in the straws} \tag{11}$$

$$\Rightarrow\ f = \frac{64}{\mathrm{Re}_S} \tag{12}$$

Substitute Eqn. (12) into Eqn. (7) and solve for the pressure drop. using the given data.

$$\boxed{\frac{\Delta p}{\rho g} = -\left(\frac{64}{\mathrm{Re}_S}\frac{L_S}{D_S}\right)\frac{\bar{V}_S^2}{2g}} \quad (L_S = 0.30\ \mathrm{m};\ \rho = 1.2\ \mathrm{kg/m^3}) \tag{13}$$

$$\boxed{\therefore \Delta p = -64.8\ \mathrm{Pa}}$$