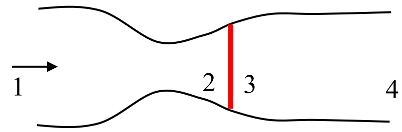
A supersonic aircraft flies at a Mach number of 2.7 at an altitude of 20 km. Air enters the engine inlet and is slowed isentropically to a Mach number of 1.3. A normal shock occurs at that location. The resulting flow is decelerated adiabatically, but not isentropically, further to a Mach number of 0.4. The final static pressure is 104 kPa (abs). Evaluate:

a. the stagnation temperature for the flow,
b. the pressure change, Δp, across the shock,
c. the final stagnation pressure, and
d. the total entropy change throughout the entire process.
e. Sketch the process on a *Ts* diagram.

SOLUTION:



The static temperature and pressure at an altitude of 20 km is, using the U.S. Standard Atmosphere, are T_1 = 216.65 K and p_1 = 5474.9 Pa (abs) (using http://www.digitaldutch.com/atmoscalc/ for example). The stagnation temperature is then:

$$\frac{T_1}{T_0} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_1^2\right)^{-1} \Rightarrow T_1/T_0 = 0.4068 \Rightarrow \boxed{T_0 = 533\text{ K}} \tag{1}$$

Note that the stagnation temperature will remain constant throughout the entire process since there is no heat transfer.

The pressure ratio across the shock wave is may be found using the normal shock relations and noting that Ma_1 = 2.7 and Ma_2 = 1.3.

$$\frac{p_3}{p_2} = \frac{2\gamma}{\gamma + 1}\mathrm{Ma}_2^2 - \frac{\gamma - 1}{\gamma + 1} \Rightarrow p_3/p_2 = 1.8050 \tag{2}$$

$$\frac{p_1}{p_{01}} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_1^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_1/p_{01} = 0.0430 \Rightarrow p_{01} = 127\text{ kPa (abs)} \tag{3}$$

$$\frac{p_2}{p_{01}} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_2^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_2/p_{01} = 0.3609 \Rightarrow p_2 = 46.0\text{ kPa (abs)} \tag{4}$$

$$\Delta p = p_3 - p_2 = p_2\left(\frac{p_3}{p_2} - 1\right) \Rightarrow \boxed{\Delta p = 37.0\text{ kPa}} \tag{5}$$

The stagnation pressure at station 4 may be found from the isentropic stagnation pressure ratio and knowing that Ma_4 = 0.4 and p_4 = 104 kPa (abs).

$$\frac{p_4}{p_{04}} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_4^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_4/p_{04} = 0.8956 \Rightarrow \boxed{p_{04} = 116\text{ kPa (abs)}} \tag{6}$$

The total entropy change throughout the process may be found using the perfect gas, entropy relation:

$$\Delta s = s_4 - s_1 = c_P \ln\frac{T_4}{T_1} - R\ln\frac{p_4}{p_1} \Rightarrow \boxed{\Delta s = 26.3\ \text{J/(kg·K)}} \qquad (7)$$

where p_1 and T_1 are 5474.9 Pa and 216.65 K, respectively (from a U.S. Standard Atmosphere), p_4 = 104 kPa (given), c_P = 1004 J/(kg·K), and R = 287 J/(kg·K). The temperature T_4 may be found from:

$$\frac{T_4}{T_{04}} = \left(1+\frac{\gamma-1}{2}\text{Ma}_4^2\right)^{-1} \Rightarrow T_4/T_{04} = 0.9690 \Rightarrow T_4 = 516\ \text{K} \qquad (8)$$

where $T_{04} = T_{01}$ = 533 K (from Eq. (1)).

Note that we could have also found Δs using stagnation conditions (refer to the Ts diagram below).

$$\Delta s = s_{04} - s_{01} = c_P \ln\frac{T_{04}}{T_{01}} - R\ln\frac{p_{04}}{p_{01}} \Rightarrow \Delta s = 26.4\ \text{J/(kg·K)} \quad \text{(Same as before, within error!)} \qquad (9)$$

where $T_{04} = T_{01}$, p_{04} = 116 kPa, and p_{01} = 127 kPa.