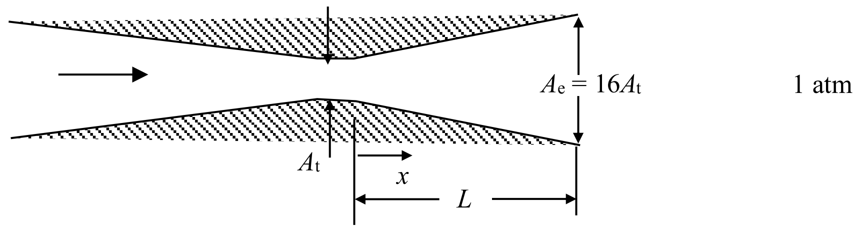
A crude converging-diverging nozzle with an exit-to-throat area ratio of $A_e/A_t = 16$ is built using a straight-sided conical diffuser as shown in the figure below.



The nozzle is supplied by an air reservoir of pressure, p_{res}, and temperature, T_{res}. The nozzle discharges into atmospheric conditions ($p_{atm} = 1$ atm).

a. If a shock wave forms half-way along the diffuser, i.e., $x/L = 0.5$, determine the reservoir pressure, p_{res}.
b. Determine over what range of reservoir pressures the flow will be choked.

SOLUTION:

First determine the area in the straight-sided nozzle as a function of position in the nozzle.

$$r = \left(r_e - r_t\right)\left(\frac{x}{L}\right) + r_t \quad (1)$$

$$A = \pi r^2 \quad (2)$$

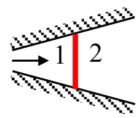
$$\frac{A}{A_t} = \left(\frac{r}{r_t}\right)^2 = \left[\left(\frac{r_e}{r_t} - 1\right)\left(\frac{x}{L}\right) + 1\right]^2 \text{ where } \frac{r_e}{r_t} = \sqrt{\frac{A_e}{A_t}} \quad (3)$$

$$\therefore \frac{A}{A_t} = \left[\left(\sqrt{\frac{A_e}{A_t}} - 1\right)\left(\frac{x}{L}\right) + 1\right]^2 \quad (4)$$

For $x/L = 1/2$ and $A_e/A_t = 16$, $A/A_t = 6.25$. (5)

Using the isentropic flow relations (or tables) for air ($\gamma = 1.4$) and noting that the throat is also the sonic area since there is a shock wave in the diverging section:

$$\frac{A_1}{A^*} = 6.25 \Rightarrow \text{Ma}_1 = 3.411 \text{ and } \frac{p_1}{p_{01}} = 0.0149 \quad (6)$$



Using the normal shock relations (or tables) for air:

$$\text{Ma}_1 = 3.411 \Rightarrow \text{Ma}_2 = 0.4547, \frac{p_{02}}{p_{01}} = 0.2300, \frac{A_2^*}{A_1^*} = 4.3474 \quad (7)$$

Now determine the sonic area ratio at the exit, downstream of the shock wave.

$$\frac{A_e}{A_2^*} = \left(\frac{A_e}{A_1^*}\right)\left(\frac{A_1^*}{A_2^*}\right) = \left(\frac{16}{1}\right)\left(\frac{1}{4.3474}\right) = 3.6804 \quad (8)$$

Using the isentropic flow relations (or tables) for air:

$$\frac{A_e}{A_2^*} = 3.6804 \Rightarrow \text{Ma}_e = 0.1597, \frac{p_e}{p_{02}} = 0.9824 \quad (9)$$

Now determine the upstream stagnation pressure using the pressure ratios. Note that $p_e = p_{\text{atm}} = 1$ atm since the exit Mach number is subsonic.

$$p_{01} = \left(\frac{p_{01}}{p_{02}}\right)\left(\frac{p_{02}}{p_e}\right)p_e = \left(\frac{1}{0.2300}\right)\left(\frac{1}{0.9824}\right)(1 \text{ atm}) \quad (10)$$

$$\boxed{\therefore p_{01} = 4.43 \text{ atm}} \quad (11)$$

For a flow that just becomes choked:

$$\frac{A_e}{A^*} = \frac{A_e}{A_t} = 16 \Rightarrow \text{Ma}_e = 0.0362, \frac{p_e}{p_0} = 0.9991 \quad (12)$$

$$p_0 = \left(\frac{p_0}{p_e}\right)p_e = \left(\frac{1}{0.9991}\right)(1 \text{ atm}) \quad (13)$$

$$\therefore p_0 = 1.001 \text{ atm} \quad (14)$$

Therefore, the flow will be choked for $p_0 \geq 1.001$ atm.