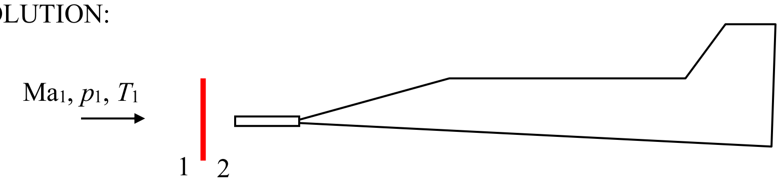
Stagnation pressure and temperature probes are located on the nose of a supersonic aircraft at 35,000 ft altitude. A normal shock stands in front of the probes. The temperature probe indicates $T_0 = 420$ °F behind the shock.

a. Calculate the Mach number and airspeed of the plane.
b. Find the static and stagnation pressures behind the shock.
c. Show the process and the static and stagnation points on a *T*-*s* diagram.

SOLUTION:



The pressure and temperature at an altitude of 35,000 ft using a U.S. Standard Atmospheric table (*e.g.* Table C.5 in Zucrow and Hoffman or using an online calculator such as http://www.digitaldutch.com/atmoscalc/) are:

$$p_1 = 3.458 \text{ psia} \tag{1}$$

$$T_1 = 393.9\ °\text{R} \tag{2}$$

The Mach number of the aircraft may be found by noting that the stagnation temperature remains constant across the shock wave ($T_{01} = T_{02} = 879$ °R) and using the adiabatic stagnation temperature ratio:

$$\frac{T_1}{T_{01}} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}_1^2\right)^{-1} \Rightarrow \boxed{\text{Ma}_1 = 2.48} \tag{3}$$

where, for air, $\gamma = 1.4$. The velocity is found from the Mach number and speed of sound:

$$V_1 = \text{Ma}_1 c_1 = \text{Ma}_1\sqrt{\gamma R T_1} \Rightarrow \boxed{V_1 = 2410 \text{ ft/s}} \tag{4}$$

where $R = 53.3\ (\text{lb}_f{\cdot}\text{ft})/(\text{lb}_m{\cdot}°\text{R})$.

The static pressure downstream of the shock, p_2, may be found from the normal shock relations.

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1}\text{Ma}_1^2 - \frac{\gamma - 1}{\gamma + 1} \Rightarrow \boxed{p_2 = 24.2 \text{ psia}} \tag{5}$$

The stagnation pressure may be found by combining the stagnation pressure upstream of the shock with the stagnation pressure ratio across the shock.

$$\frac{p_1}{p_{01}} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}_1^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_{01} = 57.4 \text{ psia} \tag{6}$$

$$p_{02} = p_{01}\left(\frac{p_{02}}{p_{01}}\right) = p_{01}\left[\frac{(\gamma + 1)\text{Ma}_1^2}{2 + (\gamma - 1)\text{Ma}_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma + 1}{2\gamma\text{Ma}_1^2 - (\gamma - 1)}\right]^{\frac{1}{\gamma-1}} \Rightarrow \boxed{p_{02} = 29.1 \text{ psia}} \tag{7}$$