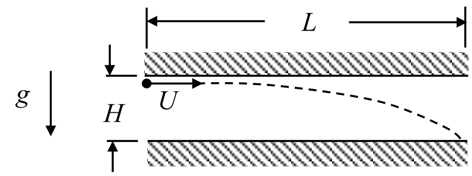
Gravity settling tanks are sometimes used to separate particles from a fluid stream. Estimate the critical length, L, for capturing a particle by gravity settling in the channel shown below. Express your answer in terms of the channel height, H, the fluid velocity, U, the fluid density, ρ_f, the fluid dynamic viscosity, μ_f, particle density, ρ_p, the particle diameter, d, and the acceleration due to gravity, g. You may assume that the particle diameter is very small and that the fluid velocity profile in the channel is uniform. How will the length L change if the particle diameter is doubled?



SOLUTION:

In order to capture the particle, we want the particle to settle on the base before passing through the device, *i.e.*:

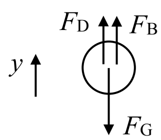
$t_{\text{settling}} < t_{\text{residence}}$

where

$t_{\text{residence time}} = L/U$ (chamber length / fluid velocity)

$t_{\text{settling time}} = H/U_{\text{pt}}$ (settling height / particle terminal velocity)

The particle terminal velocity can be determined by considering a free body diagram acting on the particle. The forces acting on the particle include a drag force, F_D, a buoyant force, F_B, and a gravitational force, F_G.



$$\sum F_y = 0 = F_D + F_B - F_G$$

$$0 = 3\pi\mu_f U_{pt} d + \rho_f \frac{\pi}{6} d^3 g - \rho_p \frac{\pi}{6} d^3 g$$

Note that Stokes drag has been assumed for the particle drag force since the particle Reynolds number is assumed to be very small. Solving the previous equation for the terminal velocity, U_{pt}, gives:

$$U_{pt} = \frac{\left(\rho_s - \rho_f\right) g d^2}{18\mu_f}$$

Since the settling time must be less than the residence time:

$$\frac{L}{U} > \frac{18\mu_f H}{\left(\rho_s - \rho_f\right) g d^2} \Rightarrow \boxed{\frac{L}{H} > \frac{18\mu_f U}{\left(\rho_s - \rho_f\right) g d^2}}$$

The length of the settling chamber, L, will decrease by a factor of four if the particle size doubles.