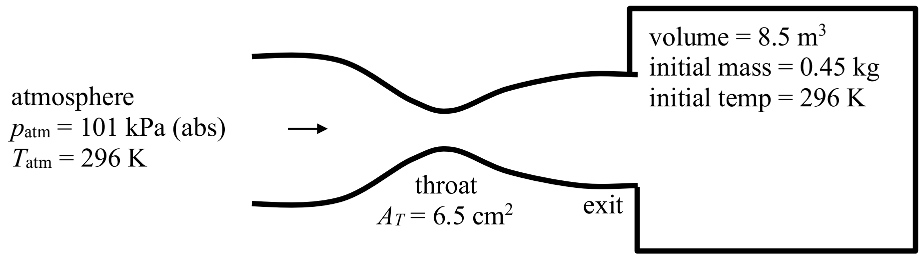
An 8.5 m^3 vacuum tank is to be used to create a flow at an exit Mach number of $Ma_E = 2.0$ (refer to the figure below). A plug is put into the nozzle and the tank is evacuated until it contains 0.45 kg of air at a temperature of 296 K. When the plug is removed, air flows from the atmosphere into the tank through the converging-diverging nozzle. The throat area is $A_T = 6.5$ cm^2.



a. Determine the design exit area.
b. Determine the initial pressure in the tank.
c. Determine the initial mass flow rate through the nozzle.
d. Determine the exit pressure, p_E, immediately after the flow begins.
e. Determine the tank pressure at which a normal shock wave will stand in the nozzle exit plane.

SOLUTION:

The design exit area may be found from the design exit Mach number, $\text{Ma}_{E,d} = 2.0$, and the isentropic flow relations.

$$\frac{A_{E,d}}{A^*} = \frac{1}{\text{Ma}_{E,d}}\left(\frac{1+\frac{\gamma-1}{2}\text{Ma}_{E,d}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow A_{E,d}/A^* = 1.6875 \Rightarrow \boxed{A_{E,d} = 11.0\text{ cm}^2} \quad (1)$$

where the sonic area is equal to the throat area, $A^* = A_T = 6.5\text{ cm}^2$, since the flow goes from stagnation conditions to supersonic conditions.

The initial pressure in the tank may be found using the ideal gas law,

$$p = \rho RT = \left(\frac{M}{V}\right)RT \Rightarrow \boxed{p_{\text{tank}}(t=0) = 4.50\text{ kPa (abs)}} \quad (2)$$

where $M = 0.45$ kg, $V = 8.5\text{ m}^3$, $R = 287$ J/(kg.K), and $T = 296$ K.

To determine the exit plane pressure and initial mass flow rate through the nozzle, first determine whether or not the flow is choked. Determine the pressure at the exit plane when the flow first becomes choked (*i.e.*, $\text{Ma}_T = 1$) by first determining the exit Mach number when the flow first becomes choked, then using this Mach number and the isentropic relations to determine the exit pressure ratio.

$$\frac{A_E}{A^*} = \frac{1}{\text{Ma}_E}\left(\frac{1+\frac{\gamma-1}{2}\text{Ma}_E^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow \text{Ma}_E = 0.372 \quad (3)$$

where $A_E/A^* = 1.6875$ from Eq. (1) (note that when the flow is choked, $A^* = A_T$). The pressure at the exit for this condition is found from the isentropic flow relation.

$$\frac{p_E}{p_0} = \left(1+\frac{\gamma-1}{2}\text{Ma}_E^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_E/p_0 = 0.9088 \Rightarrow p_E = 91.8\text{ kPa (abs)} \quad (4)$$

where $p_0 = p_{\text{atm}} = 101$ kPa (abs). Since this exit pressure is larger than the initial tank pressure, the flow must be choked and the mass flow rate is then,

$$\dot{m}_{\text{choked}} = \left(1+\frac{\gamma-1}{2}\right)^{\frac{(1+\gamma)}{2(1-\gamma)}} p_0\sqrt{\frac{\gamma}{RT_0}}A^* \Rightarrow \boxed{\dot{m}_{\text{choked}} = 0.154\text{ kg/s}} \quad (5)$$

where $p_0 = 101$ kPa (abs), $T_0 = 296$ K, $R = 287$ J/(kg.K), $\gamma = 1.4$, and $A^* = A_T = 6.5\text{ cm}^2$.

The design pressure for the nozzle is found using the isentropic relations and the design Mach number.

$$\frac{p_{E,d}}{p_0} = \left(1+\frac{\gamma-1}{2}\text{Ma}_{E,d}^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_{E,d}/p_0 = 0.1278 \Rightarrow p_{E,d} = 12.9\text{ kPa (abs)} \quad (6)$$

Since the exit pressure at design is larger than the initial tank pressure, the flow must be underexpanded and the exit pressure will be equal to the design exit pressure of $\boxed{p_{E,d} = 12.9\text{ kPa (abs)}}$.

The tank pressure at which a normal shock stands in the exit plane is found by using the design Mach number and exit pressure found in Eq. (6) just upstream of the shock, then applying the normal shock relations across the exit shock wave.

$\mathrm{Ma}_{E1} = 2.0 \Rightarrow p_{E2}/p_{E1} = 4.500$ (from the normal shock relations) $\Rightarrow p_{E2} = 58.1$ kPa (abs) (7)

where $p_{E1} = 12.9$ kPa (abs) from Eq. (6). Since the flow just downstream of the shock is subsonic, the downstream exit pressure will equal the back pressure. Thus, the tank pressure at which a normal shock just stands at the exit is **58.1 kPa (abs)**.