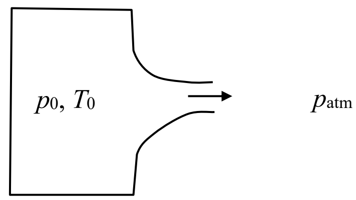
A fixed amount of gaseous fuel is to be fed steadily from a heated tank to the atmosphere through a converging nozzle. The temperature of fuel in the tank remains constant. A young engineer comes to you with the following scheme: "Pressurize the tank to a pressure considerably higher than atmospheric pressure. At the fuel nozzle outlet, the Mach number will then be equal to one. As long as the Mach number is one at the nozzle outlet, we will have the same mass flow rate." Do you agree with the young engineer? Explain your answer.

SOLUTION:



If $p_0 >> p_{atm}$, then the flow will be choked at the nozzle exit. Although the Mach number at the exit plane will remain sonic (i.e., $Ma_E = 1$) while the flow is choked, the mass flow rate will <u>not</u> remain constant since the <u>stagnation pressure within the tank will decrease as mass leaves the tank</u>. Over time, the mass flow rate from the tank will decrease.

$$\dot{m}_{\text{choked}} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma+1}{2(1-\gamma)}} p_0 \sqrt{\frac{\gamma}{RT_0}} A^* \tag{1}$$

In this expression, A^* is the throat area (while the flow is choked) and T_0 remains constant, as given in the problem statement. However, the stagnation pressure decreases since,

$$p_0 = \rho_0 R T_0 = \left(\frac{M_{\text{tank}}}{\cancel{V}_{\text{tank}}}\right) R T_0. \tag{2}$$

From Conservation of Mass applied to the tank,

$$\frac{dM_{\text{tank}}}{dt} = -\dot{m} \tag{3}$$

Thus, as mass escapes from the tank, the tank mass decreases (Eq. (3)) and, from Eq. (2), the stagnation pressure decreases. Thus, from Eq. (1), the mass flow rate decreases.