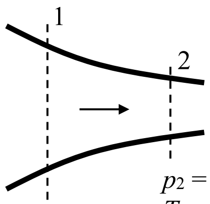
A steady flow of air passes through a converging nozzle. At the nozzle inlet, the static pressure and temperature are $p_1 = 150$ kPa (abs), $T_1 = 500$ K, and $V_1 = 150$ m/s. At the nozzle exit, $p_2 = 98.32$ kPa (abs), $T_2 = 453.2$ K, and $V_2 = 341.4$ m/s. Assume steady, uniform flow, and that the air behaves as a perfect gas with $\gamma = 1.4$, $R = 287$ J/(kg·K), and $c_p = 1005$ J/(kg·K).

a. Is the flow through the nozzle adiabatic?
b. Is the flow through the nozzle isentropic?
c. Is the flow through the nozzle frictionless?

Support all of your answers.

SOLUTION:



p_1 = 150 kPa (abs)
T_1 = 500 K
V_1 = 150 m/s

p_2 = 98.32 kPa (abs)
T_2 = 453.2 K
V_2 = 341.4 m/s

If the flow is adiabatic then the stagnation temperature will remain constant, i.e., $T_{02} = T_{01}$, where:

$$T_0 = T + \frac{V^2}{2c_p} \tag{1}$$

Using the given data:

T_1 = 500 K
V_1 = 150 m/s
T_2 = 453.2 K
V_2 = 341.4 m/s
c_p = 1005 J/(kg·K)
$\Rightarrow$ T_{01} = 511.2 K and T_{02} = 511.2 K

Since the stagnation temperatures are equal, the flow must be adiabatic.

If the flow is isentropic, then the stagnation pressure will remain constant, i.e., $p_{02} = p_{01}$ (the stagnation density will also remain constant, *i.e.* $\rho_{02} = \rho_{01}$).

$$\frac{p}{p_0} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}^2\right)^{\gamma/(1-\gamma)} \tag{2}$$

Using the given data:

p_1 = 150 kPa (abs)
V_1 = 150 m/s
T_1 = 500 K

$\Rightarrow \text{Ma}_1 = 0.34$ where $\text{Ma} = \frac{V}{c} = \frac{V}{\sqrt{\gamma R T}}$

p_2 = 98.32 kPa (abs)
V_2 = 341.4 m/s
T_2 = 453.2 K
$\Rightarrow \text{Ma}_2 = 0.80$
$\Rightarrow$ p_{01} = 162.1 kPa and p_{02} = 149.9 kPa

Since the stagnation pressures are not equal, the flow is not isentropic.

Since the flow is adiabatic but non-isentropic, then some other irreversible process must take place. Two common irreversible processes that occur in gas dynamics are frictional effects and shock waves. Shock waves cannot be the source of the entropy since shock waves only occur in supersonic flows and the flow in this converging nozzle remains subsonic throughout. Hence, we can conclude that
the flow in this nozzle is not frictionless.