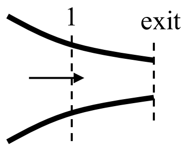
Air flows isentropically through a converging nozzle. At a section where the nozzle area is 0.013 ft^2, the local pressure, temperature, and Mach number are 60 psia, 40 °F, and 0.52, respectively. The back pressure is 30 psia. The Mach number at the exit, the mass flow rate, and the exit area are to be determined.

SOLUTION:



p_b = 30 psia

A_1 = 0.013 ft^2
p_1 = 60 psia
T_1 = 40 °F = 500 °R
Ma_1 = 0.52

First determine whether or not the flow is choked by checking the pressure ratio at the exit. In order to do this, we must first determine the flow stagnation pressure (we'll also calculate the stagnation pressure while we're at it). Note that the flow remains subsonic in the nozzle (subsonic Mach number and no minimum area) so that there will be no shock waves in the flow to modify the flow's stagnation pressure.

$$\frac{p_1}{p_0} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}_1^2\right)^{\gamma/(1-\gamma)} \quad (1)$$

$$\frac{T_1}{T_0} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}_1^2\right)^{-1} \quad (2)$$

Using the given data:

p_1 = 60 psia
T_1 = 500 °R
γ = 1.4
Ma_1 = 0.52
$\Rightarrow p_0$ = 72.2 psia
$\Rightarrow T_0$ = 527 °R

From the ideal gas law:

$$\rho_0 = \frac{p_0}{RT_0} \quad \text{(where } R = 53.3 \text{ (ft·lb}_f\text{)/(lb}_m\text{·°R)} = 1716 \text{ ft}^2\text{/(s}^2\text{·°R))} \quad (3)$$

$\Rightarrow \rho_0$ = 1.21×10^{-3} slug/ft^3

Now check to see if $p_b/p_0 < p^*/p_0$.

$$\frac{p_b}{p_0} = \frac{30.0 \text{ psia}}{72.2 \text{ psia}} = 0.4155 < \frac{p^*}{p_0} = 0.5283 \quad (4)$$

$\Rightarrow$ The flow must be sonic at the exit, i.e., $\boxed{\text{Ma}_e = 1}$!

Since the flow is sonic at the exit, we know that the exit area must be the sonic area.

$$\frac{A_1}{A_e} = \frac{A_1}{A^*} = \frac{1}{\text{Ma}_1}\left(\frac{1+\frac{\gamma-1}{2}\text{Ma}_1^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{5}$$

Using the given data:

γ = 1.4
A_1 = 0.013 ft^2
Ma_1 = 0.52

$$\frac{A_1}{A_e} = 1.3034 \quad \Rightarrow \quad \boxed{A_e = 9.97*10^{-3}\ \text{ft}^2}$$

Since the flow is choked, the mass flow rate is:

$$\dot{m}_{\text{choked}} = \left(1+\frac{\gamma-1}{2}\right)^{\frac{1+\gamma}{2(1-\gamma)}} p_0\sqrt{\frac{\gamma}{RT_0}}A^* \tag{6}$$

γ = 1.4
R = 53.3 (ft·lb$_f$)/(lb$_m$·°R) = 1716 ft^2/(s^2·°R)
A_e = 9.97*10^{-3} ft^2 ($= A^*$)
p_0 = 72.2 psia
T_0 = 527 °R

$$\Rightarrow \boxed{\dot{m} = 7.46*10^{-2}\ \text{slug/s}}$$

We could have also found the mass flow rate using:

$$\dot{m} = \rho_e V_e A_e$$

where

$$V_e = c_e = \sqrt{\gamma R T_e}$$

$$\frac{T_e}{T_0} = \frac{T^*}{T_0} = 0.8333$$

$$\frac{\rho_e}{\rho_0} = \frac{\rho^*}{\rho_0} = 0.6339$$