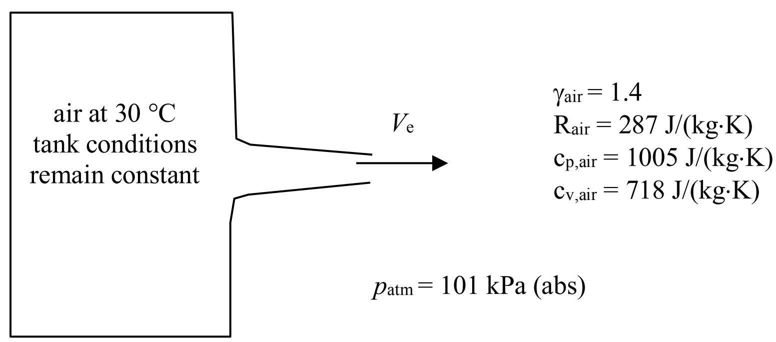
The large compressed-air tank shown in the figure exhausts from a nozzle at an exit velocity of $V_e = 235$ m/s. Assuming isentropic flow, compute:

a. the pressure in the tank
b. the exit Mach number
c. Now consider a case where the exit velocity is not given and the tank pressure is 300 kPa (abs). For these conditions, determine the exit flow speed, V_E.



SOLUTION:

First determine the exit Mach number using:

$$\text{Ma}_e = \frac{V_e}{c_e} \tag{1}$$

The exit speed of sound, assuming ideal gas behavior, is given by:

$$c_e = \sqrt{\gamma R T_e} \tag{2}$$

where, for an adiabatic flow:

$$T_0 = T_e + \frac{V_e^2}{2c_p} \tag{3}$$

Using the given data:

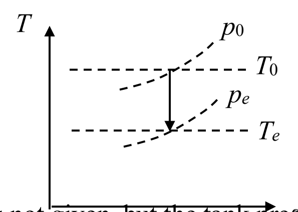
γ = 1.4
R = 287 J/(kg·K)
T_0 = 30 °C = 303 K
V_e = 235 m/s
c_p = 1005 J/(kg·K)
$\Rightarrow$ T_e = 276 K
$\Rightarrow$ c_e = 333 m/s
$\Rightarrow$ $\boxed{\text{Ma}_e = 0.71}$

Since the exit Mach number is subsonic, the exit pressure will be equal to the back pressure, *i.e.*
$p_e = p_{\text{atm}} = 101$ kPa (abs)

Assuming isentropic flow:

$$\frac{p_e}{p_0} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}_e^2\right)^{\gamma/(1-\gamma)} \tag{4}$$

Using the given data:
$\Rightarrow$ $\boxed{p_0 = 141 \text{ kPa (abs)}}$



Now consider the case where the exit velocity is not given, but the tank pressure is given as $p_0 = 300$ kPa (abs). First determine whether or not the flow is choked. For a converging nozzle, the flow is choked if,

$$\frac{p_B}{p_0} \le \frac{p^*}{p_0} = \left(1 + \frac{k-1}{2}\right)^{\frac{k}{1-k}} \underset{k=1.4}{=} 0.5283 \tag{5}$$

Using the given data ($p_0 = 300$ kPa (abs) and $p_B = 101$ kPa (abs)), $p_B/p_0 = 0.3367$. Thus, the flow is choked for the given conditions and $\text{Ma}_E = 1$.

Since the exit is at sonic conditions, the speed of the flow there is,

$$V_E = V^* = c^* \underbrace{\text{Ma}^*}_{=1} = \sqrt{kRT^*} \tag{6}$$

where

$$\frac{T^*}{T_0} = \left(1 + \frac{k-1}{2}\right)^{-1} \underbrace{=}_{k=1.4} 0.8333 \tag{7}$$

Using the given data ($T_0 = 303$ K, $k = 1.4$, $R = 287$ J/(kg.K)), $T^* = 253$ K, and $\boxed{V_E = 319 \text{ m/s}}$.