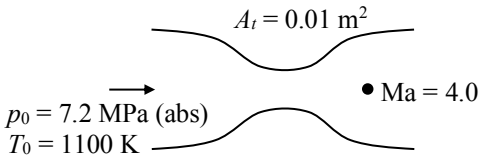
Air, at a stagnation pressure of 7.20 MPa (abs) and a stagnation temperature of 1100 K, flows isentropically through a converging-diverging nozzle having a throat area of 0.01 m^2. Determine the speed and the mass flow rate at the downstream section where the Mach number is 4.0.

SOLUTION:



At the section where Ma = 4.0:

$$\frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}^2\right)^{-1} \Rightarrow \underline{T = 261.9 \text{ K}} \quad (1)$$

where $\gamma = 1.4$, $T_0 = 1100$ K, and Ma = 4.0.

The velocity at the section may be found from the Mach number and speed of sound.

$$V = c\text{Ma} = \sqrt{\gamma RT}\text{Ma} \Rightarrow \boxed{V = 1298 \text{ m/s}} \quad (2)$$

where $R = 287$ J/(kg·K).

That mass flow rate is given by:

$$\dot{m} = \rho VA = \left(\frac{p}{RT}\right)VA \Rightarrow \boxed{\dot{m} = 87.6 \text{ kg/s}} \quad (3)$$

where

$$\frac{p}{p_0} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow \underline{p = 4.742*10^4 \text{ Pa}} \text{ (using } p_0 = 7.20 \text{ MPa)} \quad (4)$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}}\left(\frac{1 + \frac{\gamma - 1}{2}\text{Ma}^2}{1 + \frac{\gamma - 1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow \underline{A = 0.107 \text{ m}^2} \text{ (using } A^* = A_t = 0.01 \text{ m}^2) \quad (5)$$

An alternate method for determine the mass flow rate is to use the choked flow mass flow rate expression.

$$\dot{m} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma+1}{2(1-\gamma)}} p_0\sqrt{\frac{\gamma}{RT_0}}A^* \Rightarrow \boxed{\dot{m} = 87.7 \text{ kg/s}} \text{ (Same result as before, within numerical error!)} \quad (6)$$