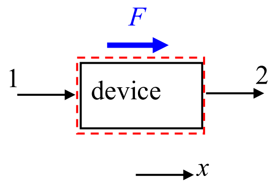
Oxygen (not air) enters a device with a cross-sectional area of 1 ft^2 (refer to this location as section 1) with a stagnation temperature of 1000 °R, stagnation pressure of 100 psia, and Mach number of 0.2. There is no heat transfer, work transfer, or losses as the gas passes through the device and expands to a pressure of 14.7 psia (section 2).

a. Determine the density, velocity, and mass flow rate at section 1.
b. Determine the Mach number, temperature, velocity, density, and area at section 2.
c. What force does the fluid exert on the device?

SOLUTION:



First determine the properties at section 1.

$$\rho_0 = \frac{p_0}{RT_0} \tag{1}$$

where

$$\rho_1 = \rho_0\left(1+\frac{\gamma-1}{2}\mathrm{Ma}_1^2\right)^{\frac{1}{1-\gamma}} \tag{2}$$

Using p_0 = 100 psia = 14400 $\text{lb}_\text{f}/\text{ft}^2$, T_0 = 1000 °R, R = 48.291 $(\text{ft}\cdot\text{lb}_\text{f})/(\text{lb}_\text{m}\cdot°\text{R})$ = 1553.7 $\text{ft}^2/(\text{s}^2\cdot°\text{R})$, ρ_0 = 0.298 $\text{lb}_\text{m}/\text{ft}^3$. In addition, with γ = 1.395, and Ma_1 = 0.2, $\boxed{\rho_1 = 0.292\ \text{lb}_\text{m}/\text{ft}^3}$.

$$V_1 = c_1\mathrm{Ma}_1 = \sqrt{\gamma RT_1}\,\mathrm{Ma}_1 \tag{3}$$

where

$$T_1 = T_0\left(1+\frac{\gamma-1}{2}\mathrm{Ma}_1^2\right)^{-1} \tag{4}$$

Using the given values, T_1 = 992.2 °R and $\boxed{V_1 = 293.3\ \text{ft/s}}$.

$$\dot{m} = \rho_1 V_1 A_1 \tag{5}$$

Using the given values, $\boxed{\dot{m} = 85.6\ \text{lb}_\text{m}/\text{s}}$.

Now use the isentropic relations to determine the properties at section 2.

$$p_2 = p_0\left(1+\frac{\gamma-1}{2}\mathrm{Ma}_2^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow \mathrm{Ma}_2 = \left\{\frac{2}{\gamma-1}\left[\left(\frac{p_2}{p_0}\right)^{\frac{1-\gamma}{\gamma}} - 1\right]\right\}^{\frac{1}{2}} \tag{6}$$

Using p_2 = 14.7 psia and p_0 = 100 psia, $\boxed{\mathrm{Ma}_2 = 1.91}$.

$$T_2 = T_0\left(1+\frac{\gamma-1}{2}\mathrm{Ma}_2^2\right)^{-1} \tag{7}$$

Using the given data, $\boxed{T_2 = 581.2\ °\text{R}}$.

$$V_2 = c_2\mathrm{Ma}_2 = \sqrt{\gamma RT_2}\,\mathrm{Ma}_2 \tag{8}$$

Using the given data, $\boxed{V_2 = 2144\ \text{ft/s}}$.

$$\rho_2 = \frac{p_2}{RT_2} \tag{9}$$

Using the given data, $\boxed{\rho_2 = 0.0754 \text{ lb}_m/\text{ft}^3}$.

$$\dot{m} = \rho_2 V_2 A_2 \Rightarrow A_2 = \frac{\dot{m}}{\rho_2 V_2} \tag{10}$$

Using the given data, $\boxed{A_2 = 0.53 \text{ ft}^2}$.

To determine the force the fluid exerts on the device, apply the linear momentum equation in the x-direction to the control volume shown in the figure.

$$\frac{d}{dt}\int_{\text{CV}} u_x \rho dV + \int_{\text{CS}} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = F_{B,x} + F_{S,x} \tag{11}$$

where

$$\frac{d}{dt}\int_{\text{CV}} u_x \rho dV = 0 \quad \text{(steady flow)} \tag{12}$$

$$\int_{\text{CS}} u_x \left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = -\dot{m}V_1 + \dot{m}V_2 = \dot{m}\left(V_2 - V_1\right) \tag{13}$$

$$F_{B,x} = 0 \tag{14}$$

$$F_{S,x} = F + p_1 A_1 - p_2 A_2 \tag{15}$$

Substitute and simplify.

$$\dot{m}\left(V_2 - V_1\right) = F + p_1 A_1 - p_2 A_2$$

$$\boxed{F = \dot{m}\left(V_2 - V_1\right) - p_1 A_1 + p_2 A} \tag{16}$$

Substitute the given values to find F = -7950 lb_f. Note that this is the force that the device exerts on the fluid. Hence, the $\boxed{\text{force the fluid exerts on the device is 7950 lb}_f \text{ acting in the } +x\text{-direction}}$.