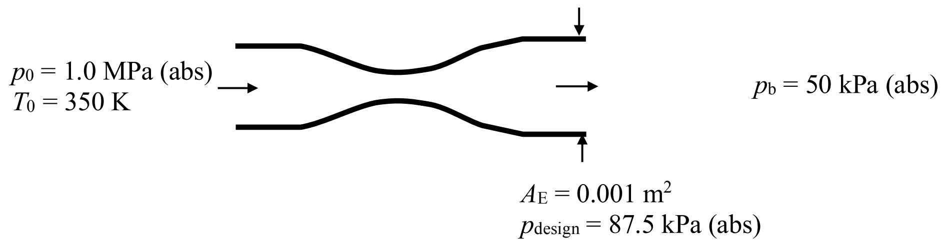
Air flows isentropically in a converging-diverging nozzle, with exit area of 0.001 m^2. The nozzle is fed from a large plenum where the stagnation conditions are 350 K and 1.0 MPa (abs). The nozzle has a design back pressure of 87.5 kPa (abs) but is operating at a back pressure of 50.0 kPa (abs). Assuming the flow within the nozzle is isentropic, determine:

a. the exit Mach number, and
b. the mass flow rate through the nozzle.

SOLUTION:



The exit Mach number may be found using the isentropic pressure ratio at the exit. Since the back pressure is less than the design pressure (underexpanded conditions), the exit pressure will be equal to the design pressure.

$$\frac{p_E}{p_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}_E^2\right)^{\frac{k}{1-k}} \tag{1}$$

Using p_E = 87.5 kPa, p_0 = 1.0 MPa, and k = 1.4, the exit Mach number is: $\boxed{\mathrm{Ma}_E = 2.24}$.

The mass flow rate through the nozzle may be found using the exit conditions. First, determine the temperature at the exit using the adiabatic stagnation temperature ratio:

$$\frac{T_E}{T_0} = \left(1 + \frac{k-1}{2}\mathrm{Ma}_E^2\right)^{-1} \tag{2}$$

Using T_0 = 350 K, k = 1.4, and Ma_E = 2.24, the exit temperature is: $\underline{T_E = 174.5\ \mathrm{K}}$.

The speed of sound at the exit is:

$$c_E = \sqrt{kRT_E} \tag{3}$$

Using k = 1.4, R = 287 J/(kg·K), and T_E = 174.5 K, the speed of sound at the exit is: $\underline{c_E = 264.8\ \mathrm{m/s}}$.

The velocity of the air at the exit is:

$$V_E = c_E \mathrm{Ma}_E \tag{4}$$

Using c_E = 264.8 m/s and Ma_E = 2.24: $\underline{V_E = 593.8\ \mathrm{m/s}}$.

The density at the exit may be found using the ideal gas law:

$$\rho_E = \frac{p_E}{RT_E} \tag{5}$$

With p_E = 87.5 kPa, R = 287 J/(kg·K), and T_E = 174.5 K: $\underline{\rho_E = 1.747\ \mathrm{kg/m^3}}$.

The mass flow rate through the nozzle is:

$$\dot{m} = \rho_E V_E A_E \tag{6}$$

Using the previous data, $\boxed{\dot{m} = 1.04\ \mathrm{kg/s}}$.