The velocity field for a steady, incompressible, two-dimensional flow through a converging duct is approximated as,

$$\mathbf{u} = \left(U_0 + bx\right)\hat{\mathbf{i}} - by\hat{\mathbf{j}},$$

where U_0 is the horizontal speed at $x = 0$ and b is a constant.

a. Derive an equation for the streamline of this flow passing through various points at the duct inlet, i.e., ($x_0 = 0$, y_0).
b. Plot the streamlines found in part (a) for $U_0 = 1$ m/s and $b = 1$ s^{-1} over the range $x = [0, 3\text{ m}]$ and $y_0 = -2$ m, -1 m, 0, 1 m, and 2 m.

SOLUTION:
The slope of the streamline is given by,

$$\frac{dy}{dx} = \frac{u_y}{u_x} = \frac{-by}{U_0 + bx}. \tag{1}$$

Solving this differential equation gives,

$$\frac{dy}{dx} = \frac{-by}{U_0 + bx} \Rightarrow \int_{y=y_0}^{y=y} \frac{dy}{y} = -b \int_{x=x_0}^{x=x} \frac{dx}{U_0 + bx} \Rightarrow \ln\left(\frac{y}{y_0}\right) = -\ln\left(\frac{U_0 + bx}{U_0 + bx_0}\right) \Rightarrow y = y_0\left(\frac{U_0 + bx_0}{U_0 + bx}\right), \tag{2}$$

$$\boxed{y = y_0\left(\frac{U_0}{U_0 + bx}\right)} \text{ using } (x_0 = 0,\ y_0). \tag{3}$$

These streamlines are plotted below for the given values of y_0 and $U_0 = 1$ m/s and $b = 1$ s^{-1} over the given range of x. Note that the flow is from left to right in the figure since $u_x > 0$ for the given range of values.