Air flows downward toward an infinitely wide horizontal flat plate. The velocity field is given by,

$$\mathbf{u} = \left(ax\hat{\mathbf{i}} - ay\hat{\mathbf{j}}\right)\left[2 + \cos(\omega t)\right],$$

where a and ω are constants.

a. Obtain an algebraic equation for a streamline at $t = 0$.
b. Plot the streamline that passes through point $(x, y) = (3, 3)$ at this instant.
c. Will the streamline change with time? Explain your answer.

SOLUTION:
The slope of the streamline is given by,

$$\frac{dy}{dx}=\frac{u_y}{u_x}=\frac{-ay\left[2+\cos(\omega t)\right]}{ax\left[2+\cos(\omega t)\right]}=-\frac{y}{x}. \qquad (1)$$

Solving this differential equation gives,

$$\frac{dy}{dx}=-\frac{y}{x}\Rightarrow\int_{y=y_0}^{y=y}\frac{dy}{y}=-\int_{x=x_0}^{x=x}\frac{dx}{x}\Rightarrow\ln\left(\frac{y}{y_0}\right)=\ln\left(\frac{x_0}{x}\right), \qquad (2)$$

$$\boxed{\frac{y}{y_0}=\frac{x_0}{x}} \text{ or } y=\frac{x_0 y_0}{x}. \qquad (3)$$

Through the point $(x_0, y_0) = (3, 3)$, Eq. (3) becomes,

$y=\frac{9}{x}$. The plot is shown below. (4)

The <u>streamlines won't change with time</u> since the time components divide out in Eq. (1).