Consider the 2D flow field defined by the following velocity:

$$\mathbf{u} = \left(\frac{1}{1+t}\right)\hat{\mathbf{i}} + \hat{\mathbf{j}}$$

- For this flow field, find the equation of: a. the streamline through the point (1,1) at t = 0, b. the pathline for a particle released at the point (1,1) at t = 0, and c. the streakline at t = 0 which passes through the point (1,1).

SOLUTION:

The slope of a streamline is tangent to the velocity vector.

$$\frac{dy}{dx} = \frac{u_y}{u_x} \tag{1}$$

where

$$u_x = \frac{1}{1+t} \tag{2}$$

$$u_y = 1 \tag{3}$$

Substitute Eqns. (2) and (3) into Eqn. (1) and solve the resulting differential equation.

$$\frac{dy}{dx} = \frac{1}{\frac{1}{1+t}} = 1+t$$

$$\int_{y_0}^{y} dy = (1+t) \int_{x_0}^{x} dx \quad \text{(where } (x_0, y_0) \text{ is a point passing through the streamline)}$$

$$y - y_0 = (1+t)(x - x_0) \tag{4}$$

For the streamline passing through the point $(x_0, y_0) = (1, 1)$ at time t = 0:

$$y = x \tag{5}$$

A streakline is a line that connects all of the fluid particles that pass through the same point in space. The equation for the streakline can be found parametrically using Eqns. (2) and (3).

$$u_x = \frac{dx}{dt} = \frac{1}{1+t} \tag{6}$$

$$u_y = \frac{dy}{dt} = 1 \tag{7}$$

Solve the previous two differential equations.

$$\int_{x_0}^x dx = \int_{t_0}^t \frac{dt}{1+t} \qquad \Rightarrow \qquad x - x_0 = \ln\left(\frac{1+t}{1+t_0}\right)$$
(8)

$$\int_{y_0}^{y} dy = \int_{t_0}^{t} dt \qquad \Rightarrow \qquad y - y_0 = t - t_0$$
(9)

where t_0 is the time at which a fluid particle passes through the point (x_0 , y_0) on the streakline. Hence, the streakline passing through the point (x_0 , y_0) = (1, 1) at time t = 0 is given parametrically (in t_0) as:

$$x - 1 = \ln\left(\frac{1}{1 + t_0}\right) \qquad \Longrightarrow \qquad x = \ln\left(\frac{1}{1 + t_0}\right) + 1 \tag{10}$$

$$y-1 = -t_0 \implies y = 1-t_0$$
(11)

Recall that t_0 is the time when a fluid particle passes through the point (x_0 , y_0).

A pathline is a line traced out by a particular fluid particle as it moves through space. The equation for the pathline can be found parametrically using Eqns. (2) and (3).

$$u_x = \frac{dx}{dt} = \frac{1}{1+t} \tag{12}$$

$$u_y = \frac{dy}{dt} = 1 \tag{13}$$

Solve the previous two differential equations.

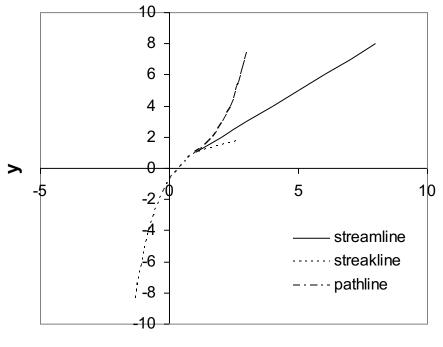
$$\int_{x_0}^{x} dx = \int_{t_0}^{t} \frac{dt}{1+t} \qquad \Rightarrow \qquad x - x_0 = \ln\left(\frac{1+t}{1+t_0}\right)$$

$$\int_{y_0}^{y} dy = \int_{t_0}^{t} dt \qquad \Rightarrow \qquad y - y_0 = t - t_0$$
(14)
(15)

where t_0 is the time at which a fluid particle passes through the point (x_0, y_0) on the pathline. Hence, the pathline for a particle passing through the point $(x_0, y_0) = (1, 1)$ at time $t_0 = 0$ is given parametrically (in *t*) as:

$$\begin{array}{ccc} x-1 = \ln(1+t) & \Rightarrow & x = \ln(1+t)+1 \\ y-1 = t & \Rightarrow & y = 1+t \end{array} \tag{16}$$

Note that the streamline, streakline, and pathline are all different. A plot of these lines through (1, 1) at t = 0 is shown below.



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