A velocity field is given by:

$$\mathbf{u} = \frac{-V_0 y}{(x^2+y^2)^{1/2}}\hat{\mathbf{i}} + \frac{V_0 x}{(x^2+y^2)^{1/2}}\hat{\mathbf{j}}$$

where V_0 is a positive constant, *i.e.* $V_0 > 0$. Determine:

a. where in the flow the speed is V_0
b. the equation and sketch of the streamlines
c. the equations for the streaklines and pathlines

SOLUTION:

The speed is given by:

$$|\mathbf{u}| = \sqrt{u_x^2 + u_y^2} \tag{1}$$

where

$$u_x = \frac{-V_0 y}{(x^2 + y^2)^{1/2}} \tag{2}$$

$$u_y = \frac{V_0 x}{(x^2 + y^2)^{1/2}} \tag{3}$$

Substituting into Eqn. (1) gives:

$$|\mathbf{u}| = \sqrt{\frac{V_0^2 y^2}{(x^2 + y^2)} + \frac{V_0^2 x^2}{(x^2 + y^2)}}$$

$$\boxed{\therefore |\mathbf{u}| = V_0} \tag{4}$$

The flow speed is everywhere equal to V_0.

The slope of the streamline is tangent to the slope of the velocity vector:

$$\frac{dy}{dx} = \frac{u_y}{u_x} \tag{5}$$

Substitute Eqns. (2) and (3) and solving the resulting differential equation.

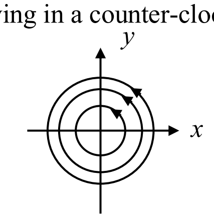
$$\frac{dy}{dx} = \frac{\dfrac{V_0 x}{(x^2 + y^2)^{1/2}}}{\dfrac{-V_0 y}{(x^2 + y^2)^{1/2}}} = \frac{x}{-y}$$

$$-\int_{y_0}^{y} y\,dy = \int_{x_0}^{x} x\,dx$$ (where (x_0, y_0) is a point located on the streamline)

$$-\tfrac{1}{2}\left(y^2 - y_0^2\right) = \tfrac{1}{2}\left(x^2 - x_0^2\right)$$

$$\boxed{x^2 + y^2 = x_0^2 + y_0^2 = \text{constant}} \tag{6}$$

The streamlines are circles! Note that when $x > 0$ and $y > 0$, Eqns. (2) and (3) indicate that $u_x < 0$ and $u_y > 0$ (note that $V_0 > 0$) so that the flow is moving in a counter-clockwise direction.



Since the flow is steady, the streaklines and pathlines will be identical to the streamlines.